Nonequilibrium Statistical Mechanics - Video course

COURSE OUTLINE

Thermal fluctuations, Langevin dynamics, Brownian motion and diffusion, Fokker-Planck equations, linear response theory, fluctuation-dissipation relations, the Boltzmann equation, critical phenomena, scaling and critical exponents.

COURSE DETAIL

S.No	Lecture Name
1	**Lecture 1: Recapitulation of equilibrium statistical mechanics** • Isolated system in thermal equilibrium • Fundamental postulate of equilibrium statistical mechanics • Microcanonical ensemble • Boltzmann's formula for the entropy • Connection with thermodynamics • Closed systems and the canonical ensemble • Canonical partition function
2	**Lecture 2: The Langevin model (Part 1)** • Brownian particle in a uid • Langevin model • Equation of motion including thermal noise • Conditional and thermal averages • The need to include a dissipative random force
3	**Lecture 3: The Langevin model (Part 2)** • Mean squared velocity • Relation between noise strength and friction: fluctuation-dissipation (FD) theorem • Velocity autocorrelation function • Stationarity of the velocity process • Mean squared displacement and diffusion constant
4	**Lecture 4: The Langevin model (Part 3)** • Variance of the displacement for a free Brownian particle • Conditional PDF of the velocity: Ornstein-Uhlenbeck (OU) distribution • Langevin equation (LE) for a Brownian particle in a magnetic field • Velocity autocorrelation tensor in a magnetic field • Explicit solution for the correlation tensor
5	**Lecture 5: The Langevin model (Part 4)** • Velocity correlation tensor for $t \geq 0$ and $t \leq 0$ • Symmetric and antisymmetric parts of the correlation tensor • Diffusion tensor in a magnetic field: longitudinal and transverse parts • Conditional PDF of the velocity in a magnetic field: modified OU distribution • Linear response theory: introductory remarks
6	**Lecture 6: Linear response theory (Part 1)**

NP-TEL

NPTEL

http://nptel.ac.in

Physics

Pre-requisites:

Basics of equilibrium statistical mechanics

Coordinators:

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	• Classical and quantum equations of motion in Hamiltonian dynamics • Liouville operator and its hermiticity • Unitarity of the time evolution operator • Density matrix; pure and mixed states • Liouville and von Neumann equations for the density operator • Expectation value of a physical observable	
7	**Lecture 7: Linear response theory (Part 2)** • Equilibrium density matrix in the canonical ensemble • Time dependent perturbation of a Hamiltonian system • First-order correction to the density operator • First-order correction to the mean value of an observable • Linear, causal, retarded response • Definition of the response function	
8	**Lecture 8: Linear response (Part 3)** • Equivalent expressions for the response function • Response to a sinusoidal force and generalized susceptibility • Symmetry properties of the frequency-dependent susceptibility • Double-time retarded Green function • Spectral function and its relation to the generalized susceptibility	
9	**Lecture 9: Linear response(Part 4)** • Susceptibility for an oscillator in a fluid • Poles of the oscillator susceptibility in the complex frequency plane • Simplification of the general expression for the response function • Simplified expression in the classical case • Kubo canonical correlation in the quantum mechanical case	
10	**Lecture 10: Linear response (Part 5)** • Canonical correlation functions • Response function as a canonical correlation • Properties of canonical correlations: stationarity, symmetry and reality • Physical implication of reality property • Analyticity of the susceptibility in the upper half frequency plane	
11	**Lecture 11: Linear response (Part 6)** • Dispersion relations for the real and imaginary parts of the susceptibility • Asymptotic behavior of the susceptibility and subtracted dispersion relations • Case of a singular DC susceptibility • Response function in terms of matrix elements of observables • Susceptibility in terms of transition frequencies	
12	**Lecture 12: Linear response theory (Part 7)** • Spectral function in terms of the transition frequencies of a system • Master analytic function from the spectral function • Boundary values of the master function: Retarded and advanced susceptibilities • Fourier representation of two-time correlation functions • Fourier representation of two-time anticommutator	
13	**Lecture 13: Quiz 1 - Questions and answers**	
14	**Lecture 14: Linear response theory (Part 8)** • Symmetry or antisymmetry of the response function under time-reversal • Spectral function as the real or imaginary part of the susceptibility • Equilibrium averages of equal-time commutators and moments of the spectral function • High-frequency expansion of the susceptibility	
15	**Lecture 15: Linear response theory (Part 9)**	

