'Tragedy of Commons'

Usage or 'exploitation' of common resources

- Forests
- Mines
- Fisheries
- Environment
- Pasturelands

2 Timber agencies
cutting trees in forest
each can use
and effort e_i

Agency 1 — e_1

Agency 2 — e_2

payoff of agency 1

$$U_1(e_1, e_2)$$

$$= e_1 \left(1 - (e_1 + e_2)\right)$$

proportional to its effort

decreases with joint effort of Agency 1 + Agency 2

$$U_2(e_2, e_1)$$

$$= e_2 \left(1 - (e_1 + e_2)\right)$$

prop to e_2

decreases with increasing $e_1 + e_2$

$$0 \le e_1 \le 1$$

$$0 \le e_2 \le 1$$

Since number of actions is infinite, can no longer draw game table!

$$\underline{u_1(e_1, e_2)} = e_1(1 - e_1 - e_2)$$

$$= e_1 - e_1^2 - e_1 e_2$$

maximize this payoff

$$\frac{du_1}{de_1} = \frac{d}{de_1}(e_1 - e_1^2 - e_1 e_2)$$

$$= 1 - 2e_1 - e_2 = 0$$

$$\boxed{e_1^* = \frac{1 - e_2}{2}}$$

$$\boxed{e_1^* = BR_1(e_2) = \frac{1-e_2}{2}}$$

Best Response effort of agency 1.

$$u_2(e_2, e_1) = e_2(1 - e_1 - e_2)$$

$$\frac{du_2}{de_2} = \frac{d}{de_2}\left\{e_2 - e_1 e_2 - e_2^2\right\}$$

$$= 1 - e_1 - 2e_2 = 0$$

$$\boxed{e_2^* = \frac{1 - e_1}{2}}$$

$BR_2(e_1)$

Best Response e_2^* of agency 2.

Remember, NE is where Best responses intersect

- Each is playing BR to others action.

$$e_1^* = BR_1(e_2)$$
$$e_2^* = BR_2(e_1)$$

$$\left.\begin{aligned} e_1^* &= BR_1(e_2^*) \\ e_2^* &= BR_2(e_1^*) \end{aligned}\right\}$$

hold only at Nash Equilibrium

$$e_1^* = BR_1(e_2^*) = \frac{1 - e_2^*}{2}$$

$$e_2^* = BR_2(e_1^*) = \frac{1 - e_1^*}{2}$$

$$e_1^* = \frac{1 - e_2^*}{2} \quad \text{——} \quad (1)$$

$$e_2^* = \frac{1 - e_1^*}{2} \quad \text{——} \quad (2)$$

$$e_1^* = \frac{1}{2} - \frac{1}{2}\left(\frac{1 - e_1^*}{2}\right)$$

$$e_1^* = \frac{1}{4} + \frac{1}{4} e_1^*$$

$$\frac{3}{4} e_1^* = \frac{1}{4}$$

$$\boxed{e_1^* = \frac{1}{3}}$$

$$e_2^* = \frac{1 - e_1^*}{2}$$

$$= \frac{1 - 1/3}{2} = \frac{1}{3}$$

$$\boxed{e_2^* = \frac{1}{3}}$$

$$\underbrace{(e_1^*, e_2^*)}_{} = \left(\frac{1}{3}, \frac{1}{3}\right)$$

Nash Equilibrium outcome.

Payoff to each at NE

$$u_1(e_1^*, e_2^*) = u_1\left(\frac{1}{3}, \frac{1}{3}\right)$$

$$= \frac{1}{3}\left(1 - \left(\frac{1}{3} + \frac{1}{3}\right)\right) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

$$u_2(e_2^*, e_1^*) = \frac{1}{9}$$

Nash Payoff to both the timber agencies is

$$u_1(e_1^*, e_2^*) = u_2(e_2^*, e_1^*)$$

$$= \frac{1}{9}.$$