Dynamic Data Assimilation: an introduction - Video course

COURSE OUTLINE

Much of the human activity is controlled by prediction of various phenomena that affect our life - be it of the man-made or natural type. Examples include prediction of weather – hurricanes, tornados, snow/ice showers, heat waves, short-term climate scenarios, prediction of revenue by local/state and national governments to develop budget priorities for the next fiscal year, prediction of the growth in GDP, prediction of IBM stock price on the day of equinox, to name a few. These predictions are generated by running a relevant class of models which may be either causality based (as in hurricane prediction) or empirically derived (as in the prediction of revenue or unemployment or GDP growth, etc).

Besides being causality based or empirically derived, models, in general, occur in various shapes and forms. A model can be (a) static or dynamic (b) deterministic or stochastic (c) operate in discrete/continuous time and (d) may be defined on continuous space or discrete space. Irrespective of its origin, models in general have several unknowns – initial conditions (IC), boundary conditions (BC), and parameters. The solutions of these models, in general, constitute the basis for generating predictions.

However, to compute the solution we need to know or estimate the values of the unknown IC, BC and/or parameters. This estimation is enabled by using the observations of the phenomenon in question – using the observed pressure distribution around the eye of the hurricane, data from satellites or ground based radars, from the time series of data on unemployment, etc. Data assimilation in large measure relates to the process of "fusing" data with the model for the singular purpose of estimating the unknowns. Once these estimates are available, we obtain an instantiation of the model which is then run forward in time to generate the requisite forecast products for public consumption.

Data assimilation as stated is intimately related to the "inverse" problems in mathematics or regression problems in statistics or retrieval problems in Geosciences. Finding the sea surface temperature distribution near the equatorial Pacific from Satellite measurements or distribution of the aerosol from satellite observations or estimating the amount of rain from radar observations are but a few of the examples of inverse problems that lie at the heart of data assimilation.

Some phenomena are intrinsically predictable (lunar/solar eclipses for the next 50 years) but certain other phenomena are only predictable for a short horizon – weather at most for a week. The length of the predictable horizon is related to predictability limit of the model. This is closely related to understanding the growth of prediction errors. This topic is closely related to analysis of chaotic systems.

Our aim is to provide a broad based background on the mathematical principles and tools from linear algebra, multivariate calculus and finite dimensional optimization theory, estimation theory, non-linear dynamics and chaos that constitute the basis for dynamic data assimilation as we know today. Our aim is to present the ideas at the level of a first year graduate/final year undergraduate student aspiring to enter this exciting area.

COURSE DETAIL

Module No:	Lecture No:	Topic
Module I: Introduction	01	Data Mining, Data Assimilation, Inverse problems and Prediction
	02	Static vs. dynamic and deterministic vs. stochastic problems- formulation & classification

NP-TEL

NPTEL

http://nptel.ac.in

Mathematics

Pre-requisites:

Good facility with Calculus, Linear Algebra, basic Probability Theory and Statistics

Additional Reading:

E. Kalnay (2003)Atmospheric Modeling, Data Assimilation, and Predictability, Cambridge University Press

Coordinators:

S. Lakshmivarahan
School of Computer ScienceIIT Madras

Module II: Mathematical tools	03	Finite dimensional vector space – basic concepts	
	04	Overview of properties and operations on matrices	
	05	Special classes of matrices, Eigen decomposition, and matrix square root	
	06	Gradient, Jacobian, Hessian, Quadratic forms and their properties	
Module III: Static, deterministic models: least Squares method – formulation and properties	07	Linear least squares (LLS) - over determined case, weighted and unweighted formulation, orthogonal and oblique projections	
	08	LLS- Underdetermined case -Lagrangian multiplier	
	09	Nonlinear least squares problem (NLS) - formulation	
	10	Approximation – first and second-order methods for solving NLS.	
	11	Examples of LLS and NLS - satellite retrieval	
Module IV: Matrix methods solving LLS	12	Normal equations – symmetric positive definite (SPD) systems – multiplicative matrix decomposition	
	13	Cholesky decomposition- matrix square root	
	14	Gramm-Schmidt orthogonalization process	
	15	QR decomposition	
	16	Singular value decomposition (SVD)	
	17	Solution of retrieval problems	
Module V: Direct minimization methods for solving LLS	18	LLS as a quadratic minimization problem	
	19	Gradient method, its properties	
	20	Convergence and speed of convergence of gradient method	
	21	Conjugate gradient and Quasi-Newton methods	

