Battle of Sexes:

Best response Dynamic

		q	$1-q$
	B \ G	C	H
p	C	10, 5	0, 0
$1-p$	H	0, 0	5, 10

Payoff of girl if she chooses C is,

$$U_G(C) = 5p + 0(1-p)$$
$$= 5p$$

Similarly, payoff of the girl if she always chooses H,

$$U_G(H) = 0p + 10(1-p)$$
$$= 10(1-p)$$

if $U_G(C) > U_G(H)$

$$5p > 10(1-p)$$

$$\Rightarrow 15p > 10$$

$$\Rightarrow \boxed{p > 2/3}$$

if $p > 2/3$

then girl is always watching cricket

Best response $q^* = 1$

if $U_G(C) < U_G(H)$

$$5p < 10(1-p)$$

$$\Rightarrow 15p < 10$$

$$\Rightarrow \boxed{p < 2/3}$$

If $p < \frac{2}{3}$, then girl chooses to always watch H, therefore best response

$$q^* = 0$$

when $U_G(C) = U_G(H)$

$$\Rightarrow 5p = 10(1-p)$$

$$\Rightarrow 15p = 10$$

$$\Rightarrow \boxed{p = 2/3}$$

at $p = 2/3$

$$U_G(C) = 5 \times 2/3 = 10/3$$

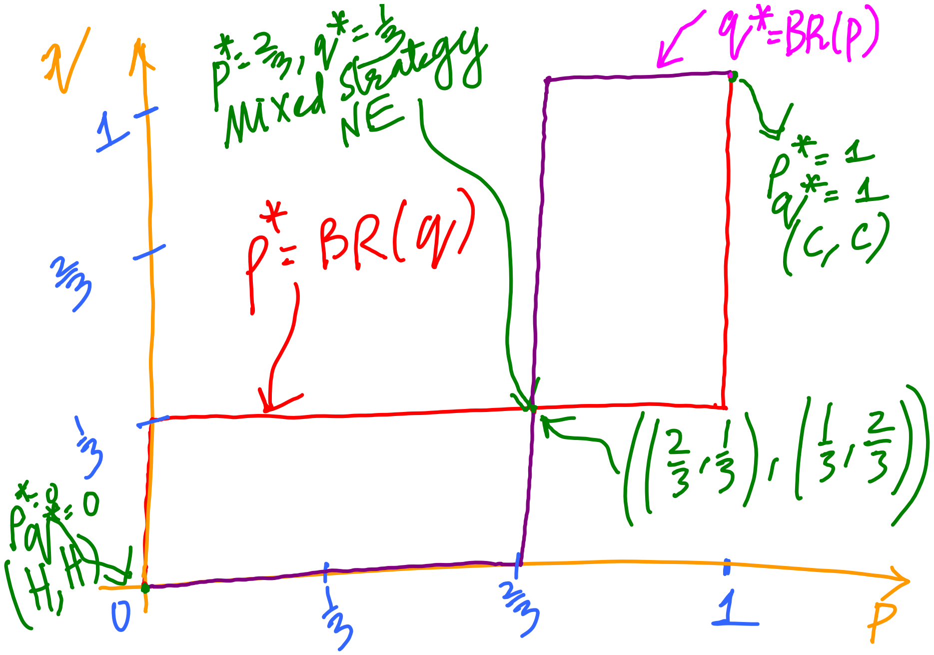
$$U_G(H) = 10(1 - 2/3) = 10/3$$

If $p = \frac{2}{3}$, best response q^* is any $0 \leq q^* \leq 1$

Since both cricket (C) and HP movie (H) yield the same payoff.

Best response q^* of girl.

$$q^* = \begin{cases} 1 & \text{if } p > \frac{2}{3} \\ 0 & \text{if } p < \frac{1}{3} \\ 0 \leq q^* \leq 1 & \text{if } p = \frac{1}{3} \end{cases}$$



Girl is employing mixture $q, 1-q$

if Boy always chooses cricket (C), his payoff $U_B(C)$ is,

$$U_B(C) = 10q + 0(1-q)$$
$$= 10q$$
$$U_B(H) = 0q + 5(1-q)$$
$$= 5(1-q)$$

If $U_B(C) > U_B(H)$

ie $10q > 5(1-q)$

$$\Rightarrow \boxed{q > 1/3}$$

Since cricket yields a higher payoff, Boy chooses to watch cricket all the time. $\Rightarrow p^* = 1$

$U_B(C) < U_B(H)$

$10q < 5(1-q)$

$\Rightarrow \boxed{q < \frac{1}{3}}$

If $q < \frac{1}{3}$, boy chooses to always watch H,

$$p^* = 0$$
$$1 - p^* = 1$$

if $q = \frac{1}{3}$, then we have

$$U_B(C) = 10q = 10/3$$
$$U_B(H) = 5(1-q) = 10/3.$$

If $q = 1/3$

Any $0 \leq p^* \leq 1$ is a best response.

$$p^* = \begin{cases} 1 & \text{if } q > \frac{1}{3} \\ 0 & \text{if } q < \frac{1}{3} \\ 0 \leq p^* \leq 1 & \text{if } q = \frac{1}{3} \end{cases}$$