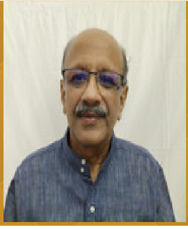

MEASURE AND INTEGRATION

PROF S. KESAVAN
Department of mathematics
The Institute of Mathematical Sciences

PRE-REQUISITES : MSc Real Analysis, Topology, Linear Algebra

INTENDED AUDIENCE : MSc (Mathematics) and above

COURSE OUTLINE :

The theory of measure and integration is now an integral part of any masters or graduate program in mathemat ics in Indian Universities. It is an important prerequisite for most analysis based courses. In this course, we will start with a study of abstract measures with the Lebesgue measure being the most important example. After looking at measurable functions and some types of convergences for such functions, we will introduce the notion of the Lebesgue integral in an abstract measure space and study its properties. We will relate the two important processes of the calculus – differentiation and integration – in the context of the Lebesgue integral. We will also study measures and integrals on product spaces and also signed measures. Finally we will study the important properties of L^p – spaces.

ABOUT INSTRUCTOR :

Prof. S. Kesavan retired as Professor from the Institute of Mathematical Sciences, Chennai. He obtained his doctoral degree from the Universite de Pierre et Marie Curie (Paris VI), France. His research interests are in Partial Differential Equations. He is the author of five books. He is a Fellow of the Indian Academy of Sciences and the National Academy of Sciences, India. He has served as Deputy Director of the Chennai Mathematical Institute (2007-2010) and two terms (2011-14, 2015-18) as Secretary (Grants Selection) of the Commission for Developing Countries of the International Mathematical Union. He was a member of the National Board for Higher Mathematics during 2000-2019.

COURSE PLAN :

Week 1: Motivation, abstract measures, Caratheodory’s method of extension, completion of a measure.
Week 2: Construction of the Lebesgue meaure, approximation properties.
Week 3: Translation invariance, nonmeasurable sets, measurable functions.
Week 4: Properties of measurable functions, Cantor function.
Week 5: Convergence, Egorov’s theorem, convergence in measure.
Week 6: Lebesgue integration, convergence theorems.
Week 7: Comparison with the Riemann integral, some applications (Weierstrass’ theorem).
Week 8: Differentiation: Monotone functions, functions of bounded variation, absolute continuity.
Week 9: Product spaces, Fubini’s theorem.
Week 10: Signed measures, Radon-Nikodym theorem.
Week 11: L^p-Spaces: Basic properties, approximation, applications.
Week 12: Duality, convolutions.