Advanced Complex Analysis - Part 1:Zeros of Analytic Functions,Analytic continuation, Monodromy, Hyperbolic Geometry and the Reimann Mapping Theorem - Video course

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Mathematics

COURSE OUTLINE

This is the first part of a series of lectures on advanced topics in Complex Analysis. By advanced, we mean topics that are not (or just barely) touched upon in a first course on Complex Analysis. The theme of the course is to study zeros of analytic (or holomorphic) functions and related theorems. These include the theorems of Hurwitz and Rouche, the Open Mapping theorem, the Inverse and Implicit Function theorems, applications of those theorems, behaviour at a critical point, analytic branches, constructing Riemann surfaces for functional inverses, Analytic continuation and Monodromy, Hyperbolic geometry and the Riemann Mapping theorem. For more details, please look at the titles, goals and keywords for each lecture given below.

COURSE DETAIL

Unit Number / Title	Lecture Number / Title
UNIT 1: Theorems of Rouche and Hurwitz **Goals & keywords**	Lecture 1: Fundamental Theorems Connected with Zeros of Analytic Functions
	Lecture 2: The Argument (Counting) Principle, Rouche's Theorem and The Fundamental Theorem of Algebra
	Lecture 3: Morera's Theorem and Normal Limits of Analytic Functions
	Lecture 4: Hurwitz's Theorem and Normal Limits of Univalent Functions
UNIT 2: Open Mapping Theorem **Goals & keywords**	Lecture 5: Local Constancy of Multiplicities of Assumed Values
	Lecture 6: The Open Mapping Theorem
UNIT 3: Inverse Function Theorem	Lecture 7:

Pre-requisites:

A first course in Topology covering the euclidean spaces (real line and real plane), and a first course in Complex Analysis covering Cauchy's Integration theory, Taylor series, Laurent series and the Residue theorem.

Coordinators:

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Goals & keywords	Introduction to the Inverse Function Theorem
	Lecture 8: Completion of the Proof of the Inverse Function Theorem: The Integral Inversion Formula for the Inverse Function
	Lecture 9: Univalent Analytic Functions have never-zero Derivatives and are Analytic Isomorphisms
UNIT 4: Implicit Function Theorem Goals & keywords	Lecture 10: Introduction to the Implicit Function Theorem
	Lecture 11: Proof of the Implicit Function Theorem: Topological Preliminaries
	Lecture 12: Proof of the Implicit Function Theorem: The Integral Formula for & Analyticity of the Explicit Function
UNIT 5: Riemann Surfaces for Multi-Valued Functions Goals & keywords	Lecture 13: Doing Complex Analysis on a Real Surface: The Idea of a Riemann Surface
	Lecture 14: F(z,w)=0 is naturally a Riemann Surface
	Lecture 15 Constructing the Riemann Surface for the Complex Logarithm
	Lecture 16 Constructing the Riemann Surface for the m-th root function
	Lecture 17 The Riemann Surface for the functional inverse of an analytic mapping at a critical point
	Lecture 18 The Algebraic nature of the functional inverses of an analytic mapping at a critical point
UNIT 6: Analytic Continuation Goals & keywords	Lecture 19 The Idea of a Direct Analytic Continuation or an Analytic Extension
	Lecture 20 General or Indirect Analytic Continuation and the Lipschitz Nature of the Radius of Convergence
	Lecture 21A Analytic Continuation Along Paths via Power Series Part A
	Lecture 21B Analytic Continuation Along Paths via Power Series Part B

	Lecture 22 Continuity of Coefficients occurring in Families of Power Series defining Analytic Continuations along Paths
UNIT 7: Monodromy **Goals & keywords**	Lecture 23: Analytic Continuability along Paths: Dependence on the Initial Function and on the Path - First Version of the Monodromy Theorem
	Lecture 24: Maximal Domains of Direct and Indirect Analytic Continuation: Second Version of the Monodromy Theorem
	Lecture 25: Deducing the Second (Simply Connected) Version of the Monodromy Theorem from the First (Homotopy) Version
	Lecture 27: Existence and Uniqueness of Analytic Continuations on Nearby Paths
	Lecture 28: Proof of the First (Homotopy) Version of the Monodromy Theorem
	Lecture 30: Proof of the Algebraic Nature of Analytic Branches of the Functional Inverse of an Analytic Function at a Critical Point
UNIT 8: Harmonic Functions, Maximum Principles, Schwarz's Lemma and Uniqueness of Riemann Mappings **Goals & keywords**	Lecture 31: The Mean-Value Property, Harmonic Functions and the Maximum Principle
	Lecture 32: Proofs of Maximum Principles and Introduction to Schwarz's Lemma
	Lecture 33: Proof of Schwarz's Lemma and Uniqueness of Riemann Mappings
	Lecture 34: Reducing Existence of Riemann Mappings to Hyperbolic Geometry of Sub-domains of the Unit Disc
UNIT 9: Pick's Lemma and Hyperbolic Geometry on the Unit Disc **Goals & keywords**	Lecture 35A: Differential or Infinitesimal Schwarz's Lemma,Pick's Lemma, Hyperbolic Arclengths, Metric and Geodesics on the Unit Disc
	Lecture 35B: Differential or Infinitesimal Schwarz's Lemma,Pick's Lemma, Hyperbolic Arclengths, Metric and Geodesics on the Unit Disc
	Lecture 36: Hyperbolic Geodesics for the Hyperbolic Metric on the Unit Disc
	Lecture 37: Schwarz-Pick Lemma for the Hyperbolic Metric on the Unit Disc
UNIT 10: Theorems of Arzela-Ascoli and Montel **Goals & keywords**	Lecture 38: Arzela-Ascoli Theorem: Under Uniform Boundedness, Equicontinuity and Uniform Sequential Compactness are Equivalent
	Lecture 39: Completion of the Proof of the Arzela-Ascoli Theorem and Introduction to Montel's Theorem
	Lecture 40: The Proof of Montel's Theorem
UNIT 11: Existence of a Riemann Mapping	Lecture 41: The Candidate for a Riemann Mapping

Mapping	Lecture 42A: Completion of Proof of The Riemann Mapping Theorem
Goals & keywords	Lecture 42B: Completion of Proof of The Riemann Mapping Theorem

References:

1. Complex Variables with Applications, by Saminathan Ponnusamy & Herb Silverman,¸ 2006, 524 pp,¸ Birkhaeuser, Boston.
2. Complex Analysis (UTM) by Theodore Gamelin, Springer, 2003.¸
3. NPTEL Video Course on Riemann Surfaces at http://nptel.ac.in/faq/111106044/