



# OPERATOR THEORY

## PROF. G. RAMESH

Department of Mathematics  
IIT Hyderabad

**PRE-REQUISITES :** Functional Analysis

**INTENDED AUDIENCE :** M. Sc Mathematics II Year students and Ph. D first year students

### COURSE OUTLINE :

In this course, we aim to study the spectral theory of normal operators and continuous functional calculus. We begin with the introduction of Hilbert space and study bounded operators on these spaces. More often we compare the results on operators with operators on finite-dimensional Hilbert spaces (or matrices). In this way, we study the spectrum and its properties, spectral theorem for compact normal operators which is a generalization of finite-dimensional operators. The further generalization is the spectral theorem for normal operators.

### ABOUT INSTRUCTOR :

Prof. G. Ramesh completed his Ph. D (Operator Theory) from IIT Madras in 2008 and did Post Doctoral studies at ISI Bangalore, Bengaluru during 2009-2010 and worked as an Assistant Professor at Hyderabad University till June 2011. In 2011 July, he joined as an assistant professor in IUT Hyderabad. Currently, he is an Associate Professor at IIT Hyderabad. His area of specialization is Functional Analysis and Operator Theory

### COURSE PLAN :

**Week 1:** Review of Hilbert space Theory , Bounded operators on Hilbert spaces, examples

**Week 2:** Adjoint an operator, examples, Self-adjoint, normal, positive, unitary, isometries, partial isometries

**Week 3:** Orthogonal projections with examples, invariant subspaces, numerical range and characterization of operators

**Week 4:** Banach Algebras, invertibility, spectrum

**Week 5:** Gelfand-Mazur theorem, spectral radius formula, spectral mapping theorem.

**Week 6:** Subdivision of the spectrum of an operator, properties of the various spectra

**Week 7:** Computing spectrum with examples

**Week 8:** Existence of square root, polar-decomposition.

**Week 9:** Compact operators, properties

**Week 10:** Spectral theorem for compact self-adjoint operators, spectral theorem for compact normal operators, Schmidt-decomposition, Monotone convergence theorem for self-adjoint operators

**Week 11:** Spectral theorem for self-adjoint operators, continuous functional calculus, spectral theorem for self-adjoint operators(multiplication form)

**Week 12:** Spectral theorem for normal operators (both integral and multiplication form), continuous functional calculus for normal operators.