

Evolutionary stable strategy (ESS)

- Incumbents and mutants in the population. ESS is a strategy that cannot be invaded by a mutant population
- In an ESS, mutants have lower fitness (reproductive success) compared with the incumbent population.
- Let us assume that each beetle is repeatedly and randomly paired off with other beetles [No same pair]
- A beetle's fitness = Average fitness from food interactions = reproductive success
- A strategy is evolutionarily stable if it cannot be invaded by a mutant strategy.

Evolutionarily stable strategies

- Strategy T invades a strategy S at level x (for small x) if:
 - x fraction of population uses T
 - $1-x$ fraction of population uses S
- Strategy S is evolutionarily stable if there is some number y such that:
 - When any other strategy T invades S at any level $x < y$, the fitness of an organism playing S is strictly greater than the fitness of an organism playing T

The Beetles' world example [Cont]

- The size game between two beetles

	Small	Large
Small	6,6	1,10
Large	10,1	4,4

- Is Small an evolutionarily stable strategy?
 - Suppose for some small number x , a $1-x$ fraction of population use Small and x use Large
 - In other words, a small population of Large beetles (x) invades the

The Beetles' world example [Cont]

	Small	Large
Small	6,6	1,10
Large	10,1	4,4

- What is the expected payoff to a Small beetle in a random interaction?
 - With prob. $1-x$, a meet another Small beetle for a payoff of 6
 - With prob. x , meet Large beetle for a payoff of 1
 - Expected payoff: $6(1-x) + 1x = 6-5x$

The Beetles' world example [Cont]

	Small	Large
Small	6,6	1,10
Large	10,1	4,4

- What is the expected payoff to a Large beetle in a random interaction?
 - With prob. $1-x$, meet a Small beetle for payoff of 10
 - With prob. x , meet another Large beetle for a payoff of 4
 - Expected payoff: $10(1-x) + 4x = 10-6x$

The Beetles' world example [Cont]

- Expected fitness of a large beetle is $10-6x$
 - Expected fitness of a small beetle is $6-5x$
-
- For small enough x , the fitness of a large beetle exceeds the fitness for a small beetle.
 - Therefore, small is not evolutionarily stable strategy.

The Beetles' world example [Cont]

- The size game between two beetles

	Small	Large
Small	6,6	1,10
Large	10,1	4,4

- Is Large an evolutionarily stable strategy?
 - Suppose for some small number x , a $1-x$ fraction of population use Large and x use Small
 - In other words, a small population of small beetles (x) invades the population

The Beetles' world example [Cont]

- The size game between two beetles

	Small	Large
Small	6,6	1,10
Large	10,1	4,4

- In case of invasion by small beetles.
 - Expected payoff to Large: $4(1-x) + 10x = 4+6x$
 - Expected payoff to Small: $1(1-x) + 6x = 1+5x$

The Beetles' world example [Cont]

In case of invasion by small beetles.

- Expected fitness of a large beetle: $4+6x$
- Expected fitness of a small beetle = $1+5x$

- Large is evolutionarily stable

Summary: The Beetles' world example

- If a few large beetles get introduced into a population of small beetles
 - Large beetles will do really well: As they rarely meet each other and they get most of the food in most competitions
 - Population of small beetles cannot drive out the large ones, so small is not evolutionarily stable.
- Conversely, if a few small beetles get introduced in a population of large beetles, will do badly
 - They will lose almost every competition for food
 - A population of large beetles *resists* the invasion of small beetles
 - Large is thus evolutionarily stable

Summary: The Beetles' world example

- The structure of the game is as that of prisoners' dilemma
- Beetles: not able to change their body size. Evolution over multiple generations achieves the analogous effect.
- Stark result: Evolution is causing the fitness of the organisms to decrease over time
- Darwin's Theory: Natural selection increases fitness in a fixed environment.
- For a beetle: environment includes all other beetles.
 - Environment is becoming increasingly more hostile for everyone.

ESS

	Hawk	Dove
Hawk	-2,-2	6,0
Dove	0,6	3,3