Sealed Bid First Price auction:

Consider a two player auction

$\underbrace{P_1, \; P_2}_{\text{Two players participating in the auction.}}$

P_1, P_2 submit their individual bids b_1, b_2 respectively for the object being auctioned.

These bids are 'sealed'.

Therefore, each player does NOT know the bid of the other player.

P_1 does NOT know b_2 of Player 2

P_2 does NOT know b_1 of Player 1

Player with highest bid wins the auction and pays an amount equal to his bid to get the object being auctioned.

If $b_1 \geq b_2$ — then P_1 wins the auction and pays his bid b_1 to get the object. Player 2 who has lost the auction does NOT pay anything.

on the otherhand, if,

$$b_2 > b_1$$

then player 2 i.e. P_2 wins the auction and pays his bid b_2 to get the object.

First Price Auction:

Player with the highest bid wins the auction and pays an amount equal to his bid value.

Each player has a private valuation for the object.

valuation is what value the player assigns to the object.

$$\left.\begin{array}{l}\text{Player } 1 - V_1 \\ \text{Player } 2 - V_2\end{array}\right\}\text{valuations}$$

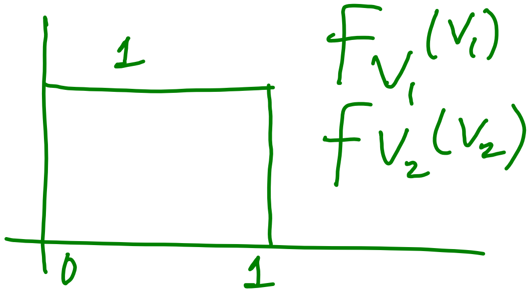
V_1, V_2 are the valuations of player 1, player 2 respectively for the object being auctioned.

These valuations a private.

$\Longrightarrow$ Player 1 does NOT know the valuation V_2 of Player 2.

— Player 2 does NOT know the valuation V_1 of player 1.

These private valuation are distributed uniformly in the interval $[0, 1]$.



This game is Bayesian in nature since there is uncertainty regarding the valuation of the other player.

Wish to analyze this game, to find the Nash equilibrium bidding strategy of each player?

We will demonstrate, that the bidding strategy

$$b_1 = \frac{1}{2} v_1$$
$$b_2 = \frac{1}{2} v_2$$

Therefore, each player bidding half his valuation is the Nash equilibrium bidding strategy.

we wish to demonstrate that $b_1 = \frac{1}{2}v_1$ and $b_2 = \frac{1}{2}v_2$ are best responses to each other.

let us assume that player 2 is bidding $b_2 = \frac{1}{2}v_2$. What is the best response bid b of player 1?

$\pi(b)$ — denotes payoff to player 1 as a function of b.

If player 1 wins the auction i.e. $b \geq b_2$, then his payoff $= V_1 - b$

Net payoff $= V_1 - b$

valuation

bid paid on winning the auction.

If player 1 loses the auction then his payoff is zero because he does NOT pay anything, neither does he get the object.

Average payoff to player 1 is given as

$$Pr(\text{win}) \times (V_1 - b) + Pr(\text{loss}) \times 0$$

$$= Pr(\text{win}) \times (V_1 - b).$$

$$\pi(b) = \underline{Pr(win)} \times (V_1 - b)$$

payoff to player 1 as a function of bid b.

What is $Pr(win)$ i.e. the probability of winning the auction?

To win $\quad b \geq b_2 = \frac{1}{2} V_2$

For player 1 to win $\quad b \geq \frac{1}{2} V_2 \Rightarrow V_2 \leq 2b$

Since V_2 is distributed uniformly in $[0,1]$, we must have

V_2 in $[0, 2b]$.

Probability V_2 lies in $[0, 2b]$

$$= \int_0^{2b} f_{V_2}(v_2)\, dv_2$$

$$= \int_0^{2b} 1 \cdot dv_2 = V_2 \Big|_0^{2b}$$

$$= 2b.$$

Pr (win) for player 1 is 2b

Therefore $\pi(b)$ is,

$$\pi(b) = 2b \times (V_1 - b)$$
$$= 2bV_1 - 2b^2$$

$$\pi(b) = 2bV_1 - 2b^2$$

$$\frac{\partial \pi(b)}{\partial b} = 2V_1 - 4b = 0$$

$$\boxed{b^* = \frac{1}{2} V_1}$$

If $b_2 = \frac{1}{2} V_2$, then the bid $b_1 = \frac{1}{2} V_1$ is the best response of player 1.

Using a similar procedure it can be shown that if $b_1 = \frac{1}{2} V_1$, then $b_2 = \frac{1}{2} V_2$ is the best response of Player 2.

Therefore $b_1 = \frac{1}{2} v_1$, $b_2 = \frac{1}{2} v_2$ are best responses to each other.

$$\boxed{\begin{aligned} b_1 &= \tfrac{1}{2} v_1 \\ b_2 &= \tfrac{1}{2} v_2 \end{aligned}}$$

Nash Equilibrium of sealed bid first price auction.