	- Derivation of the response in the Heisenberg picture - Differential and integral equations for the time-development operator - Solution to first order in the perturbation - Expression for the response function - General relation between power spectra of the response and fluctuations
16	**Lecture 16: The dynamic mobility** - Definition of the mobility of a Brownian particle - Zero-frequency mobility and diffusion constant - Dynamic mobility as a generalized susceptibility - Consistency of the Langevin model with linear response theory - Non-diffusive behaviour of a Brownian oscillator
17	**Lecture 17: Fokker-Planck equations (Part 1)** - Langevin equation (LE) for a general diffusion process - Corresponding Fokker-Planck equation (FPE) for the conditional PDF - Case of linear drift and constant diffusion coefficients - Examples: FPE for the velocity PDF, diffusion equation for the positional PDF - FPE for the phase space PDF of a Brownian particle - Generalization to three dimensions
18	**Lecture 18: Fokker-Planck equations (Part 2)** - FPE for general (nonlinear) drift and diffusion coefficients in the multi-dimensional case - Kramers' equation for phase space PDF in an applied potential - Asymptotic form of the phase space PDF - Diffusion regime (or high-friction limit): Smoluchowski equation for the positional PDF - Overdamped oscillator: OU distribution for the positional PDF
19	**Lecture 19: Fokker-Planck equations (Part 3)** - Stationary solution of the Smoluchowski equation - Thermally-assisted escape over a potential barrier - Kramers' escape rate formula - Diffusion in a constant force field: sedimentation
20	**Lecture 20: The generalized Langevin equation (Part 1)** - Inconsistency in the Langevin model: non-stationarity of the velocity - Divergence of mean squared acceleration - Generalized Langevin equation and memory kernel - Frequency-dependent friction - Dynamic mobility in the generalized model
21	**Lecture 21: The generalized Langevin equation (Part 2)** - Kubo-Green formula for the mobility: first FD theorem - Consistency of the model with stationarity and causality - Cross-correlation between the noise and the velocity - Relation between noise autocorrelation and memory kernel: second FD theorem
22	**Lecture 22: Diffusion in a magnetic field** - Langevin equations for position and velocity with a velocity-dependent force - Smoluchowski equation for positional PDF - Identification and calculation of the diffusion tensor - FPE for the radial distance PDF in Brownian motion - Corresponding LE with a drift term for the radial distance
23	**Lecture 23: The Boltzmann equation for a dilute gas (Part 1)** Single-particle phase space

