



# AN INTRODUCTION TO POINT-SET-TOPOLOGY

## PART-I

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**PRE-REQUISITES :** Anybody who has passed 12 standard can take this course though one course in real analysis available on NPTEL Portal is preferable.

**INTENDED AUDIENCE :** Anybody who has passed 12th standard and had one or two courses in Real Analysis. Students from various streams which include some modern mathematics such as B. Sc., M. Sc., Students, Ph. D. , B. Tech. and M. Tech. who had not attended any topology courses seriously, before this, will be able to benefit from this course.

**COURSE OUTLINE :** The course will start with definition of metric spaces and topological spaces and proceeds to study topological aspects of metric spaces. We prove three very important theorems on complete metric spaces and give construction of completion of metric spaces. The course then takes up the study of topological spaces, constructing new topologies from the old. Bases and subbases, I and II countability, separability, connected and path connectedness, Compactness, Lindelöfness, separation axioms etc. The course ends with the celebrated results such as Urysohn's lemma and Tietze's extension theorem and some applications. The content of this course is mandatory for any meaningful study of analysis and further topology. The lecture slides are backed up by full notes, a strong team of tutors who will handle all the queries sympathetically and also by a number of online interactive session.

**ABOUT INSTRUCTOR :** Prof. Anant R Shastri is a retired Emeritus Fellow of Department of Mathematics I.I. T. Bombay. After serving in School of Mathematics T.I.F.R. for 16 years I joined I.I.T. Bombay as a full professor in 1988. Apart from several research papers, I have published three books. Since 2004, I have constantly involved in the activities of ATM schools, The chief activity of these schools is to impart advanced training in Mathematics to Ph. D. students in various universities and research institutions in the country. These activities were initially funded by NBHM and currently adapted by National Centre for Mathematics, I.I.T. Bombay. Over the years, I have taught roughly the contents of the proposed course more than 20 times to M.Sc/ BTech / MTech and Ph. D. students. CDEEP has recorded my course on Complex Analysis MA205 for B. Tech students. I have offered a course in Algebraic Topology in two parts on NPTEL portal. I strongly believe that especially in these troubled times NPTEL mode of Knowledge-dissemination is the best.

### COURSE PLAN :

**Week 1 : Chapter I - Introduction** - Introduction, Normed linear spaces (NLS), Metric Spaces,  $\epsilon$  - Definition of continuity, Examples of continuous functions, Topological Spaces.

**Week 2 : Chapter I - Introduction** - Examples, Functions, Topology of the n-dim. Euclidean space, Equivalences on metric spaces, Equivalences continued.

**Week 3 : Chapter I - Introduction** - Counter examples, Definitions and examples, Closed sets, Interiors and boundaries, Interiors and derived sets.

**Week 4 : Chapter I - Introduction** - More examples, Metric Trinity, Baire's Category Theorem, An Application in Analysis, Completion of Metric space.

**Week 5 : Chapter II - Creating New Spaces** - Bases and subbases, Subbases, Box Topology, Subspaces, Union of spaces.

**Week 6 : Chapter II - Creating New Spaces** - Extending neighbourhoods, Quotient Spaces, Product of spaces, Study of Products - continued, Induced and co-induced topologies.

**Week 7 : Chapter III- Smallness Properties of Topological Spaces** - Path Connectivity, Connectivity, Connected components, Connectedness-continued, Local Connectivity, More Examples.

**Week 8 : Chapter III- Smallness Properties of Topological Spaces** - Compactness and Lindelöfness, Compact Metric Spaces, Compactness-continued, Countability and Separability, Types of Topological Properties.

**Week 9 : Chapter III- Smallness Properties of Topological Spaces** - Productive Properties, Productive Properties-continued, Tychonoff Theorem, Proof Alexander's Subbase Theorem.

**Week 10 : Chapter IV - Largeness properties** - Fréchet Spaces, Hausdorff spaces, Examples and Applications, Examples and Applications - continued.

**Week 11 : Chapter IV - Largeness properties** - Regularity and Normality, Characterization of Normality, Tietze's Characterization of Normal Spaces, Productiveness of Separation Axioms, The Hierarchy.

**Week 12 : Chapter V - Topological groups and Topological Vector Spaces** - Topological Groups, Topological Groups-continued, Topological Groups-continued, Topological Vector Spaces, Topological Vector Spaces-continued, Topological Vector Spaces-continued.