Bayesian Games with mixed Strategies

Bayesian BoS:

		q_1 C	$1-q_1$ H
p	C	10, 5	0, 0
$1-p$	H	0, 0	5, 10

$P(I) = \frac{1}{2}$

		$q_2 = 0$ C	$1-q_2 = 1$ H
$\frac{2}{3}$	C	10, 0	0, 10
$\frac{1}{3}$	H	0, 5	5, 0

$P(U) = \frac{1}{2}$

Payoff to girl of type I for always choosing C is,

$$5p + 0(1-p) = 5p$$

Payoff to girl of type I for always choosing H is,

$$0p + 10(1-p) = 10(1-p)$$

Therefore, girl of type I will employ mixed strategy, when,

$$5p = 10(1-p)$$

$$\Rightarrow 15p = 10$$

$$\Rightarrow p = 2/3$$

Therefore, mixed strategy employed by boy is $\frac{2}{3}, \frac{1}{3}$

Mixing C, H with probabilities $\frac{2}{3}, \frac{1}{3}$ respectively.

Payoff to girl of type U for choosing C is

$$0 \times \frac{2}{3} + 5 \times \frac{1}{3} = \frac{5}{3}$$

Payoff to girl of type U for choosing H is

$$10 \times \frac{2}{3} + 0 \times \frac{1}{3} = \frac{20}{3}$$

For girl of type U, choosing H yields a strictly greater payoff than choosing C.

Therefore $q_2 = 0$

$1 - q_2 = 1$

Payoff to boy for choosing C is,

$$\frac{1}{2}\left(10q_1 + 0(1-q_1)\right) + \frac{1}{2} \times 0 = 5q_1$$

Payoff to boy for always choosing H is,

$$\frac{1}{2}\left(0q_1 + 5(1-q_1)\right) + \frac{1}{2}\times 5 = \frac{5}{2}(1-q_1) + \frac{5}{2}$$

Since boy is using a mixed strategy, payoff to C, H must be equal.

Therefore,

$$5q_1 = \frac{5}{2}(1-q_1) + \frac{5}{2}$$

$$10q_1 = 5(1-q_1) + 5$$

$$15 q_1 = 10$$

$$\Rightarrow q_1 = \frac{2}{3}$$

$$1 - q_1 = \frac{1}{3}$$

Mixture of girl of type I is $\left(\frac{2}{3}, \frac{1}{3}\right)$.

Therefore, the mixed strategy Bayesian Nash Equilibrium of the game is

$$\left(\underbrace{\left(\frac{2}{3}, \frac{1}{3}\right)}_{\text{Mixture of Boy}}, \underbrace{\left(\frac{2}{3}, \frac{1}{3}\right)}_{\text{Mixture of Girl of Type I}}, \underbrace{(0, 1)}_{\text{Mixture of girl of type U.}} \right)$$

	$q_1 = 0$ C	$1-q_1 = 1$ H
$p = \frac{1}{3}$ C	10, 5	0, 0
$1-p = \frac{2}{3}$ H	0, 0	5, 10

$P(I) = \frac{1}{2}$

	q_2 C	$1-q_2$ H
$p = \frac{1}{3}$ C	10, 0	0, 10
$1-p = \frac{2}{3}$ H	0, 5	5, 0

$P(U) = \frac{1}{2}$

Let girl of type U be mixing.

Her payoff to always choosing C is $0p + 5(1-p) = 5(1-p)$

Her payoff to always choosing H is $10p + 0(1-p) = 10p$.

Therefore

$$5(1-p) = 10p$$

$$\Rightarrow 15p = 5$$

$$\Rightarrow p = \frac{1}{3}$$

$$1-p = \frac{2}{3}$$

Payoff to girl of type I for always choosing C is

$$5 \times \frac{1}{3} + 0 \times \frac{2}{3} = \frac{5}{3}$$

Her payoff to always choosing H is $0 \times \frac{1}{3} + 10 \times \frac{2}{3} = \frac{20}{3}$

Therefore girl of type I is always choosing H.

$$\Rightarrow \quad q_1 = 0$$

$$1 - q_1 = 1$$

Payoff to Boy corresponding to C is

$$\frac{1}{2} \times 0 + \frac{1}{2}\left(10 \times q_2 + 0(1-q_2)\right)$$

$$= 5q_2$$

Payoff to Boy for always choosing H is,

$$\frac{1}{2} \times 5 + \frac{1}{2} \times 5(1 - q_2)$$
$$= \frac{5}{2} + \frac{5}{2}(1 - q_2)$$

$$5q_2 = \frac{5}{2} + \frac{5}{2}(1 - q_2)$$
$$\Rightarrow 10q_2 = 5 + 5(1 - q_2)$$
$$= 10 - 5q_2$$
$$\Rightarrow q_2 = \frac{2}{3}$$
$$1 - q_2 = \frac{1}{3}$$

Another mixed strategy Bayesian Nash Equilibrium

$$\left(\underbrace{\left(\frac{1}{3}, \frac{2}{3}\right)}_{\text{Mixture of Boy}}, \left(\underbrace{(0, 1)}_{\text{Mixture of girl of type I}}, \underbrace{\left(\frac{2}{3}, \frac{1}{3}\right)}_{\text{Mixture of girl of type II.}} \right) \right)$$