

We have assumed that each player knows the payoffs of other players.

In several games, the payoffs of other players are NOT known.
— For example: Auction

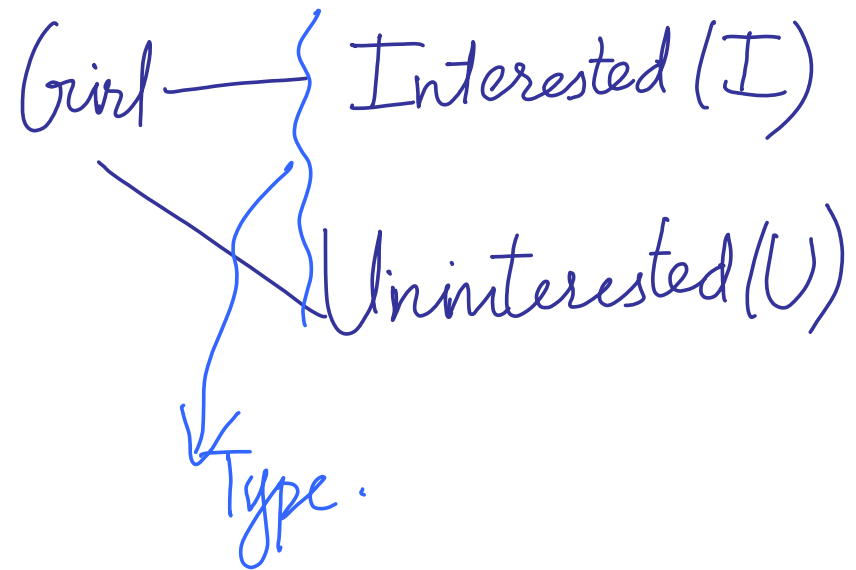
Several real life scenarios,
There is 'UNCERTAINTY'
regarding the payoffs of
other players.

These games in which
there is uncertainty
regarding payoffs of others
are known as Bayesian
Games.

Simple Bayesian Game Example:

'Bayesian' Battle of
sexes.

BoS — Game in
which P_1 (Boy) and P_2 (Girl)
can either choose C or H.



There are two types
of Girl Player P_2 .
→ I interested
U uninterested.

$$P(I) = \frac{1}{2} \left\{ \begin{array}{l} \text{Probability} \\ \text{girl is interested} \end{array} \right.$$

$$P(U) = \frac{1}{2} \left\{ \begin{array}{l} \text{Probability} \\ \text{girl is} \\ \text{uninterested.} \end{array} \right.$$

Girl of type I, is interested
in watching C or H
with boy.

		<u>Girl</u>	
		C	H
Boy	C	10, 5	0, 0
	H	0, 0	5, 10

Each prefers watching C or H with the other.

		<u>Girl</u>	
		C	H
Boy	C	10, 0	0, 10
	H	0, 5	5, 0

Corresponding to girl of type U.
 girl prefers to watch C or H alone,
 while Boy prefers to watch C or H together.

Bayesian BOS:

	C	H
C	10, 5	0, 0
H	0, 0	5, 10

$P(I) = \frac{1}{2}$

	C	H
C	10, 0	0, 10
H	0, 5	5, 0

$P(U) = \frac{1}{2}$

Girl is column player
Boy is row player.
Uncertainty regarding
payoffs of player 2 or Girl

Payoffs depend of if
girl is of type I or
Type U.

Game is Bayesian
because boy i.e P_1 is
'uncertain' regarding type
and hence payoffs of girl
i.e P_2 .

Bayesian Game:

- Assign a strategy to each player of each type.

Let us consider boy choosing C.

Let us consider girl of type I
choosing C.

- Let girl of type V also choose C.

Strategy of girl

(C, C)
action of girl of type I action of girl of type U.

$U_b(C, (C, C))$
Boy is choosing C girl of Type I Girl of Type U
Hence, we have to compute average payoff of boy, averaged with respect to probabilities of type of girl player.

$$U_b(C, (C, C))$$

$$= P(I) \times U_b(C, C) \\ + P(U) \times U_b(C, C)$$

$$= \frac{1}{2} \times 10 + \frac{1}{2} \times 10$$

$$= 10 = U_b(C, (C, C))$$

$U_b(H, (C, C))$
Boy is choosing H
Girl of Both types
is choosing C.

