



LDPC Codes: Decoding

Andrew Thangaraj
andrew@iitm.ac.in



Decoder Modifications

- Offset and scaling in minsum
 - $a(\text{minsum}) + b$ minsum - offset
 - Makes minsum as good as log-MAP
- Layered decoding
 - Faster convergence, fewer iterations
 - Codes in standards often support layering
- Most implementations
 - Offset, scaled minsum with layering



Layering in the LDPC matrix

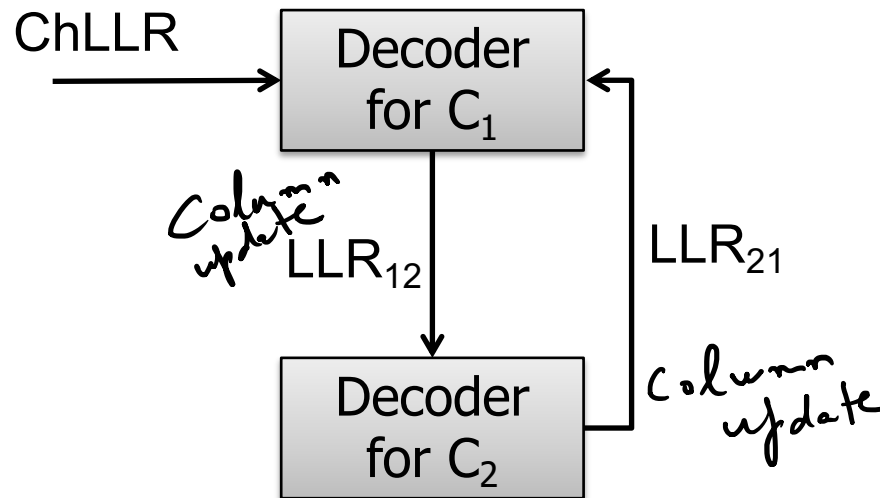
- Rows of parity check matrix grouped into several layers
 - Column weight of each layer = 1 (typically)
- one block row in 5G LDPC codes*

$$H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = \left[\begin{array}{ccccccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ \hline 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{array} \right] \begin{matrix} \text{Layer 1} \\ \text{Layer 2} \end{matrix}$$

- C_1 : Code with H_1 as parity check matrix
- C_2 : Code with H_2 as parity check matrix
 - C_1, C_2 : LDPC codes, Code = C_1 intersect C_2

Layered decoding

- Iterate between decoders for each layer



- Decoder for C_1 uses ChLLR only in first iteration
 - LLR_{21} is used afterwards
- Reduces required number of iterations by 1/2
 - Converges faster

Initialization

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ \hline 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$\mathbf{r} = [0.2 \quad -0.3 \quad 1.2 \quad -0.5 \quad 0.8 \quad 0.6 \quad -1.1]$

$L =$

Diagram illustrating the structure of the matrix L (a sparse matrix) and the vector \mathbf{r} . The vector \mathbf{r} is shown above the matrix. The matrix L is represented by a grid where blue blocks indicate non-zero entries. Arrows point from the elements of \mathbf{r} to the corresponding blocks in L . A handwritten note indicates that the zeros in the matrix are only in the PC matrix layer.

Iteration 1, Layer 1

$L =$

^{min1} 0.2	^{min2} -0.3	1.2	^{min1}	0.8	^{min2}	
			-0.5		0.6	-1.1
-	-		-			-
		-		-	-	

