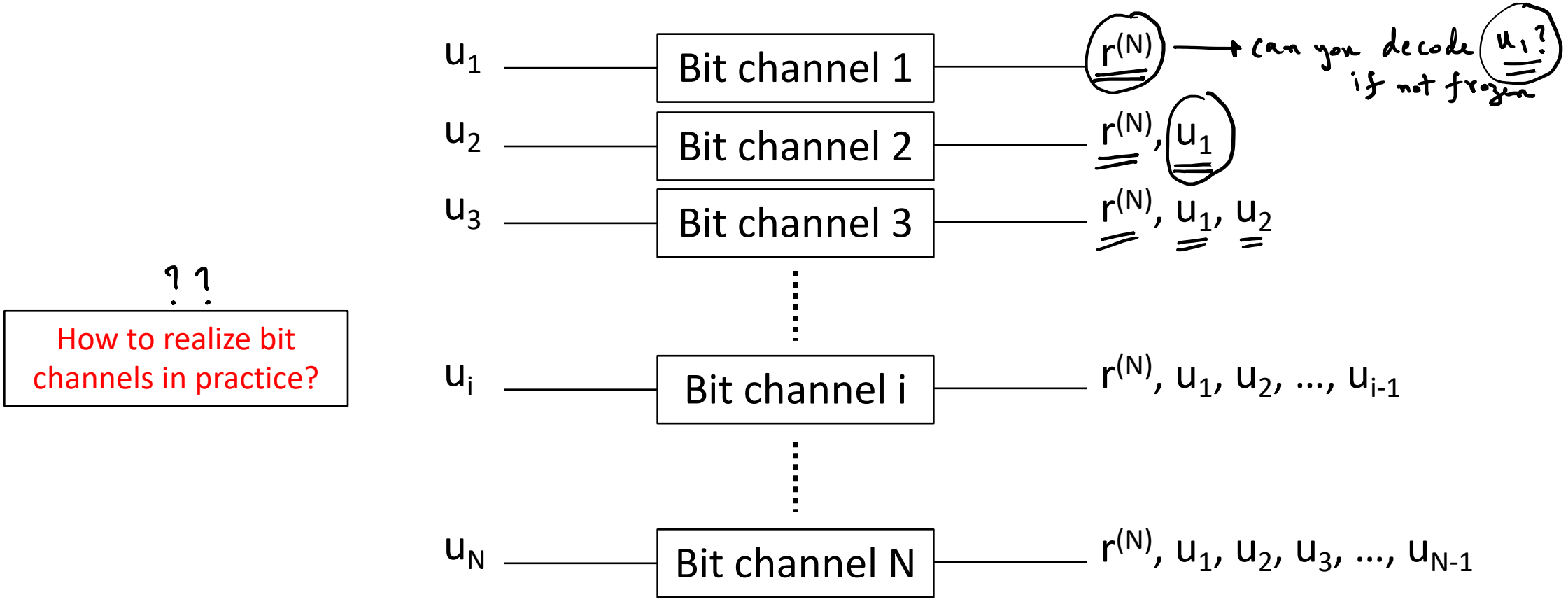
Polar Codes

Successive Cancellation Decoding

(N,K) Polar Code

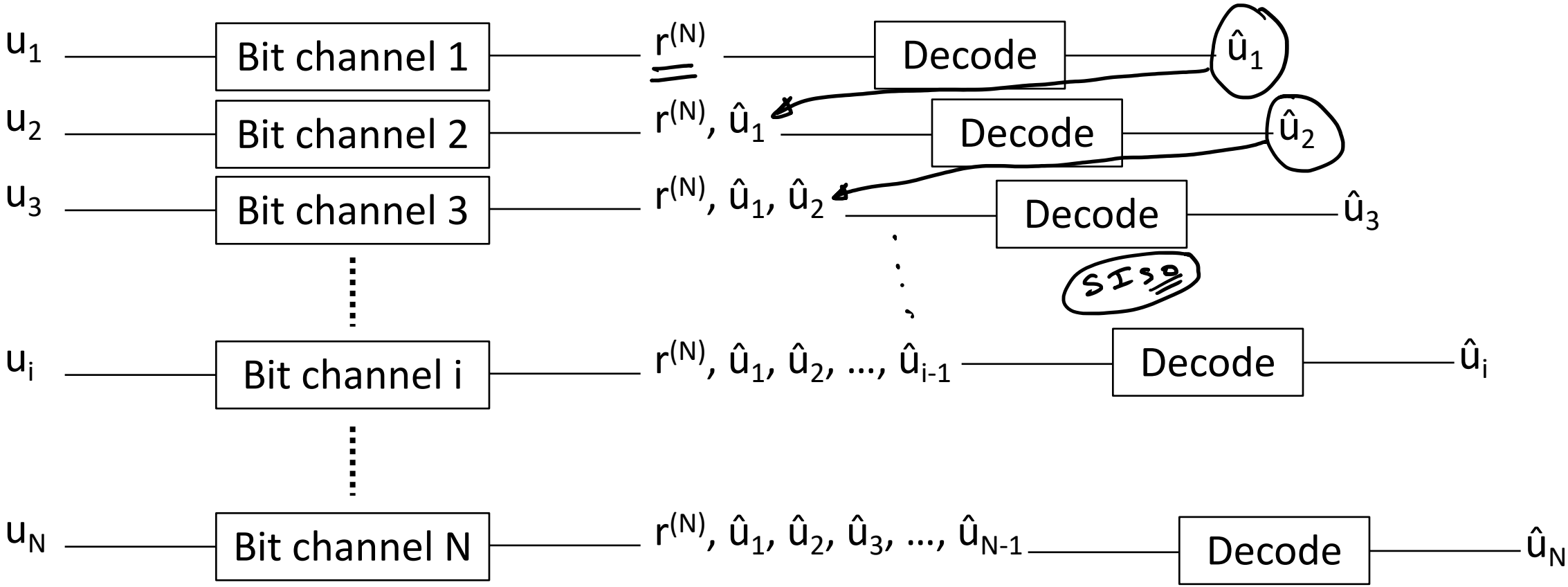
- $N = 2^n$
- Message: m of length K bits
- Form a vector u of length N bits as follows
  - Find N – K least reliable (worst) channels from reliability sequence
  - Set u_i for those N – K channels to zero (called frozen positions)
  - m: remaining K bits of u (called message positions)
- Codeword: u G_N

Bit channels and polarization



- Bit channels polarize and can be ordered based on “quality”: quality varies from very good to very bad

