'COURNOT DUOPOLY' 1838

competition between 2 firms.

Market - Strategic Interaction

2 Firms : producing 2 goods which are closely related

'Strategic Substitutes'

Quantities s_1, s_2

actions of the two firms,

Let the price function

$$\underbrace{p(s_1, s_2)}_{\text{price per unit}} = \overline{A - B(s_1 + s_2)}$$

constants

price is decreasing with quantity - "inverse Demand function"

Let the cost per unit be given by C,

Total cost of $F_1 = C s_1$

Total cost for $F_2 = C s_2$

$$U_1(S_1, S_2) = \text{Total Revenue} - \text{Total cost}$$

$$= \text{price/unit} \times \text{qty} - \text{Total cost}$$

$$= (A - B(S_1 + S_2))S_1 - CS_1$$

$$= (A - C - B(S_1 + S_2))S_1$$

$$U_1(S_1, S_2) = S_1(A - C - B(S_1 + S_2))$$

$$U_2(S_2, S_1) = S_2(A - C - B(S_1 + S_2))$$

To find best response s_1^* of Firm 1, Differentiate wrto s_1 and set equal to 0.

$$u_1(s_1, s_2) = s_1(A - C - B(s_1 + s_2))$$

$$= As_1 - Cs_1 - Bs_1^2 - Bs_1 s_2$$

$$\frac{\partial u_1}{\partial s_1} = A - C - 2Bs_1 - Bs_2 = 0$$

$$s_1^* = \frac{A - C - Bs_2}{2B}$$

Best Responses

$$s_1^* = \frac{A - C - Bs_2}{2B} \rightarrow BR_1(s_2)$$

$$s_2^* = \frac{A - C - Bs_1}{2B} \rightarrow BR_2(s_1)$$

NE — Best Responses intersect

$$s_1^* = BR_1(s_2^*)$$

$$s_2^* = BR_2(s_1^*)$$

$$s_1^* = \frac{A - C - 2Bs_2^*}{2B} \quad (1)$$

$$s_2^* = \frac{A - C - 2Bs_1^*}{2B} \quad (2)$$

only at NE

$$s_1^* = \frac{A-C}{2B} - \frac{1}{2} s_2^*$$

$$= \frac{A-C}{2B} - \frac{1}{2}\left(\frac{A-C}{2B} - \frac{1}{2} s_1^*\right)$$

$$\frac{3}{4} s_1^* = \frac{A-C}{4B}$$

$$\boxed{s_1^* = \frac{A-C}{3B}}$$

$$s_1^* = \frac{A - C}{3B}$$

$$s_2^* = \frac{A - C}{3B}$$

Nash Equilibrium Quantities.

$$NE = \left(\frac{A - C}{3B}, \frac{A - C}{3B} \right)$$

Cournot Duopoly

s_1^* s_2^*

$$A = 10 \quad B = C = 1$$

$$s_1^* = \frac{A - C - Bs_2}{2B}$$

$$= 4.5 - \frac{1}{2} s_2$$

$$s_2^* = 4.5 - \frac{1}{2} s_1$$