Calculus of Variations and Integral Equations - Web course

COURSE OUTLINE

Calculus of Variations:

Module 1: Introduction
Module 2: Variational problems with the fixed boundaries,
Module 3:, Variational problems with moving boundaries
Module 4: Sufficiency conditions

Integral Equations:

Module 1:Introduction
Module 2: Fredholm's Integral equations
Module 3: Voltera Integral equations
Module 4. FRedholm's theory - Hilbert-Schmidt theorem,
Module 5: Fredholm and Volterra Integro-Differential equation

COURSE DETAIL

Calculus of Variations:

Module No.	Topic/s	Lectures
1	Introduction, problem of brachistochrone, problem of geodesics, isoperimetric problem,Variation and its properties, functions and functionals, Comparison between the notion of extrema of a function and a functional.	4
2	Variational problems with the fixed boundaries, Euler's equation, the fundamental l e m m a of the calculus of variations, examples, Functionals in the	8

NPTEL

http://nptel.iitm.ac.in

Mathematics

Pre-requisites:

Basic knowledge of ODEs & PDEs

Additional Reading:

Bolza, O.: Lectures on the Calculus of Variations. Chelsea Publishing Company, 1904, available on Digital Mathematics library [2]. 2nd edition republished in 1961, paperback in 2005, ISBN 978-1418182014.

Hyperlinks:

http://en.wikipedia.org/wiki/

Calculus_of_variations

http://www.mathworld.wolfram.com/IntegralEquation.html

http://www.mathworld.wolfram.com/VolterraIntegralEquationoftheSecondKind.html

and related websites

Coordinators:

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	form of integrals, special cases contaning only some of the variables, examples, Functionals involving more than one dependent variables and their first derivatives, the system of Euler's equations, Functionals depending on the higher derivatives of the dependent variables, Euler-Poisson equation, examples, Functionals containing several independent variables, Ostrogradsky equation, examples, Variational problems in parametric form, applications to differential equations, examples, Variational problems with moving boundaries, pencil of extremals, Transversality condition, examples.		
3	Moving boundary problems with more than one dependent variables, transversality condition in a more general case, examples, Extremals with corners, refraction o f extremals, examples, One-sided variations, conditions for one sided variations,	4	
4	Field of extremals, central field of extremals, Jacobi's condition, The Weierstrass function, a weak	4	

extremum, a strong extremum, The Legendre condition, examples, Transforming the Euler equations to the canonical form, Variational problems involving conditional extremum, examples, constraints involving several variables and t h e i r derivatives, Isoperimetric problems, examples.

Integral Equations:

Module No.	Topic/s	Lectures
1	Introduction and basic examples, Classification, Conversion of Volterra Equation t o ODE, Conversion of IVP and BVP to Integral Equation,	4
2	Decomposition, direct computation, Successive approximation, Successive substitution methods for Fredholm Integral Equations,	4
3	A domain decomposition, series solution, successive approximation, successive substitution method for Volterra Integral Equations, Volterra Integral Equation of first kind, Integral Equations with separable Kernel,	6
4	Fredholm's first,	3

	second and third theorem, Integral Equations with symmetric kernel, Eigenfunction expansion, Hilbert-Schmidt theorem,	
5	Fredholm and Volterra Integro-Differential equation, Singular and nonlinear Integral Equation.	3

References:

- Curant, R. and D. Hilbert: Methods of Mathematical Physics, Vol I. Interscience Press, 1953.

- Elsgolc, L.E.: Calculus of Variations, Pergamon Press Ltd., 1962.

- Weinstock, Robert: Calculus of Variations with Applications to Physics and Engineering, Dover, 1974.

- Porter, D. and Stirling, D. S. G. : Integral Equations, A practical treatment from spectral theory and applications, Cambridge University Press, 1990.

- Cordumeanu, C. : Integral Equations and Applications, Cambridge University Press, 1991.