APPLIED LINEAR ALGEBRA IN AI AND ML

PROF:SWANAND KHARE
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PRE-REQUISITES : Senior undergraduate and post graduate students from CSE, EE, ECE, AI, Maths
INTENDED AUDIENCE : First course in Engineering Mathematics with some exposure to linear algebra

COURSE OUTLINE :
Linear algebra, optimization techniques and statistical methods together form essential tools for most of the algorithms in artificial intelligence and machine learning. In this course, we propose to build some background in these mathematical foundations and prepare students to take on advanced study or research in the field of AI and ML. The objective of this course is to be familiarize students with the important concepts and computational techniques in linear algebra useful for AI and ML applications. The unique objective of this course and the distinguishing point from the existing courses on the similar topics would be illustration of application of these concepts to many real life problems in AI and ML. Some of the key topics to be covered in this course are listed below: least squares solution, parameter estimation problems, concept of cost function and relation to parameter estimation, constrained least squares, multi-objective least squares, applications to portfolio optimization, sparse solutions to underdetermined systems of linear equations, applications to dictionary learning, eigenvalue eigenvector decomposition of square matrices, spectral theorem for symmetric matrices, SVD, multicollinearity problem and applications to principal component analysis (PCA) and dimensionality reduction, power method, application to Google page ranking algorithm, inverse eigenvalue problem, construction of Markov chains from the given stationary distribution, low rank approximation and structured low rank approximation problem (SLRA), Autoregressive model order selection using Hankel SLRA, approximate GCD computation and application to image de- blurring, tensors and CP tensor decomposition, tensor decomposition based sparse learning in deep networks, matrix completion problems, application to collaborative filtering

ABOUT INSTRUCTOR :
Prof. Swanand Khare obtained M.Sc. and Ph.D. degrees from IIT Bombay in 2005 and 2011 respectively. He was a post-doctoral researcher in the University of Alberta, Canada from 2011 to 2014 and then subsequently joined the Department of Mathematics at IIT Kharagpur. He currently works as an Associate Professor in the Department of Mathematics and jointly in the Centre of Excellence in AI at IIT Kharagpur. His research interests include inverse eigenvalue problems, computational linear algebra, estimation and computational issues in applied statistics. He has been actively participating in fundamental as well as applied research in these areas. He has supervised four PhD students and several masters' students in their research work. He served as an Associate Editor for a journal named Control Engineering Practice for a period of three years from 2018 to 2021. He is a recipient of Excellent Young Teacher Award 2018 at IIT Kharagpur.

COURSE PLAN:
Week 1: Vectors, operations on vectors, vector spaces and subspaces,inner product and vector norm, linear dependence and independence, Matrices, linear transformations, orthogonal matrices
Week 2: System of linear equations, existence and uniqueness, left and right inverses, pseudo inverse, triangular systems
Week 3: LU decomposition and computational complexity, rotators and reflectors, QR decomposition, Gram Schmidt Orthogonalization
Week 4: Condition number of a square matrix, geometric interpretation, norm of matrix, sensitivity analysis results for the system of linear equations
Week 5: Linear least squares, existence and uniqueness, geometrical interpretation, data fitting with least squares, feature engineering, application to Vector auto-regressive models, fitting with continuous and discontinuous piecewise linear functions
Week 6: Application of least squares to classification, two-class and multi-class least squares classifiers, Polynomial classifiers, application to MNIST data set
Week 7: Multi-objective least squares, applications to estimation and regularized inversion, regularized data fitting and application to image de-blurring, constrained least squares, application to portfolio optimization
Week 8: Eigenvalue eigenvector decomposition of square matrices,spectral theorem for symmetric matrices
Week 9:SVD, relation to condition number, sensitivity analysis of least squares problems, variation in parameter estimates in regression
Week 10: Multicollinearity problem and applications to principal component analysis (PCA) and diinensionality reduction, power method, application to Google page ranking algorithm
Week 11: Underdetermined systems of linear equations, least norm solutions, sparse solutions, applications in dictionary learning and sparse code recovery, inverse eigenvalue problem, application in construction of Markov chains from the given stationary distribution
Week 12: Low rank approximation (LRA) and structured low rank approximation problem (SLRA), application to model order selection in time series, alternating projections for computing LRA and SLRA