	22	Practice problems and programming exercises
Module VI:Deterministic, dynamic models – adjoint method	23	Dynamic models, role of observations, and least squares objective function, estimation of initial condition (IC) and parameters, adjoint sensitivity
	24	A straight line problem – a warm up
	25	Linear model, first-order adjoint dynamics and computation of the gradient of the least squares objective function
	26	Nonlinear model and first-order adjoint dynamics'
	27	Illustrative examples and practice problems, programming exercises
Module VII: Deterministic, Dynamic models – Other methods	28	Forward sensitivity method for estimation of IC and parameters, forward sensitivity dynamics - Example of Carbon dynamics
	29	Relation between adjoint and forward sensitivity, Predictability, Lyapunov index
	30	Method of nudging and overview of nudging methods
Module VIII: Static, stochastic models – Bayesian framework	31	Bayesian method – linear, Gaussian case
	32	Linear minimum variance estimation (LMVE) and prelude to Kalman filter
	33	Model space vs. observation space formulation -Duality between Bayesian and LMVE
Module IX: Dynamic, Stochastic models - Kalman filtering	34	Derivation of the Kalman filter equations
	35	Derivation of Nonlinear filter
	36	Computational requirements
	37	Ensemble Kalman filtering
Module X: Dynamic stochastic models – Other methods	38	Unscented Kalman filtering
	39	Particle filtering

	40	An overview and assessment of methods

References:

Books

1. J.M. Lewis, S. Lakshmivarahan and S. K. Dhall (2006) **Dynamic Data Assimilation: a least squares approach**, *Cambridge University Press*, 654 pages + Appendices A through F
2. E. Kalnay (2003) **Atmospheric Modeling, Data Assimilation, and Predictability**, *Cambridge University Press*

Papers (more will be added)

1. S. Lakshmivarahan and D. Stensrud (2009) Ensemble Kalman Filter: An innovative approach for meteorological data assimilation", **IEEE Control System Society, Special Issue,** Vol 29, pp 34-46,
2. S. Lakshmivarahan and John M. Lewis (2010) "Forward Sensitivity Based Approach to Dynamic Data Assimilation", **Advances in Meteorology,** Volume 2010, Article ID 375615, doi 1155/2010/375615
3. C. Gao, H. Wang, W. Weng, S. Lakshmivarahan, Y. Zhang and Y. Luo (2011) " Assimilation of multiple data sets with the ensemble Kalman filter to improve forecasts of forest carbon dynamics", **Ecological Applications**, Vol 21, pp 1461-1473
4. S. Lakshmivarahan and J. M. Lewis (2013) "Nudging Methods: A Critical Overview" a chapter in ***Data Assimilation for Atmospheric, Oceanic and Hydrologic Applications*** Vol II, edited bySeon Ki Park and L. Liang,Springer Verlag, pp 27-58
5. S. Lakshmivarahan, J. M. Lewis and Dung Phan (2013) "Data assimilation as a problem in optimal tracking: Application of Pontryagin's minimum principle", **Journal of Atmospheric Sciences**, Vol 70, pp 1257-1277
6. J. M. Lewis and S. Lakshmivarahan (2013) "A Question of Adequacy of observations in Variational DataAssimilation" a chapter in ***Data Assimilation for Atmospheric, Oceanic and Hydrologic Applications*** Vol II, edited by Seon Ki Park and L. Liang,Springer Verlag, pp 111-124