MATRIX SOLVER

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PRE-REQUISITES : PG course but senior UG students may credit it

INTENDED AUDIENCE : Basic science, Engineering

COURSE OUTLINE :

The objective of the course is to teach students about different algorithms for efficiently solving large matrix problems. One focus is discussing the mathematical background behind these schemes and the other focus is showing their implementations. The course will first try to present a basic understanding of fundamental issues of linear algebra relevant to matrix solutions. This will also discuss direct solution schemes like TDMA which has utility in scientific computing codes. The later half of the course will focus on iterative schemes for large matrices. Issues with convergence of the solvers will also be discussed. Students will be introduced to Krylov space based fast solvers. Implementations will be demonstrated using working codes. Techniques for improving convergence like preconditioning and multi-grid will also be briefly introduced. As an outcome of the course, a student will build an understanding of the matrix equations and suitable solution algorithms for them. This will help him to develop his own solver as well as to appreciate the open-source/commercial libraries of linear algebra and to utilise them efficiently.

ABOUT INSTRUCTOR :

Prof. Somnath Roy's primary area of work is computational fluid dynamics (CFD). He has been working on several applications involving heat transfer, mixing and turbulence. He also investigate CFD problems involving high computational cost and try to propose high performance computing (HPC) methodologies to address them using multi-core clusters and GPGPU platforms. In last few years, He has been mostly involved in addressing flow problems with moving boundaries. His group works on developing immersed boundary method (IBM) based computationally efficient algorithms to solve moving boundary problems and they have utilized these implementations to predict flow and heat transfer in engineering and biological applications. Over last eight years he has also been involved in teaching several courses like Fluid Mechanics, Thermodynamics, Aerodynamics Advanced Engineering Mathematics, Matrix Computing and High Performance Scientific Computing to students at different levels (UG and PG). He had earlier offered an NPTEL MOOC course titled Matrix Solvers.

COURSE PLAN :

Week 1: Introduction to matrix and equation systems (1) Symmetric matrix and transpose (1) Determinant and rank(1) , Gauss elimination(2), Row permutation (1)
Week 2: Inverse of Matrix (1), Gauss Jordon Method for Finding Inverse(2) Matrix form of difference equations (1) Tridiagonal matrix algorithm (2)
Week 3: Introduction to vector space (1) Column space and row space (1) Null space from solving Ax=0(3) Solving Ax=b (1)
Week 4: Linear independence and spanning (1) Basis and dimension (1) Four fundamental subspaces and solvability of a matrix equation (2) Linear transformation (2)
Week 5: Gram-Schmidt orthogonalization(3) QR Factorization for Normal equation (1) Eigen values, eigen vectors, spectral radius (2)
Week 6: Mid term exam
Week 7: Introduction to iterative methods, Gauss-Siedel, Jacobi and SOR (4) Convergence of iterative methods (2)
Week 8: Introduction to programming of matrix algorithms and code demonstration (2) Steepest descent algorithm and its variants (4)
Week 9: Introduction to Krylov subspaces(2) Krylov subspace for Av, Arnoldi''s Algorithm (2) GMRES (2)
Week 10: Lanczos method for symmetric matrix (1) Conjugate gradient method (1) Krylov subspace for ATv, biorthogonalization and Biconjugate gradient and BiCG-STAB(4)
Week 11: Lanczos method for symmetric matrix (1) Conjugate gradient method (1) Krylov subspace for ATv, biorthogonalization and Biconjugate gradient and BiCG-STAB(4)
Week 12: Block relaxation schemes (2) Introduction to Preconditioner and Multigrid Methods (4)