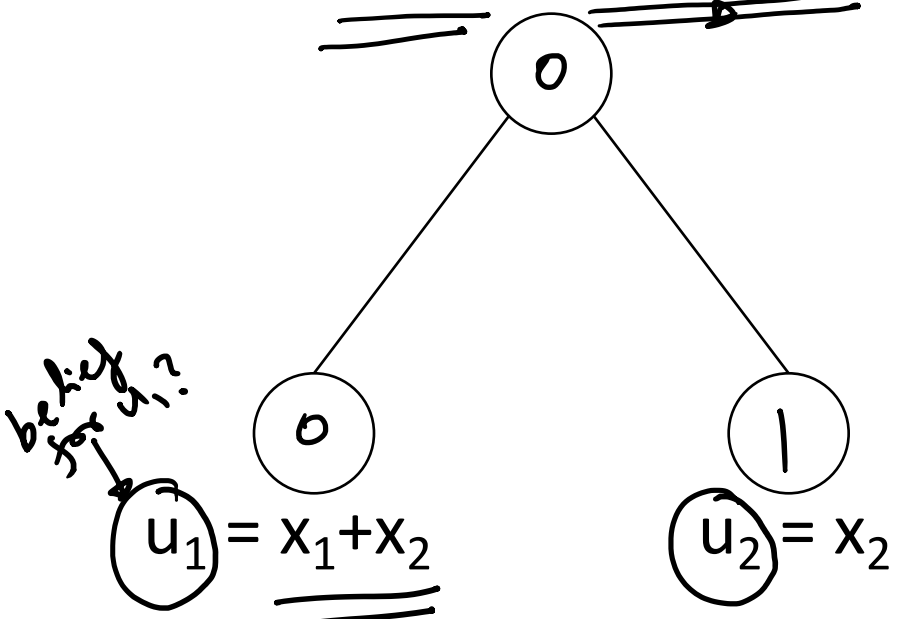
Successive Cancellation (SC) Decoding

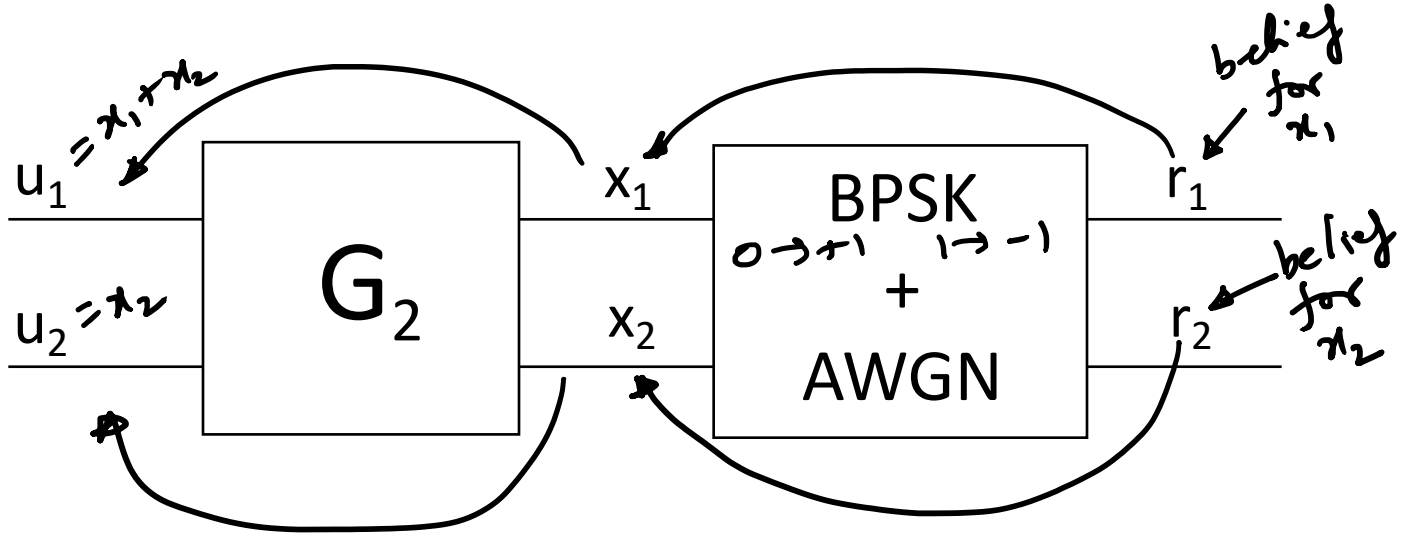


- If bit i is frozen, $\hat{u}_i = 0$
- If bit i is a message bit, $\hat{u}_i$ is found using a SISO decoder

Basic building block of SC decoder: N = 2

