'Tragedy of Commons'

$$u_1(e_1, e_2) = e_1(1-(e_1+e_2))$$

$$u_2(e_2, e_1) = e_2(1-(e_1+e_2))$$

$$e_1^* = e_2^* = 1/3$$

$$u_1(e_1^*, e_2^*) = u_2(e_2^*, e_1^*) = \frac{1}{9}.$$

Is there another outcome which yields a higher payoff to both the players?

$$\underline{u_1(e_1, e_2) + u_2(e_2, e_1)}$$

Joint payoff

$$= e_1(1-(e_1+e_2)) + e_2(1-(e_1+e_2))$$

$$= (e_1+e_2)(1-(e_1+e_2))$$

Net payoff or
Total Payoff

$$= \underline{(e_1+e_2)}\left(1-\underline{(e_1+e_2)}\right)$$

$$e_1+e_2 = e_t$$

$$= e_t(1-e_t) = u_t(e_t)$$

$\underbrace{e_1+e_2}_{\text{total effort}}$

total payoff

$$u_t(e_t) = e_t(1-e_t)$$

$$= e_t - e_t^2$$

To maximize sum payoff, differentiate u_t wrto e_t

$$\frac{du_t}{de_t} = \frac{d}{de_t}(e_t - e_t^2)$$

$$= 1 - 2e_t = 0$$

$$\boxed{e_t^* = \frac{1}{2}}$$

$$e_t^* = e_1 + e_2 = \frac{1}{2}$$

$$e_1 = \frac{1}{4} \quad e_2 = \frac{1}{4}$$

$$u_1(e_1, e_2) = u_1\left(\frac{1}{4}, \frac{1}{4}\right)$$

$$= \frac{1}{4}\left(1 - \left(\frac{1}{4} + \frac{1}{4}\right)\right)$$

$$= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} > \frac{1}{9} = u_1(e_1^*, e_2^*)$$

$$u_2\left(\frac{1}{4}, \frac{1}{4}\right) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$$e_1 = e_2 = \frac{1}{4}$$

Therefore, the Nash Equilibrium is NOT Pareto optimal!

Similar to PD.

At NE, each is using an effort $\underline{e_1^* = e_2^* = \frac{1}{3}}$ which is greater than the optimal effort $e_1 = e_2 = \frac{1}{4}$

NE leads to faster depletion of common Resource: — "Tragedy of commons".

Solution - Regulatory Framework.