Advanced Complex Analysis - Part 2: Singularity at Infinity, Infinity as a Value, Compact Spaces of Meromorphic Functions for the Spherical Metric and Spherical Derivative, Local Analysis of N - Video course

COURSE OUTLINE

This is the second part of a series of lectures on advanced topics in Complex Analysis. By advanced, we mean topics that are not (or just barely) touched upon in a first course on Complex Analysis. The theme of the course is to study compactness and convergence in families of analytic (or holomorphic) functions and in families of meromorphic functions. The compactness we are interested herein is the so-called sequential compactness, and more specifically it is normal convergence -- namely convergence on compact subsets. The final objective is to prove the Great or Big Picard Theorem and deduce the Little or Small Picard Theorem. This necessitates studying the point at infinity both as a value or limit attained, and as a point in the domain of definition of the functions involved. This is done by thinking of the point at infinity as the north pole on the sphere, by appealing to the Riemann Stereographic Projection from the Riemann Sphere. Analytic properties are tied to the spherical metric on the Riemann Sphere. The notion of spherical derivative is introduced for meromorphic functions. Infinity is studied as a singular point. Laurent series at infinity, residue at infinity and a version of the Residue theorem for domains including the point at infinity are explained. In later lectures, Marty's theorem -- a version of the Montel theorem for meromorphic functions, Zalcman's Lemma -- a fundamental theorem on the local analysis of non-normality, Montel's theorem on normality, Royden's theorem and Schottky's theorem are proved. For more details on what is covered lecturewise, please look at the titles, goals and keywords which are given for each lecture.

COURSE DETAIL

Unit Number / Title	Lecture Number / Title
UNIT 1: Theorems of Picard, Casorati-Weierstrass and Riemann on Removable Singularities **Goals & keywords**	Lecture 1: Properties of the Image of an Analytic Function: Introduction to the Picard Theorems
	Lecture 2: Recalling Singularities of Analytic Functions: Non-isolated and Isolated Removable, Pole and Essential Singularities
	Lecture 3: Recalling Riemann's Theorem on Removable Singularities
	Lecture 4: Casorati-Weierstrass Theorem; Dealing with the Point at Infinity -- Riemann Sphere and Riemann Stereographic Projection
UNIT 2: Neighborhoods of Infinity, Limits at Infinity and Infinite Limits **Goals & keywords**	Lecture 5: Neighborhood of Infinity, Limit at Infinity and Infinity as an Isolated Singularity
	Lecture 6: Studying Infinity: Formulating Epsilon-Delta Definitions for Infinite Limits and Limits at Infinity
UNIT 3: Infinity as a Point of Analyticity	Lecture 7: When is a function analytic at infinity ?

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http://nptel.ac.in

Mathematics

Pre-requisites:

A first course in Topology covering the euclidean spaces (real line and real plane), and a first course in Complex Analysis covering Cauchy's Integration theory, Taylor series, Laurent series and the Residue theorem.

Additional Reading:

http://webdoc.sub.gwdg.de/univerlag/2010/balaji.pdf

Hyperlinks:

- http://nptel.ac.in/syllabus/111106084/
- http://nptel.ac.in/syllabus/111106044/

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Goals & keywords	Lecture 8: Laurent Expansion at Infinity and Riemann's Removable Singularities Theorem for the Point at Infinity
	Lecture 9: The Generalized Liouville Theorem: Little Brother of Little Picard and Analogue of Casorati-Weierstrass; Failure of Cauchy's Theorem at Infinity
	Lecture 10: Morera's Theorem at Infinity, Infinity as a Pole and Behaviour at Infinity of Rational and Meromorphic Functions
UNIT 4: Residue at Infinity and Residue Theorem for the Extended Complex Plane **Goals & keywords**	Lecture 11: Residue at Infinity and Introduction to the Residue Theorem for the Extended Complex Plane: Residue Theorem for the Point at Infinity

References:

1. **Complex Variables with Applications, by Saminathan Ponnusamy & Herb Silverman, 2006, 524 pp,¸ Birkhaeuser, Boston.**
2. **Complex Analysis (UTM) by Theodore Gamelin, Springer, 2003.¸**
3. **NPTEL Video Course on Advanced Complex Analysis - Part I athttp://nptel.ac.in/syllabus/111106084/**
4. **NPTEL Video Course on Riemann Surfaces athttp://nptel.ac.in/syllabus/111106044/**