We have assumed that each player knows the payoffs of other players.

In general games, the payoffs of other players are NOT known.

— For example: Auction

Several real life scenarios, There is 'UNCERTAINTY' regarding the payoffs of other players.

These games in which there is uncertainty regarding payoffs of others are known as 'Bayesian Games'.

Simple Bayesian Game Example:

'Bayesian' Battle of Sexes.

BoS — Game in which P_1 (Boy) and P_2 (Girl) can either choose C or H.

Girl — Interested (I)

Girl — Uninterested (U)

Type.

There are two types of Girl Player P_2.

I interested

U uninterested.

$P(I) = \frac{1}{2}$ } Probability girl is interested

$P(U) = \frac{1}{2}$ } Probability girl is uninterested.

Girl of type I, is interested in watching C or H with boy.

Girl

Boy	C	H
C	10, 5	0, 0
H	0, 0	5, 10

Each prefers watching C or H with the other.

Girl

Boy	C	H
C	10, 0	0, 10
H	0, 5	5, 0

Corresponding to girl of type U.

girl prefers to watch C or H alone, while Boy prefers to watch C or H together.

Bayesian BoS:

	C	H
C	$10, \boxed{5}$	$0, 0$
H	$0, 0$	$5, \boxed{10}$

$P(I) = \frac{1}{2}$

	C	H
C	$10, 0$	$0, \boxed{10}$
H	$0, \boxed{5}$	$5, 0$

$P(U) = \frac{1}{2}$

Girl is column player
Boy is row player.

Uncertainty regarding payoffs of player 2 or Girl

Payoffs depend of if girl is of type I or Type U.

Game is Bayesian because boy i.e P_1 is 'uncertain' regarding type and hence payoffs of girl i.e P_2.

Bayesian Game:

Assign a strategy to each player of each type.

Let us consider boy choosing C.

Let us consider girl of type I choosing C.

Let girl of type U also choose C.

Strategy of girl

(C, C)

action of girl of type I

action of girl of type U.

$U_b(C, (C, C))$

Boy is choosing C

girl of Type I

Girl of Type U

Hence, we have to compute average payoff of boy, averaged with respect to probabilities of type of girl player.

$$U_b(C, (C, C))$$

$$= P(I) \times U_b(C, C)$$

$$+ P(U) \times U_b(C, C)$$

$$= \frac{1}{2} \times 10 + \frac{1}{2} \times 10$$

$$= 10 = U_b(C, (C, C))$$

$U_b(H, (C, C))$

Boy is choosing H

Girl of Both types is choosing C.

$$= P(I) \times U_b(H, C) + P(U) \times U_b(H, C)$$

$$= \frac{1}{2} \times 0 + \frac{1}{2} \times 0 = 0$$

$$= U_b(H, (C, C))$$

$U_b(C, (C, H))$

Boy chooses C

girl of type I chooses C

girl of type U chooses H.

$= P(I) \times U_b(C, C) + P(U) \times U_b(C, H)$

$= \frac{1}{2} \times 10 + \frac{1}{2} \times 0 = 5$

$U_b(C, (C, H)) = 5$

$$U_b(H,(C,H))$$

$$= \frac{1}{2} \times 0 + \frac{1}{2} \times 5$$

$$= \frac{5}{2}.$$

Similarly we can also consider other strategy choices for the girl.

- $(H, C).$
- (H, H)

Average Payoff Table for Boy :

	(C, C)	(C, H)	(H, C)	(H, H)
C	$\boxed{10}$	$\boxed{5}$	$\boxed{5}$	0
H	0	5/2	5/2	$\boxed{5}$

Possible Actions of Boy

Possible strategy combinations for girl.

Bayesian best response :

Payoff averaged with respect to probabilities of various types of other players.

Deduce best response of player of each type.

Bayesian Nash Equilibrium

(BNE)

$(C, (C, C))$ is this a BNE?

Boy is choosing C

girl of type I

girl of type U.

$(C, (C, C))$

C is BR of Boy

C is BR of girl of type I

However C is NOT Best response of girl of type U!

Therefore, $(C, (C, C))$ is NOT BNE since girl of type U is NOT playing her BR.

$(C, (C, C))$ is NOT a Bayesian Nash Equilibrium.

$(C, (C, H))$ is BNE?

Boy is choosing C

Girl of type I

Girl of type U

$((C, H))$ of Girl is best response of girl or P_2 of each type to C of Boy.

$$\left(C, (C, H) \right)$$

C is BR of girl of Type I

H is BR of girl of Type U.

Therefore girl of each type is playing her best response.

Therefore, $(C, (C, H))$ is a Bayesian Nash Equilibrium.

$$\boxed{(C, (C, H))}$$

C is Best Response of Boy to (C, H) of girl.

BR of girl of type I to C of Boy

H is BR of girl of type U to C of boy.

is $((C, (H, C)))$ BNE?

Girl of Type I is NOT playing BR to C of Boy.
Therefore, NOT a Bayesian Nash Equilibrium.

Is $(H, (H, \boxed{H}))$ is BNE?

Is NOT Best Response of Girl of type U to H of Boy.

NOT Bayesian Nash Equilibrium.

$(C, (C, H))$ is the Bayesian Nash Equilibrium for Bayesian BoS.