An Introduction to Riemann Surfaces and Algebraic Curves: Complex 1-Tori and Elliptic Curves - Video course

COURSE OUTLINE

The subject of algebraic curves (equivalently compact Riemann surfaces) has its origins going back to the work of Riemann, Abel, Jacobi, Noether, Weierstrass, Clifford and Teichmueller. It continues to be a source for several hot areas of current research. Its development requires ideas from diverse areas such as analysis, PDE, complex and real differential geometry, algebra---especially commutative algebra and Galois theory, homological algebra, number theory, topology and manifold theory.

The course begins by introducing the notion of a Riemann surface followed by examples. Then the classification of Riemann surfaces is achieved on the basis of the fundamental group by the use of covering space theory and uniformisation. This reduces the study of Riemann surfaces to that of subgroups of Moebius transformations. The case of compact Riemann surfaces of genus 1, namely elliptic curves, is treated in detail. The algebraic nature of elliptic curves and a complex analytic construction of the moduli space of elliptic curves is given.

COURSE DETAIL

Unit Number / Title	**Lecture Number / Title**
Unit 1: Definitions and Examples of Riemann Surfaces	Lecture 1: The Idea of a Riemann Surface
	Lecture 2: Simple Examples of Riemann Surfaces
	Lecture 3: Maximal Atlases and Holomorphic Maps of Riemann Surfaces
	Lecture 4: A Riemann Surface Structure on a Cylinder
	Lecture 5: A Riemann Surface Structure on a Torus
Unit 2: Classification of Riemann Surfaces	Lecture 6: Riemann Surface Structures on Cylinders and Tori via Covering Spaces
	Lecture 7: Moebius Transformations Make up Fundamental Groups of Riemann Surfaces

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Mathematics

Pre-requisites:

First courses in Topology and Complex Analysis. However, to help the students taking the course, most of the needed material from topology, complex analysis and algebra is recalled when necessary and their details, including exercises and suggestions for further reading, are given in the supplementary slides at the end of each lecture. These slides are also available separately as PDF files, apart from the lecture notes, for download below.

Coordinators:

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	Lecture 8: Homotopy and the First Fundamental Group	
	Lecture 9: A First Classification of Riemann Surfaces	
Unit 3: Universal Covering Space Theory	Lecture 10: The Importance of the Path-lifting Property	
	Lecture 11: Fundamental groups as Fibres of the Universal covering Space	
	Lecture 12: The Monodromy Action	
	Lecture 13: The Universal covering as a Hausdorff Topological Space	
	Lecture 14: The Construction of the Universal Covering Map	
	Lecture 15-Part A: Completion of the Construction of the Universal Covering: Universality of the Universal Covering	
	Lecture 15-Part B: Completion of the Construction of the Universal Covering: The Fundamental Group of the base as the Deck Transformation Group	
Unit 4: Classifying Moebius Transformations and Deck Transformations	Lecture 16: The Riemann Surface Structure on the Topological Covering of a Riemann Surface	
	Lecture 17: Riemann Surfaces with Universal Covering the Plane or the Sphere	
	Lecture 18: Classifying Complex Cylinders: Riemann Surfaces with Universal Covering the Complex Plane	
	Lecture 19: Characterizing Moebius Transformations with a Single Fixed Point	

	Lecture 20: Characterizing Moebius Transformations with Two Fixed Points
	Lecture 21: Torsion-freeness of the Fundamental Group of a Riemann Surface
	Lecture 22: Characterizing Riemann Surface Structures on Quotients of the Upper Half-Plane with Abelian Fundamental Groups
	Lecture 23: Classifying Annuli up to Holomorphic Isomorphism
Unit 5: The Riemann Surface Structure on the Quotient of the Upper Half-Plane by the Unimodular Group	Lecture 24: Orbits of the Integral Unimodular Group in the Upper Half-Plane
	Lecture 25: Galois Coverings are precisely Quotients by Properly Discontinuous Free Actions
	Lecture 26: Local Actions at the Region of Discontinuity of a Kleinian Subgroup of Moebius Transformations
	Lecture 27: Quotients by Kleinian Subgroups give rise to Riemann Surfaces
	Lecture 28: The Unimodular Group is Kleinian
Unit 6: Doubly-Periodic Meromorphic (or) Elliptic Functions	Lecture 29: The Necessity of Elliptic Functions for the Classification of Complex Tori
	Lecture 30: The Uniqueness Property of the Weierstrass Phe-function associated to a Lattice in the Plane
	Lecture 31: The First Order Degree Two Cubic Ordinary Differential Equation satisfied by the Weierstrass Phe-function
	Lecture 32: The Values of the Weierstrass Phe-function at the Zeros of its Derivative are nonvanishing Analytic Functions on the Upper Half-Plane
Unit 7: A Form Modular for the Congruence-mod-2 Subgroup of the Unimodular Group on the Upper Half-Plane	Lecture 33: The Construction of a Modular Form of Weight Two on the Upper Half-Plane
	Lecture 34: The Fundamental Functional Equations satisfied by the Modular Form of Weight Two on the Upper Half-Plane
	Lecture 35: The Weight Two Modular Form assumes Real Values on the Imaginary Axis in the Upper Half-plane
	Lecture 36: The Weight Two Modular Form Vanishes at Infinity
	Lecture 37A: The Weight Two Modular Form Decays Exponentially in a Neighbourhood of Infinity

	Lecture 37B: A Suitable Restriction of the Weight Two Modular Form is a Holomorphic Conformal Isomorphism onto the Upper Half-Plane
Unit 8: The Elliptic Modular J-invariant and the Moduli of Complex 1-dimensional Tori (or) Elliptic Curves	Lecture 38: The J-Invariant of a Complex Torus (or) of an Algebraic Elliptic Curve
	Lecture 39: A Fundamental Region in the Upper Half-Plane for the Elliptic Modular J-Invariant
	Lecture 40: The Fundamental Region in the Upper Half-Plane for the Unimodular Group
	Lecture 41: A Region in the Upper Half-Plane Meeting Each Unimodular Orbit Exactly Once
	Lecture 42: Moduli of Elliptic Curves
Unit 9: Complex 1-dimensional Tori are Projective Algebraic Elliptic Curves	Lecture 43: Punctured Complex Tori are Elliptic Algebraic Affine Plane Cubic Curves in Complex 2-Space
	Lecture 44: The Natural Riemann Surface Structure on an Algebraic Affine Nonsingular Plane Curve
	Lecture 45A: Complex Projective 2-Space as a Compact Complex Manifold of Dimension Two
	Lecture 45B: Complex Tori are the same as Elliptic Algebraic Projective Curves

References:

1. H. M. Farkas, I. Kra: Riemann Surfaces, Graduate Texts in Mathematics 71, Springer-Verlag, 1980
2. R. Miranda: Algebraic Curves and Riemann Surfaces, Graduate Studies in Mathematics vol.5, American Mathematical Society, 1995
3. O. Forster: Lectures on Riemann Surfaces, Graduate Texts in Mathematics 81 / Springer-Verlag, 1981
4. R. Hartshorne: Algebraic Geometry, Graduate Texts in Mathematics 52, Springer-Verlag,
5. E. Arbarello, M. Cornalba, P. A. Griffiths, J. Harris: Geometry of Algebraic Curves - I, Grundlehren der mat.Wissen.267, Springer-Verlag, 1985
6. T. E. Venkata Balaji: An Introduction to Families, Deformations and Moduli, available at

   http://webdoc.sub.gwdg.de/univerlag/2010/balaji.pdf