$x = [x_1\ x_2] = [u_1+u_2\ u_2]$





- SISO decode u_1 first (SPC)

$$L(u_1) = f(r_1,r_2) = \mathrm{sgn}(r_1)\,\mathrm{sgn}(r_2)\,\min(|r_1|, |r_2|)$$

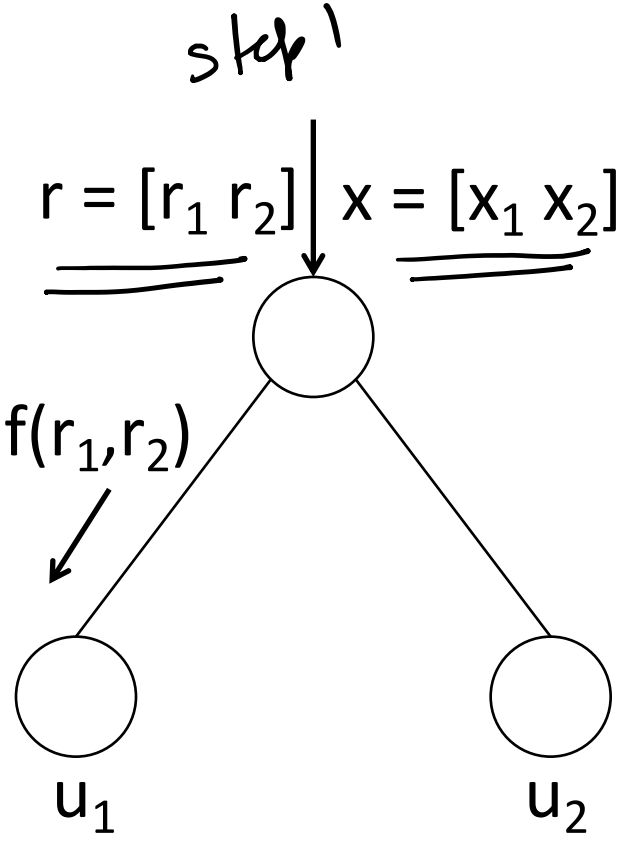
$\hat{u}_1 = 0$, if $L(u_1) \geq 0$; $\hat{u}_1 = 1$, if $L(u_1) < 0$

