SOBOLEV SPACES AND PARTIAL DIFFERENTIAL EQUATIONS

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PRE-REQUISITES : MSc; Real Analysis, Topology, Linear Algebra; Measure Theory, Functional Analysis

INTENDED AUDIENCE : MSc (Mathematics) and above

COURSE OUTLINE :

The modern theory of Partial Differential Equations relies heavily on functional analytic methods. With the advent of high speed computers, numerical methods, like the finite element method, have revolutionized areas like computational fluid dynamics and structural analysis. These rely on the study of weak solutions to PDEs and functional analysis plays a dominant role in this. At the core of the ideas involved lie the theory of distributions and the important function spaces, called the Sobolev spaces. These spaces form a natural framework in which we study generalized (i.e. weak) solutions of boundary value problems. This course will develop, in detail, the theory of distributions and study the important properties of Sobolev spaces. These will be applied to the study of weak solutions of elliptic boundary value problems. The theory of semigroups of operators on a Banach space will also be developped. This, together with Sobolev spaces will help us to study evolution equations. Prerequisites for this course are analysis, topology, linear algebra, functional analysis and measure theory (especially, the theory of L-p spaces).

ABOUT INSTRUCTOR :

Prof. S. Kesavan retired as Professor from the Institute of Mathematical Sciences, Chennai. He obtained his doctoral degree from the Universite de Pierre et Marie Curie (Paris VI), France. His research interests are in Partial Differential Equations. He is the author of five books. He is a Fellow of the Indian Academy of Sciences and the National Academy of Sciences, India. He has served as Deputy Director of the Chennai Mathematical Institute (2007-2010) and two terms (2011-14, 2015-18) as Secretary (Grants Selection) of the Commission for Developing Countries of the International Mathematical Union. He was a member of the National Board for Higher Mathematics during 2000-2019.

COURSE PLAN :

Week 1: Test functions Distributios, calculus of distributions, support and singular support of distributions.
Week 2: Convolutions of functions, convolution of distributions, Fundamental solutions.
Week 3: Fourier transform, Fourier inversion, tempered distributions.
Week 4: Sobolev spaces, definition, approximation by smooth functions.
Week 5: Extension theorems, Poincare inequality, Imbedding theorems.
Week 6: Compactness theorems, trace theory.
Week 7: Variational problems in Hilbert spaces and Lax-Milgram lemma. Examples of weak formulations of elliptic boundary value problems.
Week 8: Regularity, Galerkin's method, Maximum principles.
Week 9: Eigenvalue problems, introduction to the finite element method.
Week 10: Semigroups of operators. Examples, basic properties, Hille-Yosida theorem.
Week 11: Maximl dissipative operators, regularity
Week 12: Heat equation, wave equation, Schrodinger equation. Inhomogeneous equations.