Game Tree and Information Sets

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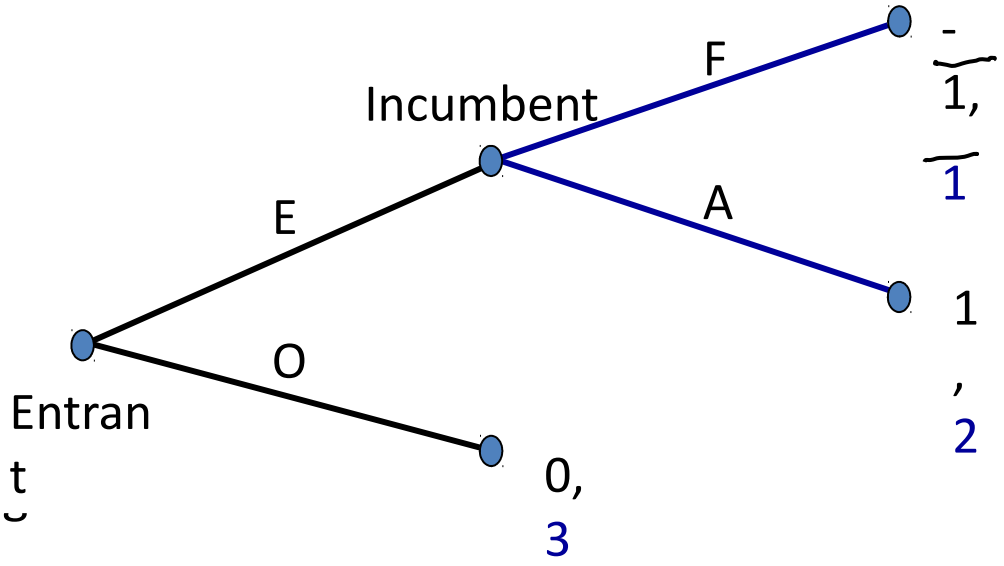
Recap: Extensive Form Games

- In strategic interactions
  - Players [Decision making units] may move simultaneously or sequentially.
  - They may move once or many times. The interactions may get repeated.
- How should we represent such interactions?
- We should consider:
  1. List of the players participating in the strategic interactions
  2. When does a player get to move in the game? [Order of Moves]
  3. What are the actions available to the player when she gets to move?
  4. How much does a player know when he gets to move? [Information]
  5. Pay-offs
- Notice: Only 1,3 and 5 were required in a normal form game.

