INTEGRAL AND VECTOR CALCULUS

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TYPE OF COURSE : Rerun | Core | UG
COURSE DURATION : 12 weeks (24 Jan' 22 - 15Apr' 22)
EXAM DATE : 23 Apr 2022

PRE-REQUISITES : Differential calculus of one and several variables.

INTENDED AUDIENCE : Mathematics students

COURSE OUTLINE :
This course will offer a detailed introduction to integral and vector calculus. We'll start with the concepts of partition, Riemann sum and Riemann Integrable functions and their properties. We then move to anti-derivatives and will look in to few classical theorems of integral calculus such as fundamental theorem of integral calculus. We'll then study improper integral, their convergence and learn about few tests which confirm the convergence. Afterwards we'll look into multiple integrals, Beta and Gamma functions, Differentiation under the integral sign. Finally, we'll finish the integral calculus part with the calculation of area, rectification, volume and surface integrals. In the next part, we'll study the vector calculus. We'll start the first lecture by the collection of vector algebra results. In the following weeks, we'll learn about scalar and vector fields, level surfaces, limit, continuity, and differentiability, directional derivative, gradient, divergence and curl of vector functions and their geometrical interpretation. We'll also study the concepts of conservative, irrotational and solenoidal vector fields. We'll look into the concepts of tangent, normal and binormal and then derive the Serret-Frenet formula. Then we'll look into the line, volume and surface integrals and finally we'll learn the three major theorems of vector calculus: Green's, Gauss's and Stoke's theorem.

ABOUT INSTRUCTOR :
Prof. Hari Shankar Mahato is currently working as an Assistant Professor in the Department of Mathematics at the Indian Institute of Technology Kharagpur. Before joining here, he worked as a postdoc at the University of Georgia, USA. He did his PhD from the University of Bremen, Germany and then he worked as a Postdoc at the University of Erlangen-Nuremberg and afterwards at the Technical University of Dortmund, both located in Germany. His research expertise are Partial Differential Equations, Applied Analysis, Variational Methods, Homogenization Theory and very recently he has started working on Mathematical Biology. He can be able to teach (both online and offline) any undergraduate courses from pre to advanced calculus, mechanics, ordinary differential equations, up to advanced graduate courses like linear and nonlinear PDEs, functional analysis, topology, mathematical modeling, fluid mechanics and homogenization theory

COURSE PLAN :

Week 1 : Partition, concept of Riemann integral, properties of Riemann integrable functions, anti-derivatives, Fundamental theorem of Integral calculus, mean value theorems.
Week 2 : Reduction formula and derivation of different types of formula, improper integrals and their convergence, tests of convergence.
Week 3 : Beta and Gamma function, their properties, differentiation under the integral sign, Leibnitz rule.
Week 4 : Double integrals. change of order of integration, Jacobian transformations, triple integrals.
Week 5 : Area of plane regions, rectification, surface integrals.
Week 6 : Volume integrals, center of gravity and moment of Inertia.
Week 7 : Collection of vector algebra results, scalar and vector fields, level surfaces, limit, continuity, differentiability of vector functions
Week 8 : Curves, Arc-length, partial derivative of vector function, directional derivative gradient, divergence and curl.
Week 9 : Irrotational, conservative and Solenoidal fields, tangent, normal, binormal, Serret-Frenet formula.
Week 10 : Application of vector calculus in mechanics, lines, surface and volume integrals. line integrals independent of path.
Week 11 : The divergence theorem of Gauss, Stokes theorem, and Green's theorem.
Week 12 : Integral definition of gradient, divergence and curl. revision of problems from Integral and Vector calculus.