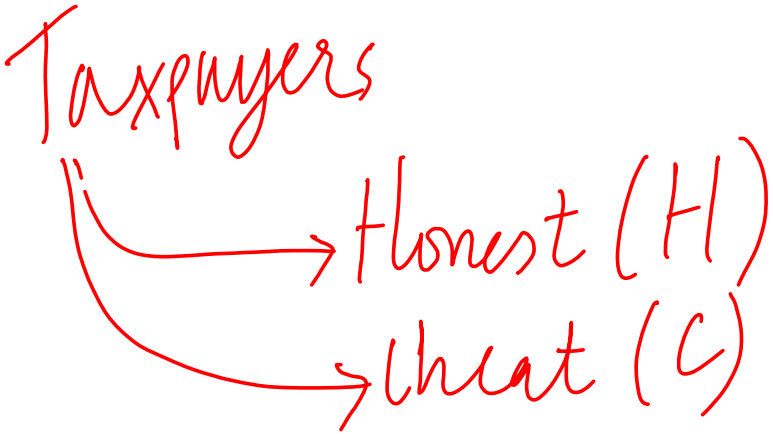
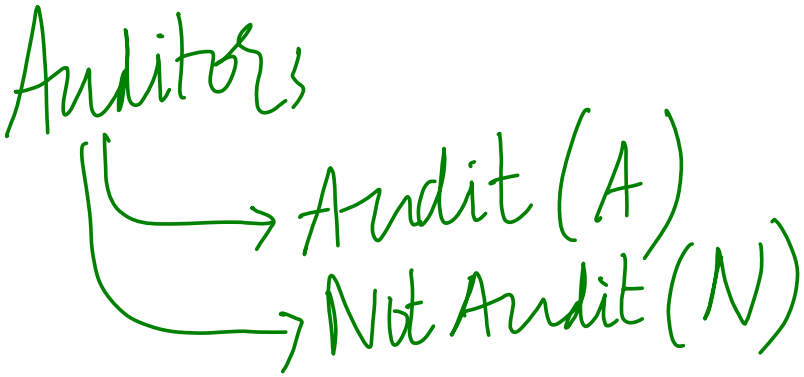
Paying Taxes:

Game between
Tax Payers (T)
Auditors (A)


Taxpayers
Honest (H)
Cheat (C)


Auditors
Audit (A)
Not Audit (N)

No intersection of best response in Pure Strategies

Tax Payer \ Auditor	A	N
H	0, 20	0, 40
C	-100, 40	40, 0

No pure strategy NE

		q	1 - q
	Tax Payer \ Auditor	A	N
p	H	0, 20	0, 40
1-p	C	-100, 40	40, 0

$$U_A(A) = 20p + 40(1-p)$$
$$= 40 - 20p$$

$$U_A(N) = 40p + 0(1-p)$$
$$= 40p$$

Auditor will employ a mixed strategy only when

$$40 - 20p = 40p$$

$$40 - 20p = 40p$$

$$\Rightarrow 60p = 40$$

$$\Rightarrow p = \frac{40}{60} = \frac{2}{3}$$

Mixture of Tax Payer is $p = \frac{2}{3}, 1-p = \frac{1}{3}$

Pay taxes with prob $p = \frac{2}{3}$

Cheat prob. $1-p = \frac{1}{3}$

We can think of this mixed strategy NE as across a population.

i.e. if you randomly pick a person, with prob $p = \frac{2}{3}$ you encounter a honest tax payer and prob $1-p = \frac{1}{3}$ you encounter a person cheating on taxes.

Therefore, the mixture across the population is

$p = \frac{2}{3}$, $1-p = \frac{1}{3}$

$U_T(H) = 0q + 0(1-q)$
$= 0$

$U_T(C) = -100q + 40(1-q)$
$= 40 - 140q$

$$0 = 40 - 140q$$

$$q = \frac{40}{140} = \frac{2}{7}$$

$$1 - q = \frac{5}{7}.$$

Therefore, auditor is using the mixed strategy

$$\left(\frac{2}{7}, \frac{5}{7}\right).$$

Therefore, mixed strategy NE of the game is,

Mixed strategy NE of Tax Payer $\left(\frac{2}{3}, \frac{1}{3}\right)$, Tax Payer $\left(\frac{2}{7}, \frac{5}{7}\right)$ Auditor