MEASURE THEORY

PROF. INDER KUMAR RANA
Department of Mathematics
IIT Bombay

PRE-REQUISITES : Basic Real Analysis

INTENDED AUDIENCE : B.Tech Dual degree in Electrical, M.Sc. Physics, mathematics

COURSE OUTLINE :
This is a course on the concepts of Measure and Integration. Normally, this is a core course for M,.Sc. Mathematics and Statistics students. The concepts find applications in advance Analysis Courses, Signal Processing, Financial Mathematics courses.

ABOUT INSTRUCTOR :
Prof. Inder K. Rana presently is an Emeritus Fellow at Department of mathematics, IIT Bombay. He has an experience of 36 years of teaching mathematics courses to undergraduate (B. Tech) and master's M.Sc. students at IIT Bombay. He has authored 4 books,namely,"Introduction to measure and Integration" American Mathematical Society, Graduate Studies in Mathematics Volume 45, 2000,"From Numbers to Analysis" World Scientific Press, 1998 ,Calculus @IITB: Concepts and Examples, math4all, India, 2007 "From Geometry to Algebra: A course in Linear Algebra" math4all, India, 2007.He has won three awards,"C. L. Chandna Mathematics Award" for the year 2000 in recognition of distinguished and outstanding contributions to mathematics research and teaching. The award is given by 'SaraswatiVishvas Canada","Excellence in Teaching" award for the year 2004 Awarded by IIT Bombay, based on the evaluations by students."Aryabhata Award" 2012 All India Ramanujan Math Club, India, for teaching and work towards math education in India.

COURSE PLAN :

Week 1
Lecture 1A Introduction, Extended Real Numbers
Lecture 1B Introduction, Extended Real Numbers
Lecture 2A Algebra and Sigma Algebra of Subsets of a Set
Lecture 2B Algebra and Sigma Algebra of Subsets of a Set
Lecture 3A Sigma Algebra generated by a Class
Lecture 3B Sigma Algebra generated by a Class
Week 2
Lecture 4A Monotone Class
Lecture 4B Monotone Class
Lecture 5A Set Functions
Lecture 5B Set Functions
Lecture 6A The Length Function and its Properties
Lecture 6B The Length Function and its Properties
Week 3
Lecture 7A Countably Additive Set Functions on Intervals
Lecture 7B Countably Additive Set Functions on Intervals
Lecture 8A Uniqueness Problem for Measure
Lecture 8B Uniqueness Problem for Measure
Week 4
Lecture 9A Extension of Measure
Lecture 9B Extension of Measure
Lecture 10A Outer Measure and its Properties
Lecture 10B Outer Measure and its Properties
Lecture 11A Measurable Sets
Lecture 11B Measurable Sets
Week 5
Lecture 12A Lebesgue Measure and its Properties
Lecture 12B Lebesgue Measure and its Properties
Lecture 13A Characterization of Lebesgue Measurable Sets
Lecture 13B Characterization of Lebesgue Measurable Sets

Week 6
Lecture 14A Measurable Functions
Lecture 14B Measurable Functions
Lecture 15A Properties of Measurable Functions
Lecture 15B Properties of Measurable Functions
Lecture 16A Measurable Functions on Measure Spaces
Lecture 16B Measurable Functions on Measure Spaces
Week 7
Lecture 17A Integral of Nonnegative Simple Measurable Functions
Lecture 17B Integral of Nonnegative Simple Measurable Functions
Lecture 18A Properties of Nonnegative Simple Measurable Functions
Lecture 18B Properties of Nonnegative Simple Measurable Functions
Lecture 19A Monotone Convergence Theorem and Fatou's Lemma
Lecture 19B Monotone Convergence Theorem and Fatou's Lemma
Week 8
Lecture 20A Properties of Integrable Functions and Dominated Convergence Theorem
Lecture 20B Properties of Integrable Functions and Dominated Convergence Theorem
Lecture 21A Dominated Convergence Theorem and Applications
Lecture 21B Dominated Convergence Theorem and Applications
Week 9
Lecture 22A Lebesgue Integral and its Properties
Lecture 22B Lebesgue Integral and its Properties
Lecture 23A Product Measure, an Introduction
Lecture 23B Product Measure, an Introduction
Lecture 24A Construction of Product Measures
Lecture 24B Construction of Product Measures
Week 10
Lecture 25A Computation of Product Measure - I
Lecture 25B Computation of Product Measure - I
Lecture 26A Computation of Product Measure - II
Lecture 26B Computation of Product Measure - II
Week 11
Lecture 27A Integration on Product Spaces
Lecture 27B Integration on Product Spaces
Lecture 28A Fubini's Theorems
Lecture 28B Fubini's Theorems
Week 12
Lecture 29A Lebesgue Measure and Integral on R2
Lecture 29B Lebesgue Measure and Integral on R2
Lecture 30A Properties of Lebesgue Measure on R2
Lecture 30B Properties of Lebesgue Measure on R2
Lecture 31A Lebesgue Integral on R2
Lecture 31B Lebesgue Integral on R2