overall parity: -1, +1

Minsum on first layer: Rows 1, 2

$r = [0.2 \quad -0.3 \quad 1.2 \quad -0.5 \quad 0.8 \quad 0.6 \quad -1.1]$

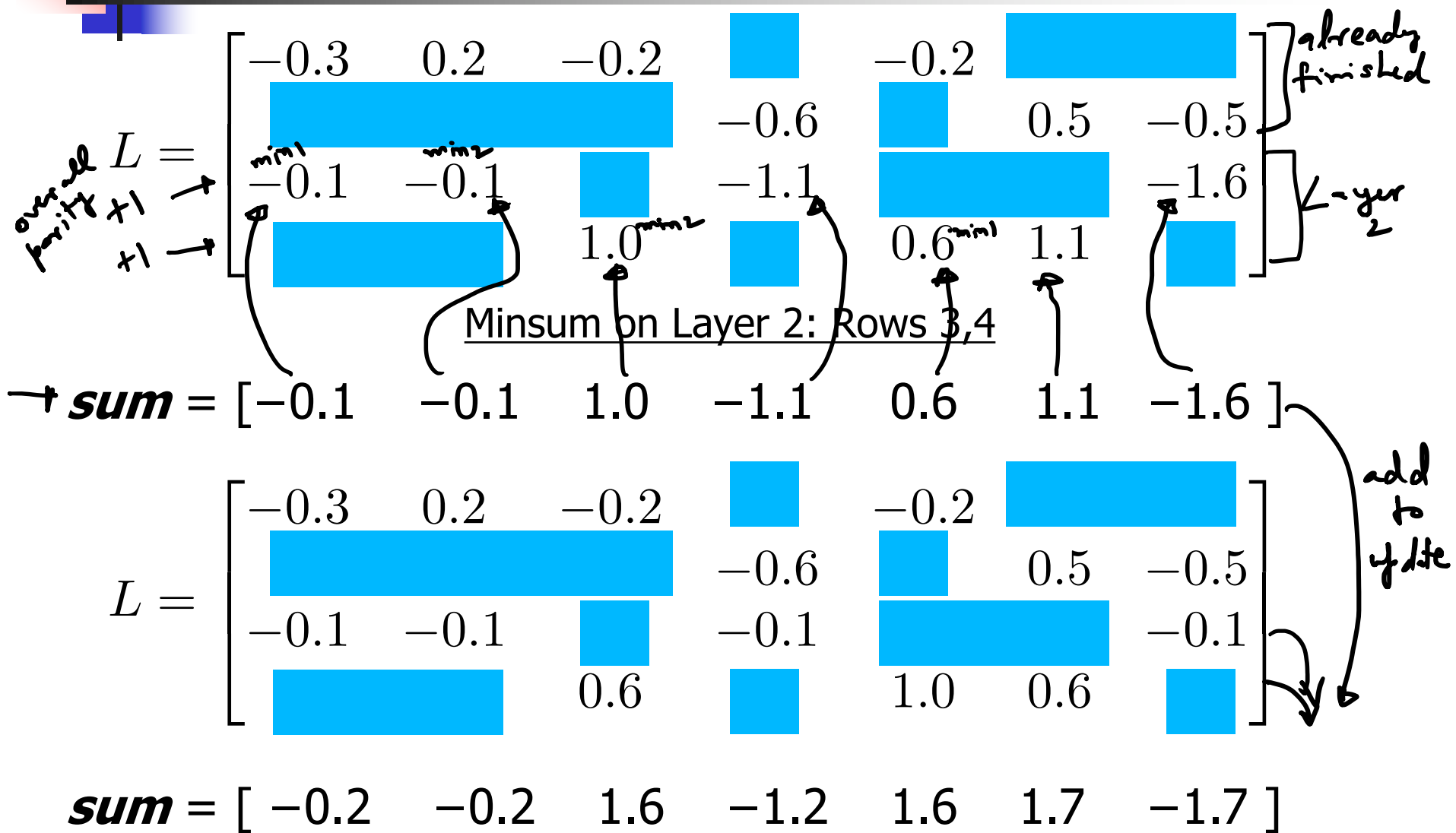
$L =$

^{min2} -0.3	^{min1} 0.2	^{min1} -0.2	^{min2}	-0.2	^{min1}	^{min1} -0.5
			-0.6		0.5	
-	-		-			-
		-		-	-	

$\text{sum} = [-0.1 \quad -0.1 \quad 1.0 \quad -1.1 \quad 0.6 \quad 1.1 \quad -1.6]$

add to update

Iteration 1, Layer 2



Iteration 2, Layer 1: Subtract

sum = $[-0.2 \quad -0.2 \quad 1.6 \quad -1.2 \quad 1.6 \quad 1.7 \quad -1.7]$ *incoming belief*

