Bayesian Cournot Game.

Market competition between 2 firms F_1, F_2.

Firm 1 has a production cost of C per unit.

Firm 2 of types.

Firm 2
- Low: $\frac{1}{2}C$ $\quad P(\text{low}) = \frac{1}{2}$
- High: C $\quad P(\text{high}) = \frac{1}{2}$

Inverse demand curve.

price $p = \left(a - (q_1 + q_2) \right)$

quantity produced by Firm 1

quantity produced by Firm 2

Payoff to each firm j

$$= \text{price} \times \text{quantity} - \text{cost of production}$$

$$= (a - (q_1 + q_2))\, q_j - c_j q_j$$

Firm 1 produces q_1

Firm 2 — type L → q_2^L

Firm 2 — type H → q_2^H

Payoff to Firm 2 of type L

$$= \left(a - (q_1 + q_2^L)\right) q_2^L - \frac{1}{2} c\, q_2^L$$

$$= a q_2^L - q_1 q_2^L - (q_2^L)^2 - \frac{1}{2} c\, q_2^L$$

$$a - q_1 - 2 q_2^L - \frac{1}{2} C = 0$$

$$a - q_1 - 2 q_2^L - \frac{1}{2} C = 0$$

$$(q_2^L)^* = \frac{a - \frac{1}{2} C - q_1}{2}$$

Best response of Firm 2 of type L

Payoff to firm 2 of type H is

$$\left(a-(q_1+q_2^H)\right)q_2^H - c\,q_2^H$$

$$= a q_2^H - q_1 q_2^H - (q_2^H)^2 - c\,q_2^H$$

$$a - q_1 - 2q_2^H - c = 0$$

$$a - q_1 - 2q_2^H - c = 0$$

$$\left(q_2^H\right)^* = \frac{a - q_1 - c}{2}$$

Best response of Firm 2 of type H.

Payoff of firm 1, corresponding to type low of firm 2 is

$$(a-(q_1+q_2^L))q_1 - C q_1$$

Payoff to firm 1 corresponding to firm 2 of type H is,

$$(a-(q_1+q_2^H))q_1 - C q_1$$

average payoff of firm 1 is

$$= \frac{1}{2}\left(\left(a - (q_1 + q_2^L)\right) q_1 - c q_1\right)$$

$$+ \frac{1}{2}\left(\left(a - (q_1 + q_2^H)\right) q_1 - c q_1\right)$$

Differentiate w.r.t.o q_1 and set equal to 0 to find the best response q_1.

$$\frac{1}{2}\left(a - 2q_1 - q_2^L - c\right)$$

$$+ \frac{1}{2}\left(a - 2q_1 - q_2^H - c\right) = 0$$

$$2q_1^* = \frac{1}{2}\left(a - c - q_2^L\right) + \frac{1}{2}\left(a - c - q_2^H\right)$$

Best response of Firm 1.

$$q_1^* = \frac{a-c}{2} - \frac{1}{4}\left(q_2^L + q_2^H\right)$$

$$\left(q_2^L\right)^* = \frac{a - \frac{1}{2}c - q_1^*}{2}$$

$$\left(q_2^H\right)^* = \frac{a - q_1^* - c}{2}$$

$$q_1^* = \frac{1}{2}(a-c) - \frac{1}{4}\left((q_2^L)^* + (q_2^H)^* \right)$$

$$= \frac{1}{2}(a-c) - \frac{1}{4}\left\{ \frac{a - \frac{1}{2}c - q_1^*}{2} + \frac{a-c-q_1^*}{2} \right\}$$

equation for q_1^*

$$q_1^* = \frac{a - \frac{5c}{4}}{3}$$

Best response quantity of Firm 1.

$$\left(q_2^L\right)^* = \frac{a - \frac{1}{2}c - q_1^*}{2}$$

$$= \frac{a - \frac{1}{2}c - \frac{1}{3}\left(a - \frac{5c}{4}\right)}{2}$$

$$= \frac{a}{3} - \frac{c}{24}$$

$$\left(q_2^H\right)^* = \frac{a - c - q_1^*}{2}$$

$$= \frac{a - c - \frac{1}{3}\left(a - \frac{5c}{4}\right)}{2}$$

$$= \frac{a}{3} - \frac{7c}{24}.$$

Bayesian NE of the Bayesian Cournot Game is/

$$\left(\underbrace{\frac{a - \frac{5c}{4}}{3}}_{\text{quantity of Firm 1}},\ \left(\underbrace{\frac{a}{3} - \frac{c}{24}}_{\text{quantity of Firm 2 of type L}},\ \underbrace{\frac{a}{3} - \frac{7c}{24}}_{\text{quantity of Firm 2 of type H}}\right)\right)$$