'Mixed Strategies'

'Matching Pennies'

$\underbrace{P_1, P_2}_{\text{2 Players}}$ — have a coin each

Head (H)

Tail (T)

If both show same face, then P_1 wins. P_2 pays 1 Rs to P_1

On the other hand, if both show a different face, then P_2 wins. P_1 has to pay 1 Rs to P_2.

	P_1 \ P_2	H	T
p	H	$\boxed{1}$, -1	-1, 1
$1-p$	T	-1, 1	$\boxed{1}$, -1

No intersection of best responses!

Pure Strategies

where each player is choosing one action

Games in which players can randomly chose a particular action with a certain probability.

- Mixed Strategy

- Randomized strategy.

p

$0 \le p \le 1$

$p = \frac{1}{4}$ — Player 1 is choosing H with probability $\frac{1}{4}$

randomly with frequency 25% he is choosing H.

Probability of Tails $= 1 - \frac{1}{4} = \frac{3}{4}$

on an average 75% of the time he is choosing T.

		q	$1-q$
	$P_1 \setminus P_2$	H	T
p	H	$1, -1$	$-1, 1$
$1-p$	T	$-1, 1$	$1, -1$

$$u_2(H) = p \times (-1) + (1-p)(1)$$
$$= 1 - 2p.$$

$$u_2(T) = p \times 1 + (1-p)(-1)$$
$$= 2p - 1$$

Therefore, P_2 would 'Mix' or 'Randomly' choose between H, T only if both payoffs are equal!

For a mixed strategy equilibrium

$$U_2(H) = U_2(T)$$

$$1 - 2p = 2p - 1$$

$$1 - 2p = 2p - 1$$

$$4p = 2$$

$$\boxed{p = \frac{1}{2}}$$

$$0 \le q \le 1$$

Player 2 chooses H with probability q ✓

chooses T with prob $1 - q$ ✓

$$U_1(H) = q \times 1 + (1-q)(-1)$$
$$= 2q - 1$$

$$U_1(T) = q \times (-1) + (1-q) \times 1$$
$$= 1 - 2q$$

P_1 will 'mix' only when,

$$U_1(H) = U_1(T)$$

$$\Rightarrow 2q - 1 = 1 - 2q$$

$$2q - 1 = 1 - 2q$$

$$4q = 2$$

$$\boxed{q = \frac{1}{2}}$$

$q = \frac{1}{2}$, $1 - q = \frac{1}{2}$

P_2 is mixing H, T with prob. $\frac{1}{2}$, $\frac{1}{2}$.

Similarly, $p = \frac{1}{2} \Rightarrow 1-p = \frac{1}{2}$

Therefore P_1 is mixing H, T with prob $\frac{1}{2}, \frac{1}{2}$.

Mixed Strategy NE

$$\Big(\underbrace{\left(\tfrac{1}{2}, \tfrac{1}{2}\right)}_{\text{Mixed Strategy } P_1}, \underbrace{\left(\tfrac{1}{2}, \tfrac{1}{2}\right)}_{\text{Mixed Strategy } P_2} \Big)$$