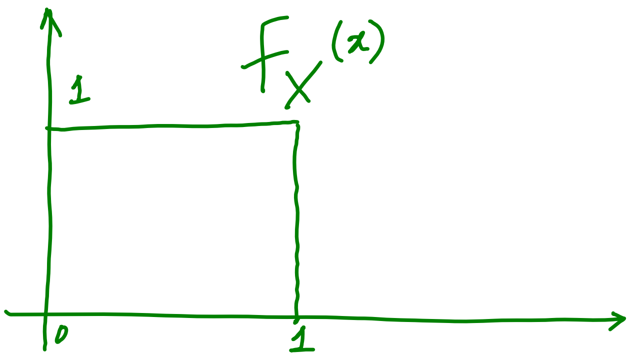
Auctions:

Auctions as Bayesian Games.

Random variable which is uniformly distributed in $[0, 1]$.



Probability density function of the uniform random variable in $[0,1]$ is defined as

$$f_X(x) = \begin{cases} 1 & \text{if } 0 \le x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

The probability that this random variable takes a value in the interval $[a,b]$ is given as

$$\int_a^b f_X(x)\,dx$$

Example: What is the probability that the uniform random variable takes a value between $[\frac{1}{4}, \frac{1}{2}]$?

$$= \int_{\frac{1}{4}}^{\frac{1}{2}} f_X(x)\,dx = \int_{\frac{1}{4}}^{\frac{1}{2}} 1.\,dx = x\Big|_{\frac{1}{4}}^{\frac{1}{2}}$$

$$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

The probability that the uniform random variable in $[0,1]$ takes a value between $[\frac{1}{4}, \frac{1}{2}]$ is $\frac{1}{4}$.

What is the probability that it lies in the interval $[0, \frac{1}{2}]$?

$$= \int_0^{1/2} f_X(x)\,dx = \int_0^{1/2} 1\,dx = x\Big|_0^{1/2}$$

$$= \frac{1}{2}.$$

Consider any interval $[a,b]$ which lies in $[0,1]$. The probability that the random variable takes any value in $[a,b]$ is,

$$\int_a^b f_X(x)\,dx = \int_a^b 1\,dx = x\Big|_a^b$$

$$= b-a$$

Therefore, the probability that it lies in any interval $[a,b]$ fully contained in $[0,1]$ $= b-a$ i.e. the length of the interval.

Therefore, the probability that it takes a value in [0, 1]

$$= \int_0^1 f_X(x)\,dx = \int_0^1 1 . dx = 1$$

For example, consider a discrete probability event such as the tossing of a fair die — The probability of occurrence of any face is $1/6$.

Random variable which is distributed uniformly in the interval [0, 1].