

We have assumed that each player knows the payoffs of other players.

In several games, the payoffs of other players are NOT known.  
— For example: Auction

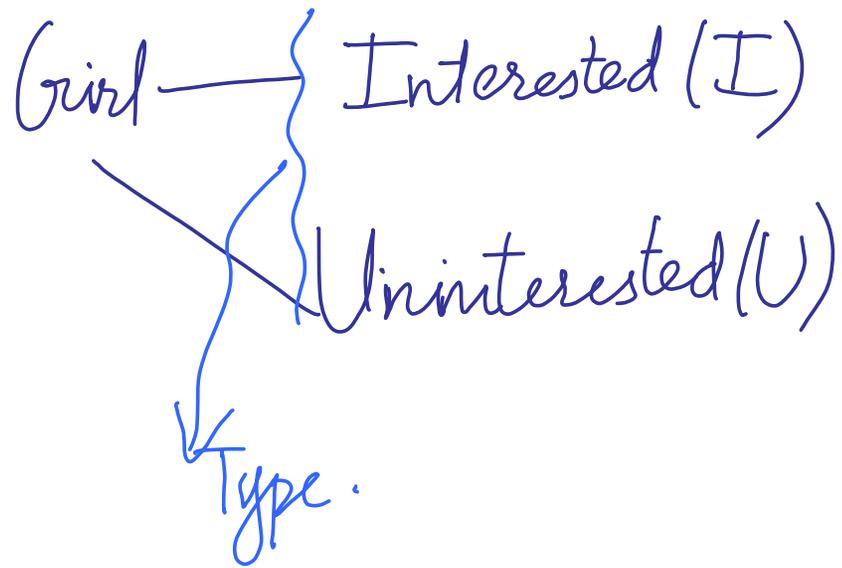
Several real life scenarios,  
There is 'UNCERTAINTY'  
regarding the payoffs of  
other players.

These games in which  
there is uncertainty  
regarding payoffs of others  
are known as Bayesian  
Games.

Simple Bayesian Game  
Example:

'Bayesian' Battle of  
sexes.

BoS — Game in  
which  $P_1$  (Boy) and  $P_2$  (Girl)  
can either choose C or H.



There are two types  
of Girl Player  $P_2$ .

I interested  
U uninterested.

$$P(I) = \frac{1}{2} \left. \begin{array}{l} \text{Probability} \\ \text{girl is interested} \end{array} \right\}$$

$$P(U) = \frac{1}{2} \left. \begin{array}{l} \text{Probability} \\ \text{girl is} \\ \text{uninterested.} \end{array} \right\}$$

Girl of type I, is interested  
in watching C or H  
with boy.

		Girl	
		C	H
Boy	C	10, 5	0, 0
	H	0, 0	5, 10

Each prefers watching C or H with the other.

		Girl	
		C	H
Boy	C	10, 0	0, 10
	H	0, 5	5, 0

Corresponding to girl of type U.  
 Girl prefers to watch C or H alone,  
 while Boy prefers to watch C or H together.

## Bayesian BOS:

	C	H
C	10, 5	0, 0
H	0, 0	5, 10

$P(I) = \frac{1}{2}$

	C	H
C	10, 0	0, 10
H	0, 5	5, 0

$P(U) = \frac{1}{2}$

Girl is column player  
Boy is row player.  
Uncertainty regarding  
payoffs of player 2 or girl

Payoffs depend of if  
girl is of type I or  
Type U.

Game is Bayesian  
because boy i.e.  $P_1$  is  
'uncertain' regarding type  
and hence payoffs of girl  
i.e.  $P_2$ .

Bayesian Game:  
↓ Assign a strategy to each player of each type.

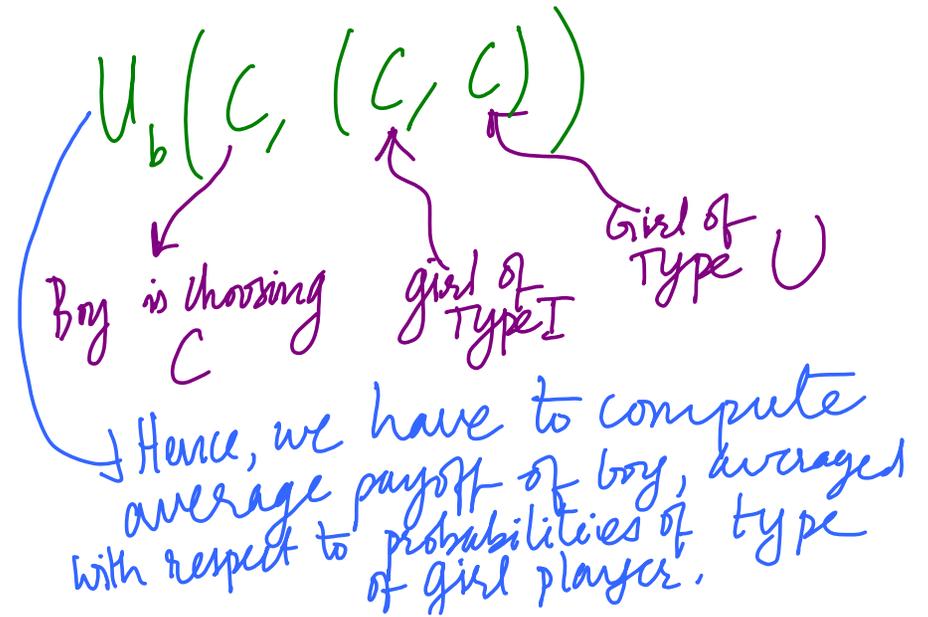
Let us consider boy choosing C.  
Let us consider girl of type I choosing C.  
↓ Let girl of type V also choose C.

