VARIATIONAL CALCULUS AND ITS APPLICATIONS IN CONTROL THEORY AND NANOMECHANICS

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TYPE OF COURSE : Rerun | Elective | UG/PG
COURSE DURATION : 12 weeks (24 Jan' 22 - 15 Apr' 22)
EXAM DATE : 24 Apr 2022

PRE-REQUISITES : 1) Multivariable Calculus 2) Ordinary Differential Equations (optional)

INDUSTRIES APPLICABLE TO : Industries in areas of (1) Control System and Instrumentation Engineering, (2) Nanoscience, (3) Software Development

COURSE OUTLINE :
This course is designed as an introduction to the theory and applications of Variational Calculus to problems in geometry, differential equations and physics, particularly mechanics. This course assumes very limited knowledge of vector calculus, ordinary differential equations and basic mechanics. Many new applications in applied mathematics, physics, chemistry, biology and engineering are included. This course will serve as a reference for advanced study and research in this subject as well as for its applications in the fields of industrial control systems and instrumentation engineering, nanoscience and software development.

ABOUT INSTRUCTOR :
Prof. Sarthok Sircar is currently an Assistant Professor in the Department of Mathematics at Indraprastha Institute for Information Technology, Delhi. His prior academic appointments include (a) Lectureship in the Division of Mathematical Sciences, University of Adelaide, Australia, (b) Research Associate in Division of Applied Mathematics, University of Colorado, Boulder, (c) Research Fellow in Biomathematics in the University of Utah, (d) Visiting Scholar in the Center for Nanophase Material Science at Oak Ridge National Laboratory, and (e) Research Scientist in Corning Inc. at Ithaca, NY. His main mathematical interests are in the development and analysis of nonlinear hyperbolic and elliptic partial differential equations, with applications which lie at the interface of applied mathematics and biology. He is particularly interested in solving problems involving soft matter and fluid flow using asymptotic and perturbation methods, numerical approximation and statistical mechanics.

COURSE PLAN :

Week 1: Introduction: Problems involving Calculus of Variations: Gold-diggers Problem, Catenary, Brachystochrone, Dido's problem, Geodesics, minimal surface, optimal harvest, Revision: Extremals in Finite Dim Calculus (Functions of one and several variables), Euler Lagrange equation (E-L eqns)

Week 2: Special cases E-L eqns: (1) Functions depending on y', (2) Functions with no explicit 'x' dependence, (3) Functions with no explicit 'y' dependence, (4) degenerate functions. Invariance of E-L eqns, existence, uniqueness of solutions, Generalization : (1) Functionals containing higher derivatives

Week 3: Generalization: (2) Functionals containing several dependent variables, (3) Functionals containing two independent variables. Numerical solution: (1) Euler's FD Method, (2) Ritz Method, (3) Kantorovich's Method

Week 4: Isoperimetric Problem: Finite dim case/ Lagrange Multipliers including (a) single constraint, (b) multiple constraints, (c) Abnormal problems. Isoperimetric Problems involving functional including cases of generalization in higher dimension, multiple isoperimetric constraints, several dependent variables

Week 5: Holonomic and non-Holonomic Constraints, Problems with Variable endpoints: Natural BCs, Solution of Elastica

Week 6: Problems with Variable endpoints: case of several dependent variable, Transversality conditions, Broken extremals (Weierstrass Erdmann Condition), Newton's Aerodynamic Problem. Hamiltonian formulation of E-L Eqns.

Week 7: Hamiltonian formulation: Case of several dependent variables, Symplectic Maps, Hamilton-Jacobi Equations (HJ Eqns), Method of seperation of variables for HJ Eqns.

Week 8: Variational Symmetries, Noether's Theorem, Finding Variational Symmetries. Second Variation: Finite dim case, Legendre Condition

Week 9: Conjugate points, Jacobi necessary condition, Jacobi Accessory Eqns (JA Eqns), Sufficient Conditions, finding Conjugate points, saddle points. Optimal Control Theory (OC): solving OC systems via Variational Techniques

Week 10: OC Theory: Constrained Optimization, Pontrygin Minimum Principle (PMP), Hamilton-Jacobi-Bellmann Eqns (HJB), Penalty function method, Slack Variable Method.

Week 11: Nanomechanics: Oscillatory motion of Carbon Nanotube (CNT), Basics (special functions): Pochammer symbol, Hypergeometric Function (HF). Basics (Physical Chemistry): van der Waal Interaction Energy, Lennard Jones Potential. Oscillatory Motion of DWCNT via Hamilton's Principle

Week 12: Additional problem solving sessions.