Stackelberg Model

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Market Structure and Oligopoly

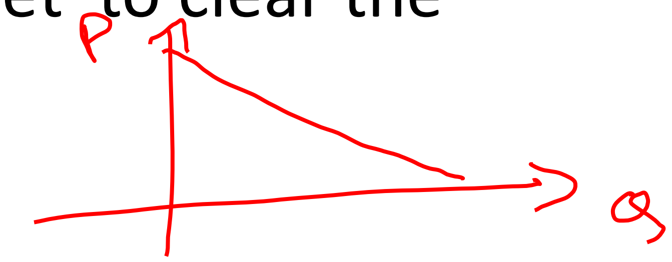
- Markets differ along following criterion
  - number of firms
  - barrier of entry and exit
  - ability of firms to differentiate their products
- Oligopoly
  - small number of firms in a market with relatively high barriers to entry
- because relatively few firms compete in an oligopoly,
  - each firm faces a downward-sloping demand curve
  - each firm can set its price: $p > MC$
  - each affects rival firms

Duopoly as a Special Case of Oligopoly

- Basic Duopoly Model
  - Only 2 firms (no other firms can enter)
  - firms sell identical products
  - market that lasts only 1 period (product or service cannot be stored and sold later)
- Two Types
  - firms choose quantities: ~~Cournot model~~, Stackelbergmodel
  - firms set prices: ~~Bertrand model~~

Stackelberg model

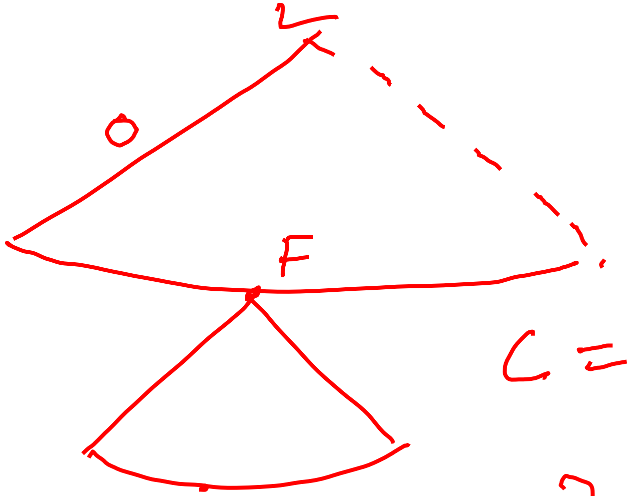
- Cournot model: both firms make their output decisions simultaneously
- Stackelberg'smodel: firms act sequentially
  - A firm sets its output first [Leader]
  - then its rival sets its output [Follower]
  - Once the two quantities are chosen, price is set to clear the market. For example, take $P = a - b(q_L + q_F)$



How do we solve this game?

$$\frac{\partial \pi_F}{\partial q_F}$$

- Work backwards -- use backward induction
- Start at the last step: $P = a - b(q_L + q_F)$, setting price to clear the market
- Next step before that -- follower chooses quantity to maximize profit given leader's choice.
- $\pi_F = (a - b(q_L + q_F) - c)\, q_F$
- Take derivative and set = 0 to get BR
- $a - bq_L - 2bq_F - c = 0$
- $q_F^* = (a - bq_L - c)/2b$



$$C = c\, q_F$$

$$R = [a - b(q_L + q_F)]\, q_F$$

- Now go the first step -- leader chooses quantity to maximize profit
- $\pi_L = (a - b(q_L + q_F) - c)\, q_L$
- However, leader knows how follower will respond -- leader can figure out follower's BR, so:
- $\pi_L = (a - b(q_L + (a - bq_L - c)/2b) - c)\, q_L$
- Simplify to get $\pi_L = (a - bq_L - c)/2\, q_L$
- Take derivative and set equal to 0 to get BR:

  $(a - 2bq_L - c)/2 = 0 \rightarrow q_L = (a - c)/2b$

- And $q_F^* = (a - bq_L - c)/2b = (a - b(a-c)/2b - c)/2b = (a-c)/4b$

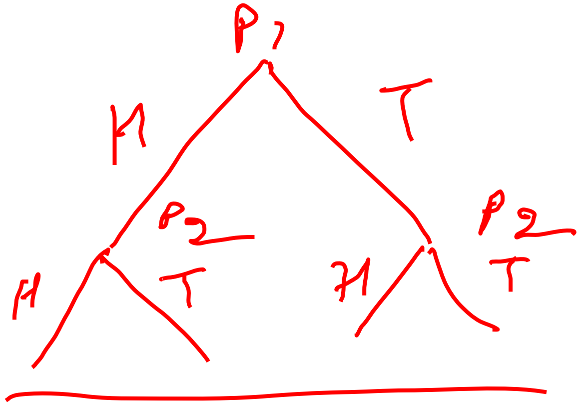
$\pi_L = TR - TC$

$= (a - b(Q_L + Q_F))Q_L - CQ_L$

$\frac{a-c}{2b}, \frac{a - bQ_L - c}{2b}$

$\frac{a-c}{2b}, \frac{a-c}{4b}$

- Leader has the advantage -- he sets higher quantity and gets a higher profit than the follower
- Often called the “first-mover” advantage
- Total output = (a-c)/2b + (a-c)/4b = 3(a-c)/4b
- Greater than total Cournot output of 2(a-c)/3b



Depicting the Stackelberg outcome