

(6,3) code.

$$H = \begin{matrix} & \begin{matrix} m_1 & m_2 & m_3 & p_1 & p_2 & p_3 \end{matrix} \\ \begin{matrix} 3 \\ 1 \\ 0 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

parity bits: computed using H and \underline{m}

$$\underline{m} = [m_1, m_2, m_3]$$

$$\underline{c} = [m_1, m_2, m_3, \underbrace{p_1, p_2, p_3}_{\text{parity bits}}]$$

$$H \underline{c}^T = \underline{0} \quad \rightarrow \quad \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{p} = [p_1, p_2, p_3]$$

$$\text{Row 1: } m_1 + m_2 + p_1 = 0 \rightarrow p_1 = m_1 + m_2$$

$$\text{Row 2: } m_2 + m_3 + p_2 = 0 \rightarrow p_2 = m_2 + m_3$$

$$\text{Row 3: } m_1 + m_3 + p_3 = 0 \rightarrow p_3 = m_1 + m_3$$

$$H = \begin{bmatrix} P & I \end{bmatrix}$$

3x3 identity

$$\begin{bmatrix} P & I \end{bmatrix} \begin{bmatrix} \underline{m}^T \\ \underline{p}^T \end{bmatrix} = \underline{0}$$

$$\underline{p}^T + \underline{p}^T = \underline{0}$$

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$P^T = P \overline{x}^T$$

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} P \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$k=2 \quad \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \\ \overline{x_5} \\ x_1 \\ x_2 \end{bmatrix} \begin{matrix} (k+1) \\ \vdots \\ \text{end} \\ \vdots \\ k \end{matrix}$$