MEASURE THEORETIC PROBABILITY 1

PROF. SUPRIO BHAR
Department of Mathematics and Statistics
Indian Institute of Technology Kanpur

TYPE OF COURSE : New | Elective | PG
COURSE DURATION : 8 Weeks (26-Jul' 21 - 17-Sep' 21)
EXAM DATE : 26 Sep 2021

PRE-REQUISITES : A good background of Real Analysis and Basis Probability Theory (covering Probability distributions and standard Random variables).

INTENDED AUDIENCE : Students who have already learnt about basic Probability distributions and Random variables, and are interested in learning the Mathematical formulation of Probability. PG/Ph.D students and senior UG students are welcome.

INDUSTRIES APPLICABLE TO : This is a course focused on the Mathematical foundations of Probability and not on applications. However, this course is useful prerequisite towards advanced courses such as Stochastic Calculus and Financial Mathematics. As such, most industries should recognize this course.

COURSE OUTLINE :

This course is aimed at the students who have already learnt about basic Probability distributions and Random variables, and are interested in learning the Mathematical formulation of Probability. We first discuss the modeling of sample space and events into a probability space and, Random variables as functions on such probability spaces. Then, using Measure Theoretic techniques, we discuss a theory of integration which unifies the formulas for the expectations of discrete and absolutely continuous Random variables. Towards the end, we look at various convergence theorems for expectations of Random variables and related inequalities. Students, who complete the course, should be able to apply these results towards advanced studies.

ABOUT INSTRUCTOR :

Assistant Professor, Department of Mathematics and Statistics, Indian Institute of Technology Kanpur; Ph.D (Indian Statistical Institute, 2015); Research Interests: Stochastic PDEs

COURSE PLAN :

Week-1: Introduction to the course: A review of basic Probability and motivation towards the Mathematical formulation of Probability Theory; Fields and Sigma-fields of subsets of a non-empty set; Examples (emphasis on Borel sigma-fields on Euclidean spaces); Limits of sequences of events/sets, Monotone Class Theorem (Statement only)
Week-2: Measures and Measure spaces (Emphasis on Probabililty measures and Probability spaces); Examples and Properties
Week-3: Measurable functions (Emphasis on Random Variables); Examples:Properties (composition, algebraic properties, pointwise limits, measurability of components in higher dimensions); More examples using the above properties
Week-4: Caratheodery Extension Theorem (Emphasis on Uniqueness part, Existence part is statement only): Law/distribution of Random Variables; Distribution functions of Random Variables, properties
Week-5: Correspondence between Distribution functions and Probability measures on the real line; Extension of the above correspondence to higher dimensions (in brief); Lebesgue measure on the real line; Lebesgue measure on higher dimensions (in brief)
Week-6: Integration of measurable functions with respect to a measure (Emphasis on Expectation and Moments of Random Variables); Law/distribution of discrete Random variables, Example of measure theoretic integration: Expectation of discrete Random Variables; Convergence Theorems (Monotone Convergence Theorem, Fatou’s Lemma, Dominated Convergence Theorem), applications
Week-7: Convergence Theorems (Monotone Convergence Theorem, Fatou’s Lemma, Dominated Convergence Theorem), applications - Continued; Connection between Riemann and Lebesgue integration
Week-8: Radon-Nikodym Theorem (Statement only); Interpretation of the probabililty density function for absolutely continuous Random Variables as a Radom-Nikodym density, Law/ distribution of absolutely continuous Random variables; Example of measure theoretic integration: Expectation of absolutely continuous Random Variables; inequalities involving Expectation and Moments of Random Variables; Conclusion of the course