

Bayesian Cournot
Game.

Market competition between
2 firms F_1, F_2 .

Firm 1 has a production cost of C per unit.

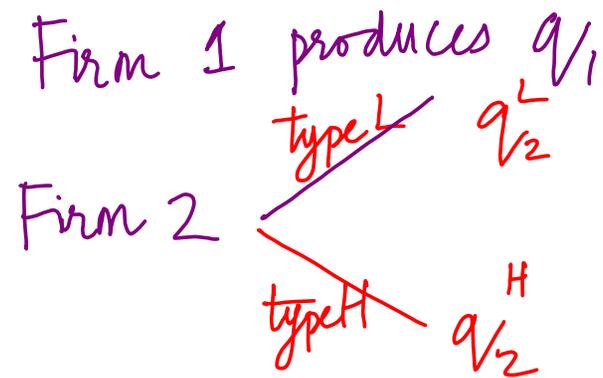
Firm 2 of types:
Low $\frac{1}{2}C$ $P(\text{low}) = \frac{1}{2}$
High C $P(\text{high}) = \frac{1}{2}$

Inverse demand curve.
price $p = (a - (q_1 + q_2))$
quantity produced by Firm 1 quantity produced by Firm 2

Payoff to each firm j

= price \times quantity - cost of production

$$= (a - (q_1 + q_2)) q_j - c_j q_j$$



Payoff to Firm 2 of type L

$$= (a - (q_1 + q_2^L)) q_2^L - \frac{1}{2} C q_2^L$$

$$= a q_2^L - q_1 q_2^L - (q_2^L)^2 - \frac{1}{2} C q_2^L$$

$$a - q_1 - 2 q_2^L - \frac{1}{2} C = 0$$

$$a - q_1 - 2 q_2^L - \frac{1}{2} C = 0$$

$$(q_2^L)^* = \frac{a - \frac{1}{2} C - q_1}{2}$$

Best response of Firm 2
of type L

Payoff to firm 2 of type H is

$$\left(a - (q_1 + q_2^H) \right) q_2^H - C q_2^H$$
$$= a q_2^H - q_1 q_2^H - (q_2^H)^2 - C q_2^H$$

$$a - q_1 - 2q_2^H - C = 0$$

$$a - q_1 - 2q_2^H - C = 0$$

$$(q_2^H)^* = \frac{a - q_1 - C}{2}$$

Best response of Firm 2
of type H.

Payoff of firm 1, corresponding to type low of firm 2 is

$$(a - (q_1 + q_2^L))q_1 - Cq_1$$

Payoff to firm 1 corresponding to firm 2 of type H is,

$$(a - (q_1 + q_2^H))q_1 - Cq_1$$

Average payoff of firm 1 is

$$= \frac{1}{2} \left((a - (q_1 + q_2^L)) q_1 - C q_1 \right)$$

$$+ \frac{1}{2} \left((a - (q_1 + q_2^H)) q_1 - C q_1 \right)$$

Differentiate wrt to q_1 and
set equal to 0 to find
the best response q_1 .

$$\frac{1}{2} \left(a - 2q_1 - q_2^L - C \right) + \frac{1}{2} \left(a - 2q_1 - q_2^H - C \right) = 0$$

Best response
of Firm 1

$$2q_1^* = \frac{1}{2} \left(a - c - q_2^L \right) + \frac{1}{2} \left(a - c - q_2^H \right)$$
$$q_1^* = \frac{a-c}{2} - \frac{1}{4} \left(q_2^L + q_2^H \right)$$

$$(q_2^L)^* = \frac{a - \frac{1}{2}c - q_1^*}{2}$$

$$(q_2^H)^* = \frac{a - q_1^* - c}{2}$$

$$\begin{aligned}
 q_1^* &= \frac{1}{2}(a-c) - \frac{1}{4} \left((q_2^L)^* + (q_2^H)^* \right) \\
 &= \frac{1}{2}(a-c) - \frac{1}{4} \left\{ \frac{a - \frac{1}{2}c - q_1^*}{2} + \frac{a - c - q_1^*}{2} \right\}
 \end{aligned}$$

equation for q_1^*

$$q_1^* = \frac{a - 5c}{4}$$

Best response quantity of Firm 1.

$$\begin{aligned}\left(q_2^L\right)^* &= \frac{a - \frac{1}{2}c - q_1^*}{2} \\ &= \frac{a - \frac{1}{2}c - \frac{1}{3}\left(a - \frac{5c}{4}\right)}{2} \\ &= \frac{a}{3} - \frac{c}{24}\end{aligned}$$

$$\begin{aligned}\left(q_2^H\right)^* &= \frac{a - c - q_1^*}{2} \\ &= \frac{a - c - \frac{1}{3}\left(a - \frac{5c}{4}\right)}{2} \\ &= \frac{a}{3} - \frac{7c}{24}.\end{aligned}$$

Bayesian NE of the Bayesian Cournot Game is,

$$\left(\frac{a - \frac{bc}{4}}{3}, \left(\frac{a - c}{3} - \frac{c}{24}, \frac{a - \frac{7c}{4}}{3} - \frac{c}{24} \right) \right)$$

quantity of Firm 1 quantity of Firm 2 of type L quantity of Firm 2 of type H