	• Equation for number density in the absence of collisions • Binary collisions and two-particle scattering • The collision integral • The Boltzmann equation
24	**Lecture 24: The Boltzmann equation for a dilute gas (Part 2)** • The equilibrium distribution: sufficiency condition • Boltzmann's *H*-Theorem • The equilibrium distribution: necessary condition • The Maxwell-Boltzmann distribution • Equilibrium distribution in a potential
25	**Lecture 25: The Boltzmann equation for a dilute gas (Part 3)** • Remarks on the *H*-Theorem • Detailed balance and equilibrium distribution • Collision invariants and equations of continuity • Linearization of the Boltzmann equation close to equilibrium
26	**Lecture 26: The Boltzmann equation for a dilute gas (Part 4)** • Single relaxation time approximation to the collision integral • Relaxation of the velocity • Equivalence to a Kubo-Anderson Markov process • Relaxation of a non-uniform distribution in the position variable
27	**Lecture 27: The Boltzmann equation for a dilute gas (Part 5)** • Relaxation of a non-uniform gas • Frequency-dependent diffusion coefficient • The diffusion constant • Shift of the equilibrium velocity distribution under a uniform force
28	**Lecture 28: Quiz 2 - Questions and answers**
29	**Lecture 29: Critical phenomena (Part 1)** • Recapitulation of thermodynamics • Intensive and extensive variables • Phase diagram for a single component substance • Liquid-gas coexistence line and the critical point
30	**Lecture 30: Critical phenomena (Part 2)** • Extensivity of thermodynamic potentials • Some convexity properties of thermodynamic potentials • Divergence of specific heat at the critical point • Simplest magnetic equation of state • Fluid-magnet analogy
31	**Lecture 31: Critical phenomena (Part 3)** • Fluid-magnet analogy (contd.): phase diagrams • Ising model with nearest-neighbour interaction • Mean field theory (MFT) for the Ising model • Critical temperature in MFT • Critical exponents in MFT
32	**Lecture 32: Critical phenomena (Part 4)** • Definition of specific heat, order parameter, susceptibility and critical isotherm exponents • Difference between actual and MFT values of critical exponents • Static susceptibility formula • Correlation length • Critical exponent for the divergence of the correlation length

33	**Lecture 33: Critical phenomena (Part 5)** • Equation of state in the Ising model • Magnetization versus magnetic field for different temperatures • Landau expansion for the free energy • Criterion for the validity of MFT • Upper critical dimensionality in the Ising universality class
34	**Lecture 34: Critical phenomena (Part 6)** • Scaling functions • Relations between critical exponents • Landau free energy functional • Equilibrium configuration of the order parameter • Relaxation to equilibrium configuration
35	**Lecture 35: Critical phenomena (Part 7)** • Time-dependent Landau-Ginzburg equation • Langevin equation for the order parameter • Fokker-Planck equation for configuration probability • Linearized LE and relaxation to equilibrium • Critical slowing down • Dynamic scaling hypothesis
36	**Lecture 36: The Wiener process (standard Brownian motion)** • The Wiener process (standard Brownian motion) • Sample path properties • Iterated logarithm law and arcsine law • Functionals of the Wiener process • Itô calculus: basic rules • The Feynman-Kac formula and generalizations

References:

1. **V. Balakrishnan, Elements of Nonequilibrium Statistical Mechanics, Ane Books, Delhi & CRC Press, 2008.(Chapters 1-4, 6, 9, 11-13, 15-17.)**
2. **N. Goldenfeld, Lectures on Phase Transitions and the Renormalization Group, Levant Books, Kolkata, India, 2005. (Chapters 1, 5, 8.)**
3. **K. Huang, Statistical Mechanics, 2nd edition, Wiley, New York, 1987. (Chapters 3, 4, 16, 17.)**
4. **M. Kardar, Statistical Physics of Fields, Cambridge University Press, Cambridge, 2007. (Chapters 3, 4.)**
5. **R. Kubo, M. Toda and N. Hashitsume, Statistical Physics II: Nonequilibrium Statistical Mechanics, Springer-Verlag, Berlin, 1985. (Chapters 1, 2, 4.)**
6. **L. D. Landau and E. M. Lifshitz, Statistical Physics, Part 1, 3rd edition, Pergamon, New York, 1980. (Chapter 12.)**
7. **G. F. Mazenko, Nonequilibrium Statistical Mechanics, Wiley-VCH, Weinheim, 2006. (Chapters 1, 2, 7, 8.)**
8. **H. Risken, The Fokker-Planck Equation, Springer-Verlag, New York, 1996. (Chapters 2-4, 6.)**
9. **H. E. Stanley, Introduction to Phase Transitions and Critical Phenomena, Oxford University Press, Oxford, 1989. (Chapters 1, 3, 5, 6, 10-12.)**