Strategy of girl

$(C, C)$

action of  
girl of type I

action of  
girl of type U.



$$U_b(C, (C, C))$$

$$= P(I) \times U_b(C, C)$$

$$+ P(U) \times U_b(C, C)$$

$$= \frac{1}{2} \times 10 + \frac{1}{2} \times 10$$

$$= 10 = U_b(C, (C, C))$$

$U_b(H, (C, C))$   
Boy is choosing H  
Girl of Both types  
is choosing C.

$$\begin{aligned} &= P(I) \times U_b(H, C) \\ &\quad + P(U) \times U_b(H, C) \\ &= \frac{1}{2} \times 0 + \frac{1}{2} \times 0 = 0 \\ &= U_b(H, (C, C)) \end{aligned}$$

$$U_b(C, (C, H))$$

Boy chooses C      girl of type I chooses C  
 girl of type U chooses H.

$$= P(I) \times U_b(C, C) + P(U) \times U_b(C, H)$$

$$= \frac{1}{2} \times 10 + \frac{1}{2} \times 0 = 5$$

$$U_b(C, (C, H)) = 5$$

$$U_b(H, (C, H))$$

$$= \frac{1}{2} \times 0 + \frac{1}{2} \times 5$$

$$= \frac{5}{2}$$

Similarly we can also consider other strategy choices for the girl.

(H, C).  
(H, H)

## Average Payoff Table for Boy:

	(C, C)	(C, H)	(H, C)	(H, H)
C	10	5	5	0
H	0	$5/2$	$5/2$	5

Possible Actions of Boy

Possible strategy combinations for girl.

## Bayesian best response:

Payoff averaged with respect to probabilities of various types of other players.

Deduce best response  
of player of each type.

Bayesian Nash Equilibrium  
(BNE)

$(C, (C, C))$  is this  
 a BNE?  
 Boy is choosing  
 $C$   
 girl of type I  
 girl of type U.

$(C, (C, C))$  However  $C$  is NOT  
 Best response of  
 girl of type U.  
 $C$  is BR of Boy  
 $C$  is BR of  
 girl of type I  
 Therefore,  $(C, (C, C))$  is NOT BNE  
 since girl of type U is NOT playing  
 her BR.

$(C, (C, C))$  is NOT  
a Bayesian Nash  
Equilibrium.

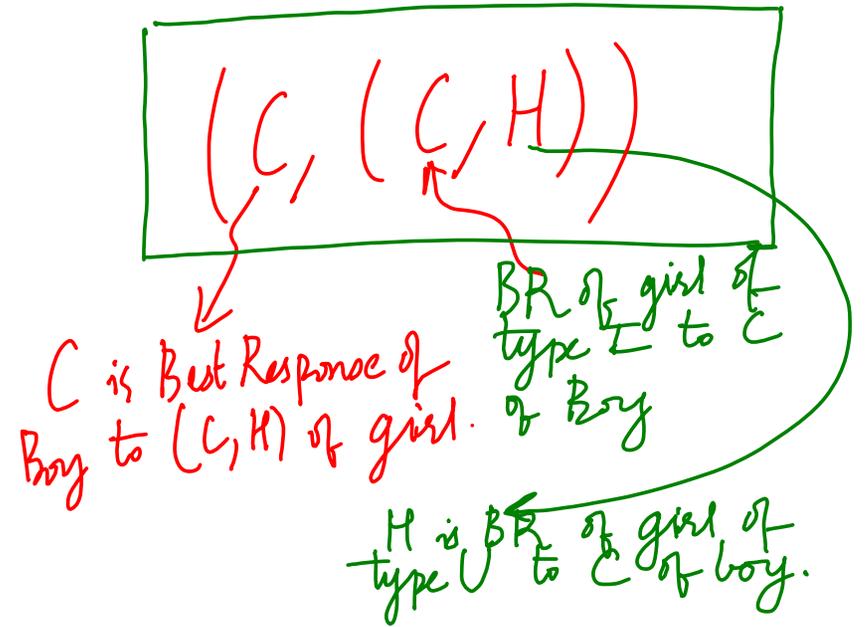
$(C, (C, H))$  is BNE?  
Bay is choosing C  
Grist of type I  
Grist of type U

$(C, H)$  of Girl is best response  
of girl or  $P_2$  of each type  
to  $C$  of Boy.

$(C, (C, H))$

$C$  is BR of girl of type I  
 $H$  is BR of girl of type U.  
Therefore girl of each type is  
playing her best response.

Therefore,  $(C, (C, H))$  is  
a Bayesian Nash Equilibrium.



is  $(C, (H, C))$  BNE?

Girl of Type I is NOT  
playing BR to C of Boy.  
Therefore, NOT a Bayesian  
Nash Equilibrium.

Is  $(H, (H, H))$  is BNE?

Is NOT Best Response  
of Girl of type U to H  
of Boy.  
NOT Bayesian Nash Equilibrium.

$(C, (C, H))$  is the  
Bayesian Nash Equilibrium  
for Bayesian BOS.