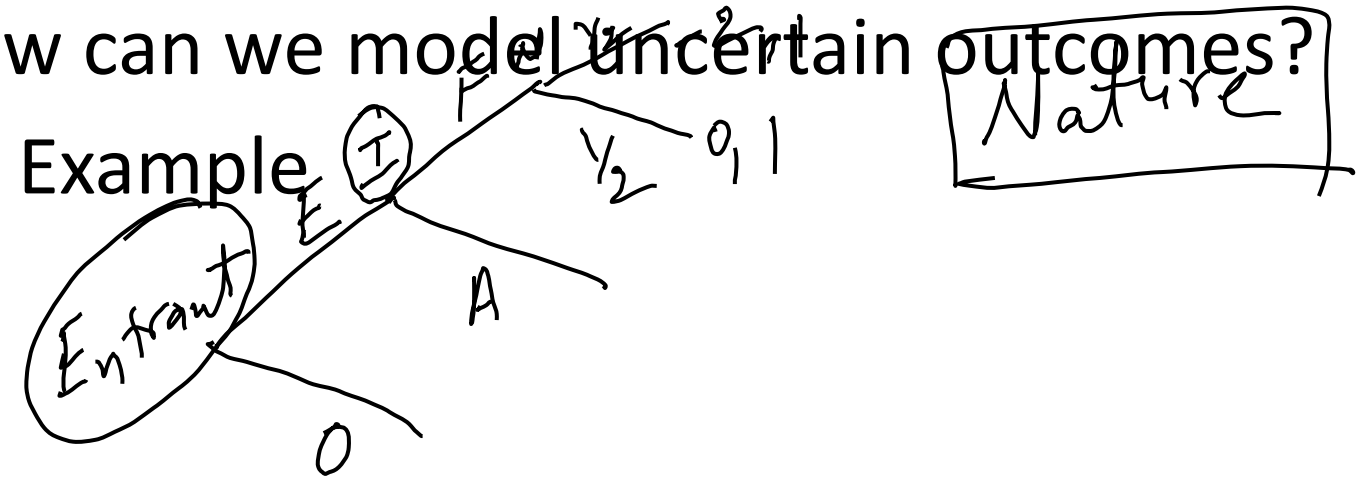
Game Tree

- Game Tree: A simple and useful way of representing an extensive form game.
- A game tree is a graph.
- It consists of
  - Nodes
  - Branches
- Nodes -> Labels
  - Initial Nodes: beginning of the
  - Decision Nodes: Player labels
  - Terminal Nodes: Payoffs
- Nodes -> Information
- Branches-> Actions.



Introducing Nature as a player

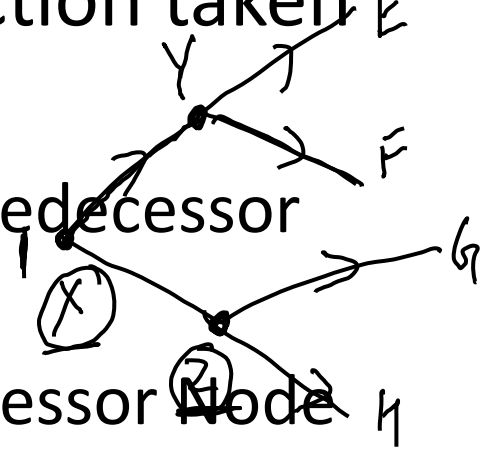
- How can we model uncertain outcomes?
- An Example



- Nature to model uncertain outcomes.

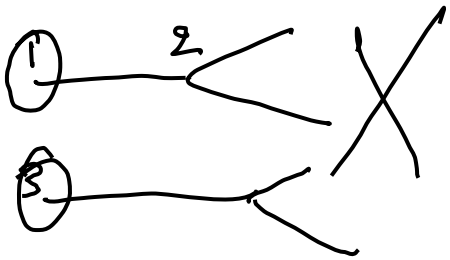
Nodes and Branches

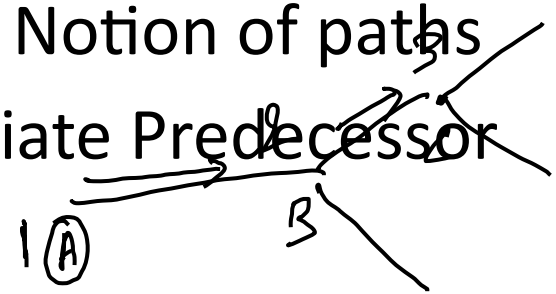
- Each node indicates
  - either the beginning of the game, or
  - a player's turn to make a decision, or
  - the end of the game.
- Each branch always indicates an action taken by one of the players.
  - Predecessor Node and Immediate Predecessor Node
  - Successor Node and Immediate Successor Node

