GALOIS THEORY

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PRE-REQUISITES : Linear Algebra ; Algebra – First Course

INTENDED AUDIENCE : BS / BSc / BE / ME / MSc / PhD

INDUSTRIES APPLICABLE TO : R & D Departments of IBM / Microsoft Research Labs SAP /TCS / Wipro/ Infosys

COURSE OUTLINE :

Galois Theory is showpiece of a mathematical unification which brings together several differentbranches of the subject and creating a powerful machine for the study problems of considerablehistorical and mathematical importance. This course is an attempt to present the theory in such alight, and in a manner suitable for undergraduate and graduate students as well as researchers.This course will begin at the beginning. The quadratic formula for solving polynomials of degree2 has been known for centuries and is still an important part of mathematics education. Thecorresponding formulas for solving polynomials of degrees 3 and 4 are less familiar. Theseexpressions are more complicated than their quadratic counterpart, but the fact that they exist comesas no surprise. It is therefore altogether unexpected that no such formulas are available for solvingpolynomials of degree ≥ 5. A complete answer to this intriguing problem is provided by Galoistheory. In fact Galois theory was created precisely to address this and related questions aboutpolynomials. This feature might not be apparent from a survey of current textbooks on universitylevel algebra.This course develops Galois theory from historical perspective and I have taken opportunity to weavehistorical comments into lectures where appropriate. It provides a platform for the developmentof classical as well as modern core curriculum of Galois theory. Classical results by Abel, Gauss,Kronecker, Legrange, Ruffini and Galois are presented and motivation leading to a modern treatmentof Galois theory. The celebrated criterion due to Galois for the solvability of polynomials by radicals.The power of Galois theory as both a theoretical and computational tool is illustrated by a study ofthe solvability of polynomials of prime degree.The participant is expected to have a basic knowledge of linear algebra, but other that the course islargely self-contained. Most of what is needed from fields and elementary theory polynomials ispresented in the early lectures and much of the necessary group theory is also presented on the way.Classical notions, statements and their proofs are provided in modern set-up. Numerous examplesare given to illustrate abstract notions. These examples are sort of an airport beacon, shining aclear light at our destination as we navigate a course through the mathematical skies to get there.Formally we cover the following topics :Galois extensions and Fundamental theorem of Galois Theory.Finite Fields, Cyclic Groups, Roots of Unity, Cyclotomic Fields.Splitting fields, Algebraic closureNormal and Separable extensionsSolvability of equations.Inverse Galois Problem

ABOUT INSTRUCTOR :

Prof. Dilip P. Patil received B. Sc. and M. Sc. in Mathematics from the University of Pune in 1976 and 1978, respectively. From 1979 till 1992 he studied Mathematics at School of Mathematics, Tata Institute of Fundamental Research, Bombay and received Ph. D. through University of Bombay in 1989. Currently he is a Professor of Mathematics at the Departments of Mathematics, Indian Institute of Science, Bangalore. At present he is a Visiting Professor at the Department of Mathematics, IIT Bombay. He has been a Visiting Professor at Ruhr-Universität Bochum, Universität Leipzig, Germany and several universities in Europe and Canada. His research interests are mainly in Commutative Algebra and Algebraic Geometry

COURSE PLAN :

Week 1 : Prime Factorisation in Polynomial Rings, Gauss's Theorem
Week 2 : Algebraic Extensions
Week 3 : Group Actions
Week 4 : Galois Extensions
Week 5 : Finite Fields, Cyclic Groups, Roots of Unity, Cyclotomic Fields
Week 6 : Splitting Fields, Algebraic Closure
Week 7 : Normal and Separable Extensions
Week 8 : Norms and Trace
Week 9 : Fundamental Theorem on Symmetric
Week 10 : Proof of the Fundamental Theorem Polynomial, of Algebra
Week 11 : Orbits of the action of Galois group
Week 12 : Inverse Galois Problem