Random Matrix Theory and Applications - Web course

COURSE OUTLINE

1. General considerations.
2. Maximum entropy approach of complex systems.
3. Random matrix ensembles: general introduction.
4. Correlations and fluctuation measures.
5. System dependent random matrix ensembles.
6. Application of random matrices to quantum systems.
7. Applications of random matrices to classical systems.

COURSE DETAIL

A Web course shall contain 40 or more 1 hour lecture equivalents.

Sl. No.	Topic/s	No. of Hrs.
1.	**General considerations:** ▪ Complexity in physical systems: various forms. ▪ Statistical behavior of physical properties. ▪ Distribution of eigen-values and eigen-functions of various operators; connection to physical properties. ▪ Observed universality of physical properties: need of random matrix models. ▪ Deviation from standard random matrix models: need of system dependent models.	3
2.	**Maximum entropy approach of complex systems: an information theoretic viewpoint:** • Probability and information entropy: the role of the relevant physical parameters as constraints. • The role of symmetries in motivating a natural probability measure e.g Gaussian or uniform etc. • The maximum entropy criterion in the context of statistical inferences.	3

NP-TEL

NPTEL

http://nptel.iitm.ac.in

Physics

Pre-requisites:

Necessary: Matrix Algebra, Differential Equations, Statistical Physics

Also preferred: Basic Quantum Mechanics

Additional Reading:

Various papers on applications of random matrices appeared in various archives at:

http://xxx.lanl.gov

Hyperlinks:

1. RMTool A MATLAB based Random Matrix Calculator

   http://www.eecs.umich.edu/~rajnrao/rmtool/

2. RMTool A MATLAB based Random Matrix Calculator

   http://mathworld.wolfram.com/RandomMatrix.html

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3.	**Random matrix ensembles: General:** • Nature of ensemble: Role of symmetry, interactions and other system conditions. • Basis invariance vs Basis dependence of the ensemble and their transformation properties. • Stationary vs non-stationary ensembles. • Conservative systems and Gaussian ensembles of Hermitian matrices: ten standard types. • Non-conservative systems and ensembles of non-Hermitian matrices: Ginibre ensembles. • Time-periodic systems and circular ensembles of unitary matrices. • Laguerre ensembles. • Multi-cut ensembles.	12	
4.	**Correlations and fluctuation measures:** • Fluctuation measures of eigenvalues e.g. number variance, spacing distribution, spectral rigidity, gap probabilities etc. • Fluctuations measures of eigenfunctions e.g local intensity distribution, inverse participation ratio, local density of states etc. • Level density and level repulsion: role of global symmetries. • Eigenfunction localization: role of interactions and disorder. • Behavior at the edge of the spectrum. • Critical level statistics and multifractality of eiegnfunctions. • Universality of fluctuations measures.	7	
5.	**System dependent random matrix ensembles:** • Varying system conditions and transition between stationary ensembles. • Common mathematical formulation of fluctuation measures for multi-parametric Gaussian ensembles. • Connection to one and two dimensional Calogero-Sutherland Hamiltonian of interacting particles. • Phase transition and critical ensembles.	6	

	• Correlated random matrix ensembles.	
6.	**Application of random matrices to quantum systems:** • Random matrix theory of quantum transport. • Random matrix theory of quantum chaotic systems. • Disordered systems. • Quantum gravity. • Nuclear resonances, Atoms, molecules etc..	4
7.	**Application of random matrices to classical systems:** • Financial systems e.g stock market fluctuations. • Biological systems e.g signals received by brain. • Atmospheric correlations. • Complex networks e.g traffic systems. • Light propagation through random media. • Elastomechanics. • Number theoretic systems e.g. Reimann-zeta function.	5

References:

1. Random matrices, by M.L. Mehta, 3rd edition. Pure and Applied Mathematics (Amsterdam), 142. Elsevier/Academic Press, Amsterdam, 2004.
2. Random Matrix Theories in Quantum Physics: Common Concepts, by T. Guhr, A. Mueller-Groeling, H. A. Weidenmueller, Phys.Rept.299:189-425,1998.
3. Random-matrix physics: spectrum and strength fluctuations, by T.A.Brody, J. Flores, J.B.French, P.A.Mello, A.Pandey, S.S.M.Wong, Rev. Mod. Phys. 53, 385 - 479 (1981).
4. Applications of Random Matrices in Physics, E. Brezin, V. Kazakov, D. Serban, P. Wiegmann, A. Zabrodin, Kluwer Academic Publications (2006).
5. Quantum Signatures of Chaos, by F.Haake, 2nd edition., Springer Verlag TELOS, (2007).