$L =$ $\begin{bmatrix} -0.3 & 0.2 & -0.2 & \text{blue} & -0.2 & \text{blue} \\ \text{blue} & \text{blue} & \text{blue} & -0.6 & \text{blue} & 0.5 & -0.5 \\ -0.1 & -0.1 & \text{blue} & -0.1 & \text{blue} & \text{blue} & -0.1 \\ \text{blue} & \text{blue} & 0.6 & \text{blue} & 1.0 & 0.6 & \text{blue} \end{bmatrix}$ *Layer 1*

0.2 - (-0.3)

sum = $[0.1 \quad -0.4 \quad 1.8 \quad -0.6 \quad 1.8 \quad 1.2 \quad -1.2]$ *incoming belief - layer 1*
Corrected input for layer 1

$L =$ $\begin{bmatrix} 0.1 & -0.4 & 1.8 & \text{blue} & 1.8 & \text{blue} \\ \text{blue} & \text{blue} & \text{blue} & -0.6 & \text{blue} & 1.2 & -1.2 \\ -0.1 & -0.1 & \text{blue} & -0.1 & \text{blue} & \text{blue} & -0.1 \\ \text{blue} & \text{blue} & 0.6 & \text{blue} & 1.0 & 0.6 & \text{blue} \end{bmatrix}$

Iteration 2, Layer 1: Minsum

sum = [0.1 -0.4 1.8 -0.6 1.8 1.2 -1.2] *← beliefs*

$L =$ $\begin{bmatrix} \overset{\text{min1}}{0.1} & \overset{\text{min2}}{-0.4} & 1.8 & \text{blue} & 1.8 & \text{blue} & \text{blue} \\ \text{blue} & \text{blue} & \text{blue} & -0.6 & \text{blue} & 1.2 & -1.2 \\ -0.1 & -0.1 & \text{blue} & -0.1 & \text{blue} & \text{blue} & -0.1 \\ \text{blue} & \text{blue} & 0.6 & \text{blue} & 1.0 & 0.6 & \text{blue} \end{bmatrix}$ *Layer 1*

Minsum in Layer 1: Rows 1,2

$L =$ $\begin{bmatrix} \overset{\text{min2}}{-0.4} & \overset{\text{min1}}{0.1} & \overset{\text{min1}}{-0.1} & \text{blue} & \overset{\text{min2}}{-0.1} & \text{blue} & \text{blue} \\ \text{blue} & \text{blue} & \text{blue} & -1.2 & \text{blue} & 0.6 & -0.6 \\ -0.1 & -0.1 & \text{blue} & -0.1 & \text{blue} & \text{blue} & -0.1 \\ \text{blue} & \text{blue} & 0.6 & \text{blue} & 1.0 & 0.6 & \text{blue} \end{bmatrix}$

Iteration 2, Layer 1: Update

$$\mathbf{sum} = [0.1 \quad -0.4 \quad 1.8 \quad -0.6 \quad 1.8 \quad 1.2 \quad -1.2]$$

corrected
version

Update in Layer 1: Rows 1,2

$$L = \begin{bmatrix} -0.4 & 0.1 & -0.1 & \text{blue} & -0.1 & \text{blue} & \text{blue} \\ \text{blue} & \text{blue} & \text{blue} & -1.2 & \text{blue} & 0.6 & -0.6 \\ -0.1 & -0.1 & \text{blue} & -0.1 & \text{blue} & \text{blue} & -0.1 \\ \text{blue} & \text{blue} & 0.6 & \text{blue} & 1.0 & 0.6 & \text{blue} \end{bmatrix}$$

add
to
update

$$\mathbf{sum} = [-0.3 \quad -0.3 \quad 1.7 \quad -1.8 \quad 1.7 \quad 1.8 \quad -1.8]$$

Subtract \rightarrow minsum \rightarrow update

Iteration 2, Layer 2: Subtract

sum = $\begin{bmatrix} -0.3 & -0.3 & 1.7 & -1.8 & 1.7 & 1.8 & -1.8 \end{bmatrix}$ *incoming belief*

L = $\begin{bmatrix} -0.4 & 0.1 & -0.1 & \text{blue} & -0.1 & \text{blue} & \text{blue} \\ \text{blue} & \text{blue} & \text{blue} & -1.2 & \text{blue} & 0.6 & -0.6 \\ -0.1 & -0.1 & \text{blue} & -0.1 & \text{blue} & \text{blue} & -0.1 \\ \text{blue} & \text{blue} & 0.6 & \text{blue} & 1.0 & 0.6 & \text{blue} \end{bmatrix}$ *ignore*