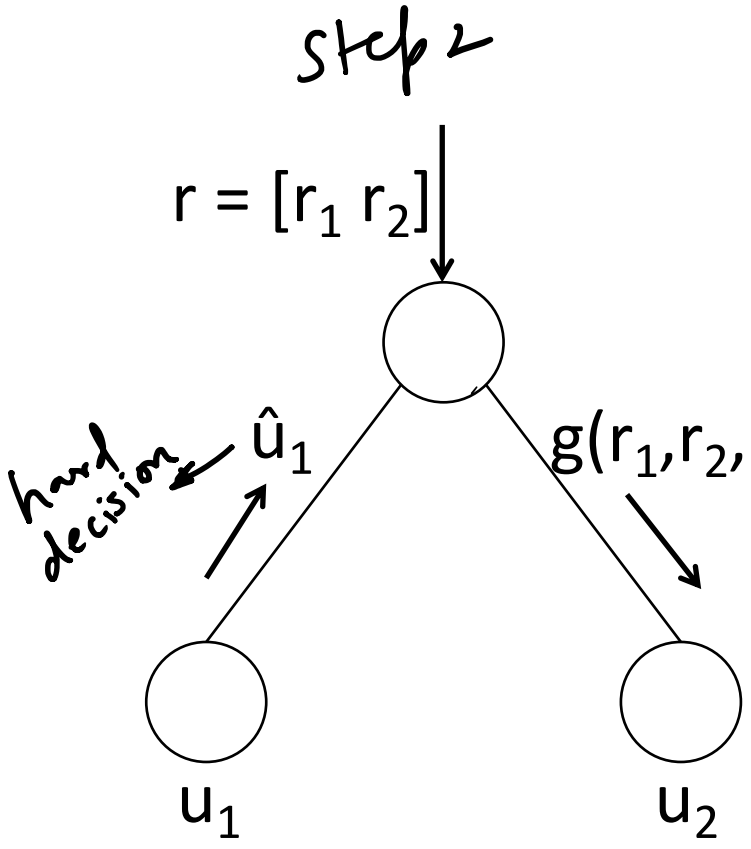
- Given $\hat{u}_1$, decode u_2 (Rep)

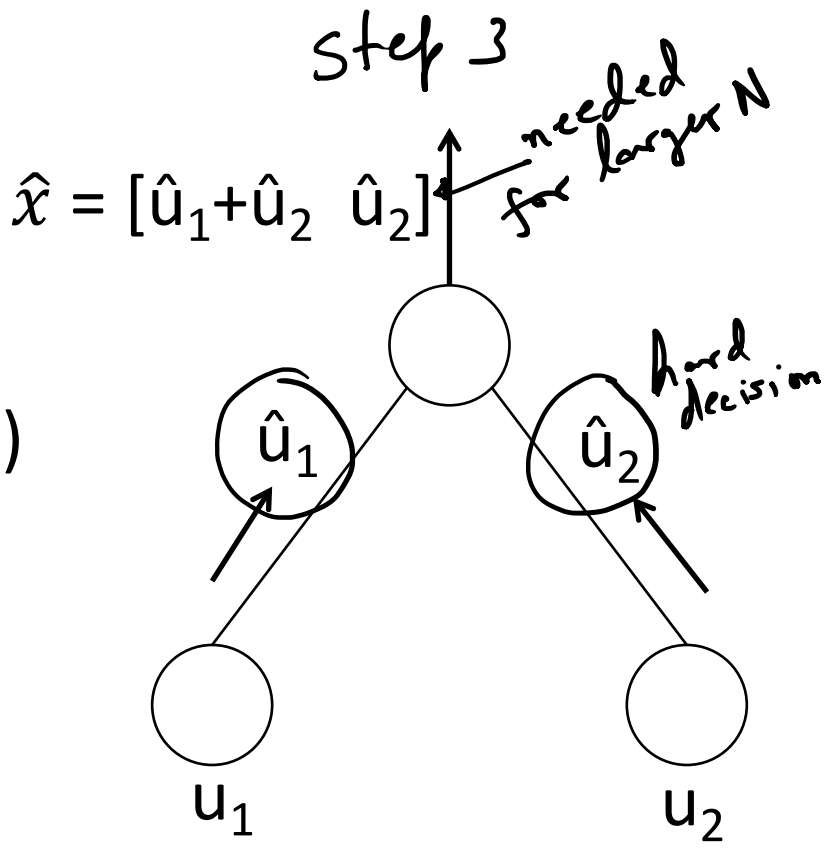
If $\hat{u}_1 = 0$, $L(u_2) = r_2 + r_1$ $\quad (x = [u_2\ u_2])$