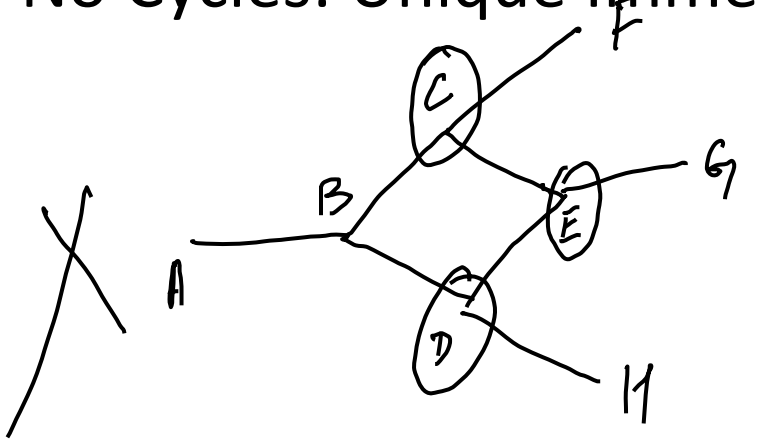


Important Rules

- Three important requirements
  - Unique Initial Node
  - Only one way to proceed: Notion of paths
  - No Cycles: Unique Immediate Predecessor

