

Bayesian Cournot  
Game.

Market competition between  
2 firms  $F_1, F_2$ .

Firm 1 has a production cost of  $C$  per unit.

Firm 2 of types:  
Low  $\frac{1}{2}C$   $P(\text{low}) = \frac{1}{2}$   
High  $C$   $P(\text{high}) = \frac{1}{2}$

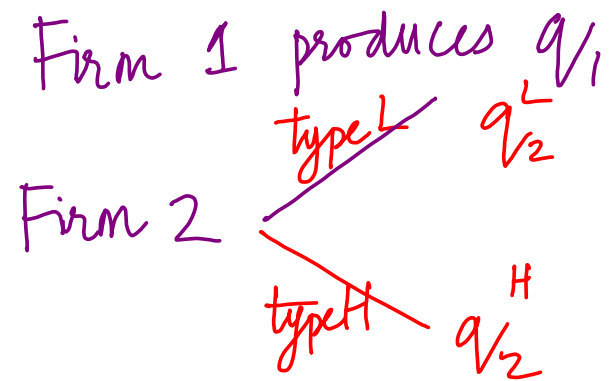
Inverse demand curve.

$$\text{price } p = (a - (q_1 + q_2))$$

quantity produced by Firm 1      quantity produced by Firm 2

Payoff to each firm  $j$

$$= \text{price} \times \text{quantity} - \text{cost of production}$$
$$= (a - (q_1 + q_2)) q_j - c_j q_j$$



$$\begin{aligned}
 & \text{Payoff to Firm 2 of type L} \\
 &= (a - (q_1 + q_2^L)) q_2^L - \frac{1}{2} C q_2^L \\
 &= a q_2^L - q_1 q_2^L - (q_2^L)^2 - \frac{1}{2} C q_2^L \\
 & a - q_1 - 2 q_2^L - \frac{1}{2} C = 0
 \end{aligned}$$

$$\begin{aligned}
 a - q_1 - 2 q_2^L - \frac{1}{2} C &= 0 \\
 (q_2^L)^* &= \frac{a - \frac{1}{2} C - q_1}{2}
 \end{aligned}$$

Best response of Firm 2 of type L

Payoff to firm 2 of type H is

$$\begin{aligned} & (a - (q_1 + q_2^H)) q_2^H - C q_2^H \\ &= a q_2^H - q_1 q_2^H - (q_2^H)^2 - C q_2^H \end{aligned}$$

$$a - q_1 - 2q_2^H - C = 0$$

$$a - q_1 - 2q_2^H - C = 0$$

$$(q_2^H)^* = \frac{a - q_1 - C}{2}$$

Best response of Firm 2 of type H.

Payoff of firm 1, corresponding to type low of firm 2 is

$$(a - (q_1 + q_2^L))q_1 - Cq_1$$

Payoff to firm 1 corresponding to firm 2 of type H is,

$$(a - (q_1 + q_2^H))q_1 - Cq_1$$

average payoff of firm 1 is

$$= \frac{1}{2} \left( (a - (q_1 + q_2^L)) q_1 - c q_1 \right)$$

$$+ \frac{1}{2} \left( (a - (q_1 + q_2^H)) q_1 - c q_1 \right)$$

↑ Differentiate wrto  $q_1$  and  
set equal to 0 to find  
the best response  $q_1$ .

$$\frac{1}{2} \left( a - 2q_1 - q_2^L - c \right) + \frac{1}{2} \left( a - 2q_1 - q_2^H - c \right) = 0$$

$$2q_1^* = \frac{1}{2} \left( a - c - q_2^L \right) + \frac{1}{2} \left( a - c - q_2^H \right)$$

Best response  
of Firm 1

$$q_1^* = \frac{a-c}{2} - \frac{1}{4} \left( q_2^L + q_2^H \right)$$

$$(q_2^L)^* = \frac{a - \frac{1}{2}c - q_1^*}{2}$$

$$(q_2^H)^* = \frac{a - q_1^* - c}{2}$$



$$\begin{aligned}
 q_1^* &= \frac{1}{2}(a-c) - \frac{1}{4} \left( (q_2^L)^* + (q_2^H)^* \right) \\
 &= \frac{1}{2}(a-c) - \frac{1}{4} \left\{ \frac{a - \frac{1}{2}c - q_1^*}{2} + \frac{a - c - q_1^*}{2} \right\}
 \end{aligned}$$

equation for  $q_1^*$

$$q_1^* = \frac{a - 5c/4}{3}$$

Best response quantity of Firm 1.

$$\begin{aligned}
 (q_2^L)^* &= \frac{a - \frac{1}{2}c - q_1^*}{2} \\
 &= \frac{a - \frac{1}{2}c - \frac{1}{3}\left(a - \frac{5c}{4}\right)}{2} \\
 &= \frac{a}{3} - \frac{c}{24}
 \end{aligned}$$

$$\begin{aligned}
 (q_2^H)^* &= \frac{a - c - q_1^*}{2} \\
 &= \frac{a - c - \frac{1}{3}\left(a - \frac{5c}{4}\right)}{2} \\
 &= \frac{a}{3} - \frac{7c}{24}.
 \end{aligned}$$

Bayesian NE of the Bayesian Cournot Game is,

$$\left( \frac{a - \frac{5c}{4}}{3}, \left( \frac{a - \frac{c}{24}}{3}, \frac{a - \frac{7c}{24}}{3} \right) \right)$$

quantity of Firm 1      quantity of Firm 2 of type L      quantity of Firm 2 of type H