Subtraction in Layer 2: Rows 3,4

sum = $\begin{bmatrix} -0.2 & -0.2 & 1.1 & -1.7 & 0.7 & 1.2 & -1.7 \end{bmatrix}$ *corrected belief*

L = $\begin{bmatrix} -0.4 & 0.1 & -0.1 & \text{blue} & -0.1 & \text{blue} & \text{blue} \\ \text{blue} & \text{blue} & \text{blue} & -1.2 & \text{blue} & 0.6 & -0.6 \\ -0.2 & -0.2 & \text{blue} & -1.7 & \text{blue} & \text{blue} & -1.7 \\ \text{blue} & \text{blue} & 1.1 & \text{blue} & 0.7 & 1.2 & \text{blue} \end{bmatrix}$ *ignore*

Iteration 2, Layer 2: Minsum

$$\mathbf{sum} = \begin{bmatrix} -0.2 & -0.2 & 1.1 & -1.7 & 0.7 & 1.2 & -1.7 \end{bmatrix}$$

$$L = \begin{bmatrix} -0.4 & 0.1 & -0.1 & \text{redacted} & -0.1 & \text{redacted} \\ \text{redacted} & \text{redacted} & \text{redacted} & -1.2 & \text{redacted} & 0.6 & -0.6 \\ -0.2 & -0.2 & \text{redacted} & -1.7 & \text{redacted} & \text{redacted} & -1.7 \\ \text{redacted} & \text{redacted} & 1.1 & \text{redacted} & 0.7 & 1.2 & \text{redacted} \end{bmatrix}$$

Minsum in Layer 2: Rows 3,4

$$L = \begin{bmatrix} -0.4 & 0.1 & -0.1 & \text{redacted} & -0.1 & \text{redacted} \\ \text{redacted} & \text{redacted} & \text{redacted} & -1.2 & \text{redacted} & 0.6 & -0.6 \\ -0.2 & -0.2 & \text{redacted} & -0.2 & \text{redacted} & \text{redacted} & -0.2 \\ \text{redacted} & \text{redacted} & 0.7 & \text{redacted} & 1.1 & 0.7 & \text{redacted} \end{bmatrix}$$



Iteration 2, Layer 2: Update

$$\mathbf{sum} = \begin{bmatrix} -0.2 & -0.2 & 1.1 & -1.7 & 0.7 & 1.2 & -1.7 \end{bmatrix}$$

Update in Layer 2: Rows 3,4

$$L = \begin{bmatrix} -0.4 & 0.1 & -0.1 & \text{blue} & -0.1 & \text{blue} \\ \text{blue} & \text{blue} & \text{blue} & -1.2 & \text{blue} & 0.6 & -0.6 \\ -0.2 & -0.2 & \text{blue} & -0.2 & \text{blue} & \text{blue} & -0.2 \\ \text{blue} & \text{blue} & 0.7 & \text{blue} & 1.1 & 0.7 & \text{blue} \end{bmatrix}$$

$$\mathbf{sum} = \begin{bmatrix} -0.4 & -0.4 & 1.8 & -1.9 & 1.8 & 1.9 & -1.9 \end{bmatrix}$$



Decision

- If $\text{Sum}_j > 0$, Decision on Bit $j = 0$
- If $\text{Sum}_j < 0$, Decision on Bit $j = 1$
 - Assuming BPSK 0 \rightarrow +1 and 1 \rightarrow -1

After two iterations,

$\text{Sum} = [-0.4 \quad -0.4 \quad 1.8 \quad -1.9 \quad 1.8 \quad 1.9 \quad -1.9]$

belief



$\text{Dec} = [\quad 1 \quad \quad 1 \quad \quad 0 \quad \quad 1 \quad \quad 0 \quad \quad 0 \quad \quad 1]$

- For more iterations, continue...



Summary

- LDPC codes provide close-to-capacity performance
- Adopted in several standards
- VLSI architectures and implementations are available today
- Interesting issues
 - Optimized implementations
 - Speed and efficiency
 - Optimal layering?