$$\begin{aligned} &= P(I) \times U_b(H, C) \\ &\quad + P(U) \times U_b(H, C) \\ &= \frac{1}{2} \times 0 + \frac{1}{2} \times 0 = 0 \\ &= U_b(H, (C, C)) \end{aligned}$$

$$\begin{aligned}
 & U_b(C, (C, H)) \\
 & \text{Boy chooses } C \quad \text{girl of type } I \text{ chooses } C \\
 & \text{girl of type } U \text{ chooses } H. \\
 & = P(I) \times U_b(C, C) + P(U) \times U_b(C, H) \\
 & = \frac{1}{2} \times 10 + \frac{1}{2} \times 0 = 5
 \end{aligned}$$

$$U_b(C, (C, H)) = 5$$

$$\begin{aligned}U_b(H, (C, H)) \\&= \frac{1}{2} \times 0 + \frac{1}{2} \times 5 \\&= \frac{5}{2}.\end{aligned}$$

Similarly we can also consider other strategy choices for the girl.

$(H, C).$
 (H, H)

Average Payoff Table for Boy:

	(C, C)	(C, H)	(H, C)	(H, H)
C	10	5	5	0
H	0	$5/2$	$5/2$	5

Possible Actions of Boy

Possible strategy combinations for girl.

Bayesian best response:

Payoff averaged with respect to probabilities of various types of other players.

Deduce best response
of player of each type.

Bayesian Nash Equilibrium
(BNE)

$(C, (C, C))$ is this
 a BNE?

Boy is choosing C
 girl of type I
 girl of type U.

$(C, (C, C))$

However C is NOT
 Best response of
 girl of type U.

C is BR of Boy
 C is BR of girl of type I

Therefore, $(C, (C, C))$ is NOT BNE
 since girl of type U is NOT playing
 her BR.

$(C, (C, C))$ is NOT
a Bayesian Nash
Equilibrium.

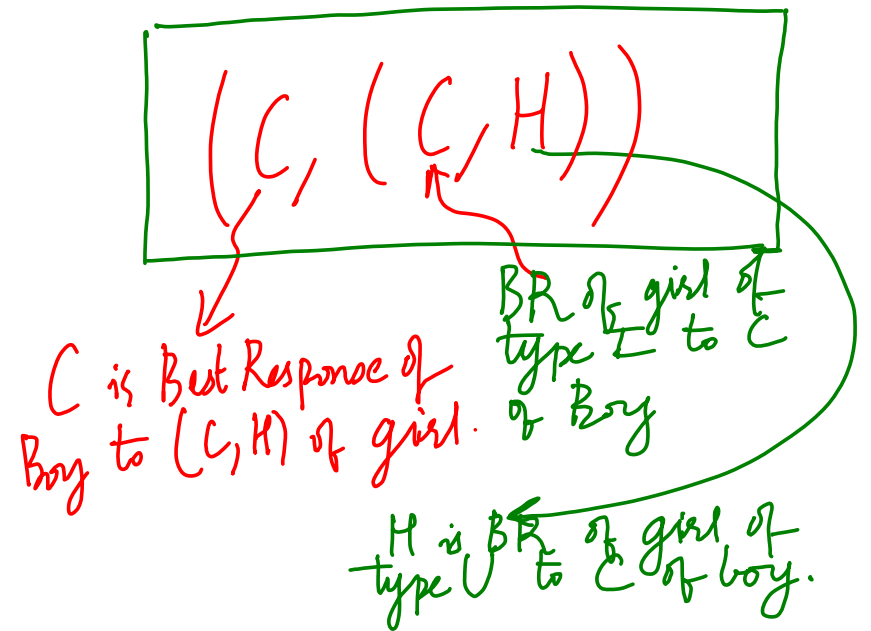
$(\underline{C}, (C, H))$ is BNE?
Boy is choosing \underline{C}
Girl of type I
Girl of type U

(C, H) of Girl is best response
of girl or P_2 of each type
to C of Boy.

$(C, (C, H))$

C is BR of girl of type I
 H is BR of girl of type U.
Therefore girl of each type is
playing her best response.

Therefore, $(C, (C, H))$ is
a Bayesian Nash Equilibrium.



is $(C, (H, C))$ BNE?

Girl of Type I is NOT
playing BR to C of Boy.
Therefore, NOT a Bayesian
Nash Equilibrium.

Is $(H, (H, \boxed{H}))$ is BNE?

Is NOT Best Response
of Girl of type U to H
of Boy.
NOT Bayesian Nash Equilibrium.

$(C, (C, H))$ is the
Bayesian Nash Equilibrium
for Bayesian BOS.

