# FOURIER ANALYSIS AND ITS APPLICATIONS


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**PRE-REQUISITES :** Basic real analysis. A basic knowledge of functional analysis would help

**COURSE OUTLINE :**

Fourier analysis is a fundamental component in the tool-kit of every pure and applied mathematician with numerous applications to signal processing, image processing, tomography and several other areas of engineering. In this course we shall look at the most basic theoretical foundations of this subject with several applications. Along the way we shall have to recapitulate some of the requisite results from functional analysis. We offer some Selected applications such as Hurwitz's proof of the isoperimetric theorem and Weyl's theorem on equidistribution modulo one. Some applications to differential equations will be indicated.

**ABOUT INSTRUCTOR :**

Prof. Gopala Krishna Srinivasan is Department of Mathematics Indian Institute of Technology Bombay Powai, Mumbai 400076 Personal Details: • Gender: Male • Age: 52 • Phone number: 9819807454 (Mobile), 022 256 7454 (Office) Academic Qualifications: • Bachelor of Science, University of Bombay, 1987. • Master of Science, University of Bombay, 1989. • Doctor of Philosophy, Mathematics, University of Minnesota, Minneapolis, in 1995. Thesis: WTC Expansions and Painlevé Analysis. Areas of Specialization/interest: 1. Partial Differential Equations, Shock waves in Hyperbolic Systems of Conservation Laws. 2. Dynamical systems. 3. Classical Analysis, Special Functions and History of Mathematics. Publications: 1. S. Kichenassamy and G. K. Srinivasan, The Structure of WTC Expansions and Applications, Jour. Physics - A, 28 (2005) 1977-2004. 2. N. Joshi and G. K. Srinivasan,Well-Posedness of Painlevé Expansions, Nonlinearity, 10 (1997) 71-79. 3. G. K. Srinivasan and V. D. Sharma, Modulation Equations for Weakly nonlinear Geometrical Optics in Media Exhibiting Mixed Nonlinearity, Studies in Applied Mathematics, 110 (2003) 103-122. 4. G. K. Srinivasan and V. D. Sharma, A Note on the Jump Conditions For Systems of Conservation Laws, Studies in Applied Mathematics, 110 (2003) 391-396. 5. G. K. Srinivasan and V. D. Sharma, On Weakly Nonlinear Waves in Media Exhibiting Mixed Nonlinearity, Journal of Mathematical Analysis and Applications, 285 (2003) 629-641. 6. G. K. Srinivasan and V. D. Sharma, Energy Dissipated Across Shocks in Weak Solutions of a Scalar conservation Law, Studies in Applied Mathematics, 112 (2004) 281-291. 7. G. K. Srinivasan and V. D. Sharma, Implosion-Time for Converging Spherical and Cylindrical Shells, Zeitschrift für Angewandte Mathematik und Physik, 55 (2004) 974-982. 8. V. D. Sharma and G. K. Srinivasan, Wave Interaction in a Non-equilibrium Flow, International Journal of Non-Linear Mechanics, 40 (2005) 1031-1040. 9. G. K. Srinivasan, The gamma function - an eclectic tour, American Mathematical Monthly 114 (2007) 297-315. 10. G. K. Srinivasan, A Note on Lagrange's Method of Variation of Parameters, Missouri Journal of Mathematical Sciences, 19 (2007) 11-14. 11. G. K. Srinivasan, V. D. Sharma and B. S. Desale, An Integrable System of ODE Reductions of the stratified Boussinesq Equations, Compt. Math. Appl, 53 No. 2,(2007) 296-304. 12. B. S. Desale and G. K. Srinivasan, Singular analysis of the system of ODE reductions of the stratified Boussinesq equations, IAENG International journal of applied mathematics, 38 (2008) no. 4, 184-191. 13. G. K. Srinivasan and P. Zvengrowski, On the horizontal monotonicity of $|\Gamma(s)|$, Canadian Math. Bulletin, 54 (2011), no. 3, 538-543. 14. G. K. Srinivasan, Dedekind's proof of Euler's reflection formula via ODEs, Mathematics Newsletter (published by Ramanujan Mathematical Socity) 21 (2011) no. 3, 82-83. 15. D. Chakrabarty and G. K. Srinivasan, On a remarkable formula of Ramanujan, Arch. Math. (Basel) 99 (2012), 125-135. 16. G. K. Srinivasan, A unified approach to the integrals of Mellin–Barnes–Hecke type, To appear in Expositiones Mathematicae. Ph.D student: 1. Title of Thesis: An Integrable System of ODE Reductions of the stratified Boussinesq Equations. Defended in January 2007. 2> sunil Kumar Yadav. Books I have completed a web-book on Algebraic Topology under the NPTEL Scheme.

The link is http://nptel.iitm.ac.in/courses/111101002/ Proficiency in foreign languages: (i) Have completed four levels of basic German at the prestigious Göethe Institute (Max Muller Bhavan) upto Mittelstuffe - I. (ii) Completed the first two levels of French at Alliance Française de Mumbai. Awards and Recognitions: 1. Award for Excellence in Teaching, Indian Institute of Technology, Bombay (Sept 5, 2011) 2. Award for Excellence in Teaching, Indian Institute of Technology, Bombay (Sept 5, 2007) 3. Award for Excellence in Teaching, Indian Institute of Technology, Bombay (Sept 5, 2002) 4. Citation For Excellence in Teaching, awarded by the Department of Mathematics at the University of Minnesota, Minneapolis (1995). 5. Shri Pandharinath Moroba Mungre Prize, for the year 1989 for passing MSc examination with highest number of marks on the aggregate. 6. The Late Shri Balkrishna Waman Deshpande Prize, for the year 1989 for passing MSc examination with highest number of marks on the aggregate. 7. Principal V. K. Joag Memorial Prize, for the year 1989 for passing MSc examination with highest number of marks on the aggregate. 8. Professor Laxman Vasudeo Gurjar Prize, for the year 1989 for passing MSc examination with highest number of marks on the aggregate. 9. Certificate of Merit for Outstanding Performance and Being Among the Top 5% of Candidates Qualifying in C.S.I.R - U.G.C Fellowship in Mathematics (1988) 2

**COURSE PLAN :**

**Week 1:** Introduction and the basic stage set up. The basic convergence theorem and its applications

**Week 2:** Cesaro summability and Fejer's theorem. Uniform distribution modulo one.

**Week 3:** Mean convergence and its applications. Parseval's theorem and its applications to The isoperimetric theorem. Maximum modulus theorem for holomorphic functions.

**Week 4:** Pointwise convergence of Fourier series. Failure of pointwise convergence for Continuous functions – application of the Banach Steinhaus theorem

**Week 5:** Some rudiments of functional analysis – Review of basic theorems

**Week 6:** Schwartz space of rapidly decreasing functions and its basic properties. Examples. Density in classical Lebesgue spaces.

**Week 7:** Fourier transforms of Schwartz functions. Fourier inversion theorem and some applications. Fourier transforms of absolutely summable functions.

**Week 8:** Plancheral's theorem and Fourier transforms of square summable functions. Fourier transform as a unitary operator.

**Week 9:** The Fourier transform as a bounded linear map L^p --> L^q if $1 \leq p \leq 2$, Riemann Lebesgue lemma, examples and applications.

**Week 10:** Solution of the Heat equation in the half space via Fourier transform, Poisson summation formula and Jacobi theta function identity

**Week 11:** Energy conservation in wave equations, Paley Weiner theorem for Fourier transforms of smooth functions with compact support

**Week 12:** Principle of equipartition of energy, Hermite equation, Eigenfunctions of Fourier transform