

Cournot Duopoly

$$\begin{aligned} u_1(s_1, s_2) &= s_1(A - C - B(s_1 + s_2)) \\ u_2(s_2, s_1) &= s_2(A - C - B(s_1 + s_2)) \end{aligned}$$

utility or payoff to Firm 2

Payoff to firm 1

$$s_1^* = BR_1(s_2) = \frac{A - C - Bs_2}{2B}$$

$$= \frac{A - C}{2B} - \frac{1}{2} s_2$$

$$s_2^* = BR_2(s_1) = \frac{A - C}{2B} - \frac{1}{2} s_1$$

$$A = 10 \quad B = C = 1$$

$$s_1^* = 4.5 - \frac{1}{2} s_2$$

$$s_2^* = 4.5 - \frac{1}{2} s_1$$

Nash Payoff in
Cournot Duopoly is,

$$U_1(s_1^*, s_2^*) = U_2(s_2^*, s_1^*) = 9$$

Is there any outcome
which yields a higher
payoff for both the
players?

$$U_1(s_1, s_2) = s_1(A - C - B(s_1 + s_2))$$

$$U_2(s_2, s_1) = s_2(A - C - B(s_1 + s_2))$$

$$U_1(s_1, s_2) + U_2(s_2, s_1)$$

$$= \underline{(s_1 + s_2)} (A - C - \underline{B(s_1 + s_2)})$$

$$= \underbrace{s_t}_{s_1 + s_2} (A - C - Bs_t)$$

$$U_t(s_t) = s_t(A - C - Bs_t)$$

$$= (A - C)s_t - Bs_t^2$$

$$\frac{\partial U_t(s_t)}{\partial s_t} = (A - C) - 2Bs_t = 0$$

$$s_t^* = \frac{A - C}{2B}$$

$$s_1 = s_2 = \frac{A - C}{4B}$$

$$A = 10 \quad B = C = 1$$

$$\frac{A - C}{4B} = \frac{9}{4} = 2.25$$

$$U_1\left(\frac{9}{4}, \frac{9}{4}\right) = \frac{9}{4} \cdot \left(9 - 1 \times \frac{9}{2}\right)$$

$$= \frac{9}{4} \times \frac{9}{2} = \frac{81}{8}$$

$$\begin{aligned}
 U_1\left(\frac{9}{4}, \frac{9}{4}\right) &= \frac{81}{8} = \frac{81}{9} \times \left(\frac{9}{8}\right) \\
 &= 9 \times (>1) \\
 &> 9
 \end{aligned}$$

outcome $s_1 = \frac{9}{4} = 2.25$
 $\left\{ \begin{array}{l} s_2 = \frac{9}{4} = 2.25 \end{array} \right.$
 yields a higher
 payoff for both!

