Real Analysis - Video course

COURSE OUTLINE

Real number system and its order completeness, sequences and series of real numbers. Metric spaces: Basic concepts, continuous functions, completeness, contraction mapping theorem, connectedness, Intermediate Value Theorem, Compactness, Heine-Borel Theorem. Differentiation, Taylor's theorem, Riemann Integral, Improper integrals Sequences and series of functions, Uniform convergence, power series, Weierstrass approximation theorem, equicontinuity, Arzela-Ascoli theorem.

COURSE DETAIL

MODULE 1: REVIEW OF SET THEORY. **LECTURES 1 - 5.**
Operations on sets, family of sets, indexing set, functions, axiom of choice, relations, equivalence relation, partial order, total order, maximal element, Zorn's lemma, finite set, countable set, uncountable set, Cantor's theorem, cardinal numbers, Continuum Hypothesis.ns
MODULE 2: SEQUENCES AND SERIES OF REAL NUMBERS. **LECTURES 6 - 13.**
Real Number System, algebraic properties, order properties, absolute value function, LUB axiom, Archemedian property. Sequences of real numbers, convergent sequence, subsequence, Sandwich theorem, monotonic sequence, limsup, liminf, Bolzano-Weierstrass Theorem, Cauchy sequence, infinite series, convergent series, absolute convergence, series of nonnegative real numbers, comparison test, root test, ratio test, power series, extended real line, conditional convergence, rearrangement.
MODULE 3: METRIC SPACES - BASIC CONCEPTS. **LECTURES 14 - 19.**
Metric, metric space, metric induced by norm, open ball, closed ball, sphere, interval, interior, exterior, boundary, open set, topology, closure point, limit point, isolated point, closed set, Cantor set.
MODULE 4: COMPLETENESS. **LECTURES 20 - 22.**
Sequences in metric spaces, complete metric space, Cantor's Intersection Theorem, Baire Category Theorem.
MODULE 5: LIMITS AND CONTINUITY. **LECTURES 23 - 25.**
Limit and continuity of a function defined on a metric space, uniform continuity, homeomorphism, Lipschitz continuous function, contraction, isometry, Banach's contraction mapping principle.
MODULE 6: CONNECTEDNESS AND COMPACTNESS. **LECTURES 26 - 32.**

NP-TEL

NPTEL

http://nptel.iitm.ac.in

Mathematics

Pre-requisites:

Familiarity with the calculus of functions of one variable

Additional Reading:

1. T. M. Apostol, Mathematical Analysis
2. G. F. Simmons, Topology and Modern Analysis

Coordinators:

Prof. S.H. Kulkarni
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Connectedness: Connected set, interval, Intermediate Value Theorem, connected component, totally disconnected set.
Compactness: Compact set, finite intersection property, totally bounded set, Bolzano - Weierstrass theorem, sequential compactness, Heine - Borel theorem, continuous functions on compact sets, types of discontinuity.

MODULE 7: DIFFERENTIATION.
LECTURES 33 - 37.

Derivative, differentiable function, chain rule, derivative of a composite function, local minimum, local maximum, Rolle's theorem, Lagrange's mean value theorem, Cauchy's mean value theorem, indeterminate forms, L' Hospital's rule, intermediate value property, higher order derivatives, Taylor's theorem, Taylor series, infinitely differentiable function, Maclaurin series, differentiation of vector valued functions.

MODULE 8: INTEGRATION.
LECTURES 38 - 45.

Riemann integral, Riemann -Stieltjes integral, Riemann -Stieltjes integrable function, First Mean Value Theorem of Integral Calculus, Integration as a Limit of Sum, Fundamental Theorem of Integral Calculus, integration of vector valued function, function of bounded variation, total variation, Integration by part, second Mean Value Theorem of Integral Calculus, change of variable formula, improper integral.

MODULE 9: SEQUENCES AND SERIES OF FUNCTIONS.
LECTURES 46 - 52.

Sequence of functions, pointwise convergence, series of functions, uniform convergence, uniformly bounded sequence, Cauchy's criterion for uniform convergence, uniformly Cauchy sequence, Weierstrass' M test, Dini's theorem, uniform convergence and continuity, uniform convergence and integration, uniform convergence and differentiation, Weierstrass theorem, equicontinuous family of functions, Arzela - Ascoli Theorem.

References:

1. W. Rudin, Principles of Mathematical Analysis.
2. C. C. Pugh, Real Mathematical Analysis.