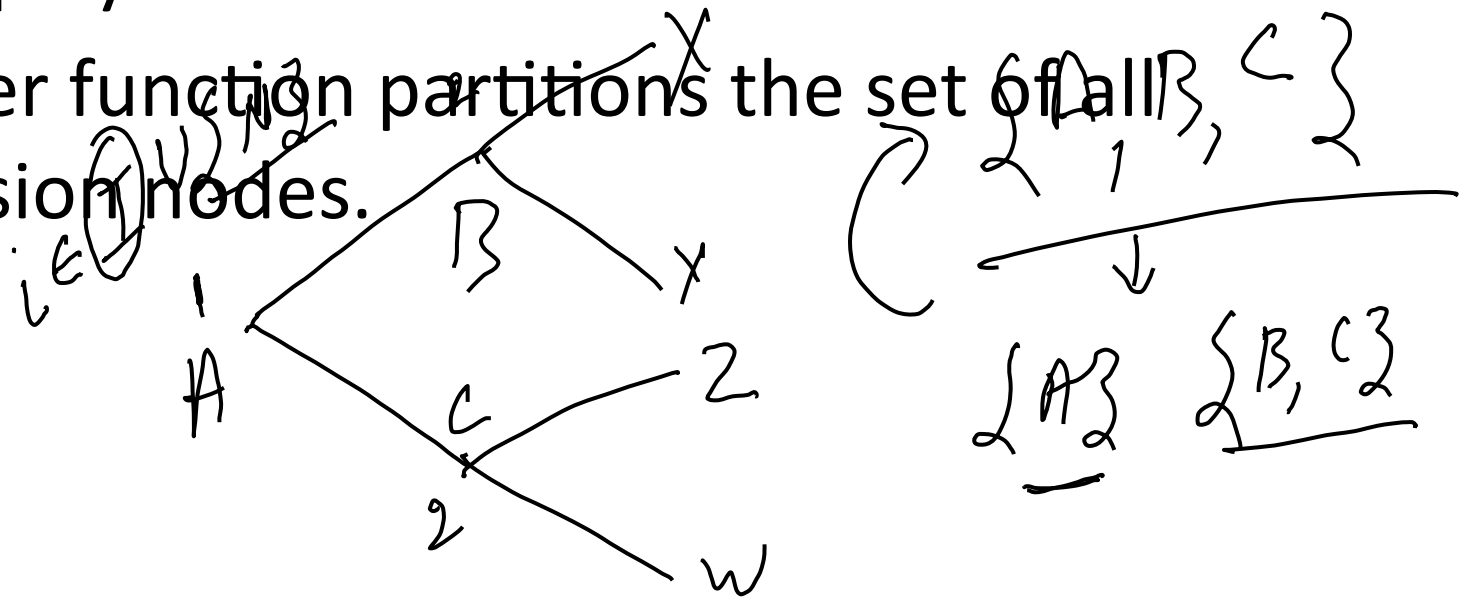






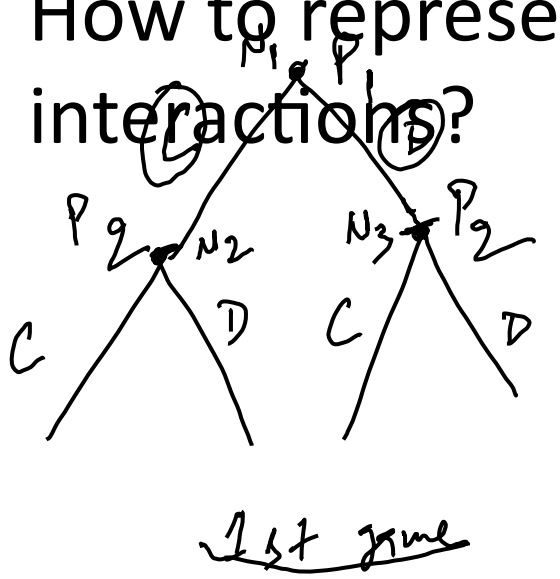
Player Function

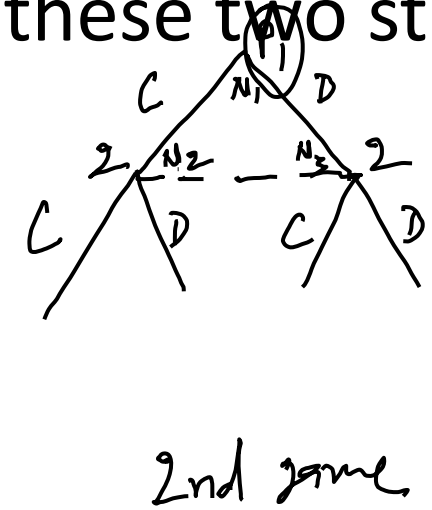
- A function which assigns each decision node to a player.
- Player function partitions the set of all decision nodes.



Knowledge

- Consider Prisoners' Dilemma and variant:
  - Both prisoners move simultaneously.
  - Prisoner 1 moves first, prisoner 2 observes 1's action and then decides his action.
- How to represent these two strategic interactions?





1st 2nd

$\{N_1\}$ $\{N_1\}$

$\{N_2\}\{N_3\}$ $\{N_2, N_3\}$

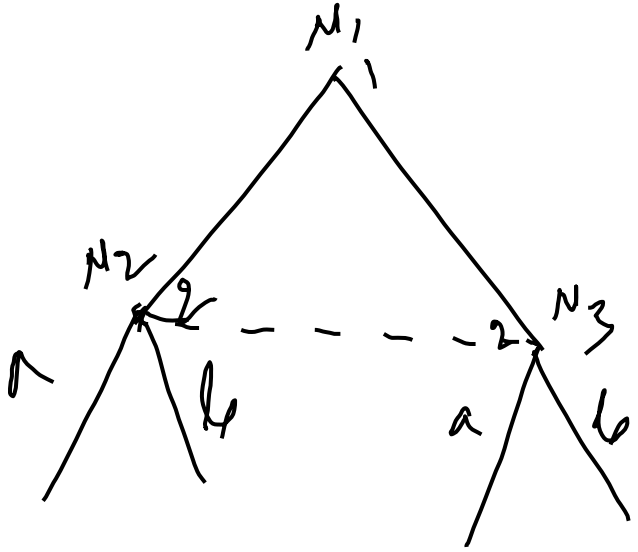
$P_1 \to \{C, D\}$ ...

$2 \to \{C, D\}$

Information Sets

- An information set belongs to a particular player and contains decision nodes satisfying following criterion
  - the player gets to play/ make a move at every node in that information set, and
  - when a node belonging to the information set is reached, the player does not know which node in the information set has been reached.
- Each decision node is in exactly one information set.
- At each decision node in an information set, the player must have
  - the same set of feasible actions, and
  - ultimately choose the same action.

Information Sets: Example



Perfect vs. Imperfect Information

- Common Knowledge: Players know the game structure
- Perfect Information
  - Players, when making any decision, know of all the events that have previously occurred.
  - All information sets are singleton.
- Imperfect Information:
  - Players when making any decision, may not be perfectly informed about some (or all) of the events that have already occurred.
  - At least one of the information sets is not singleton.