If $\hat{u}_1 = 1$, $L(u_2) = r_2 - r_1$ $\quad (x = [\bar{u}_2\ u_2])$

Basic building block: message-passing on tree







minsum

$$f(r_1,r_2) = \mathrm{sgn}(r_1)\,\mathrm{sgn}(r_2)\,\min(|r_1|,\,|r_2|)$$

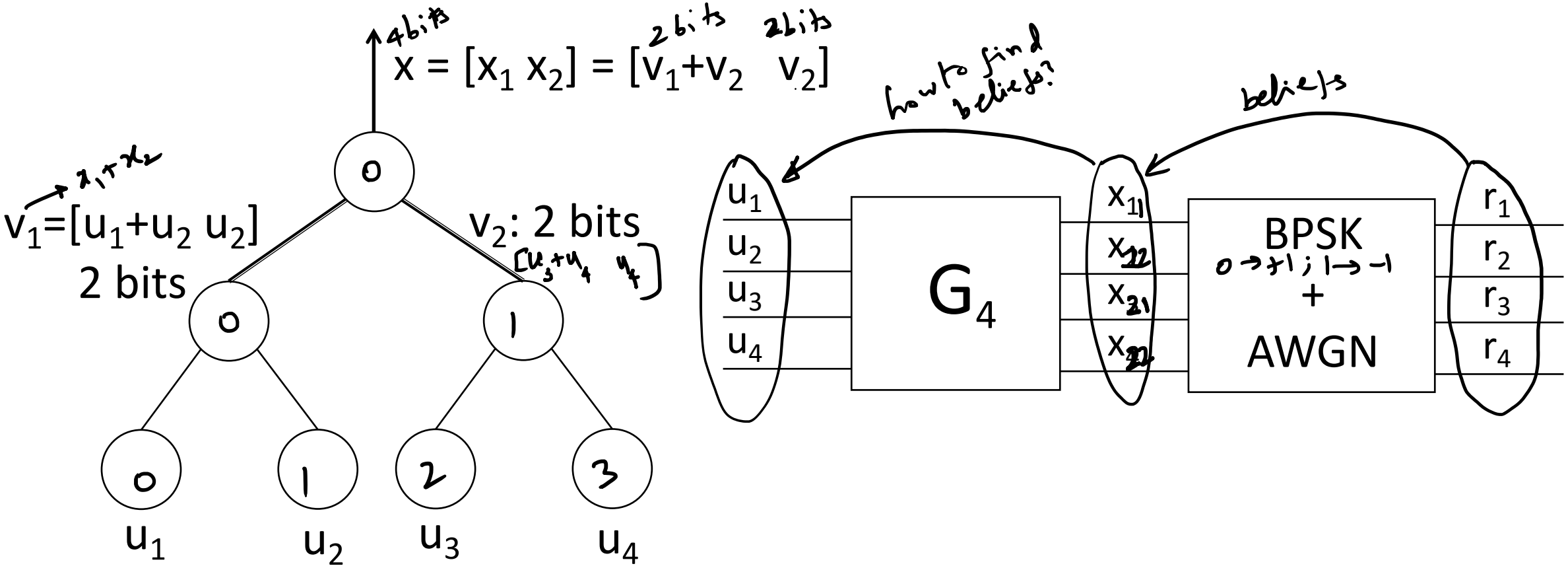
$$g(r_1,r_2,b) = r_2 + (1-2b)\,r_1$$

b: bit

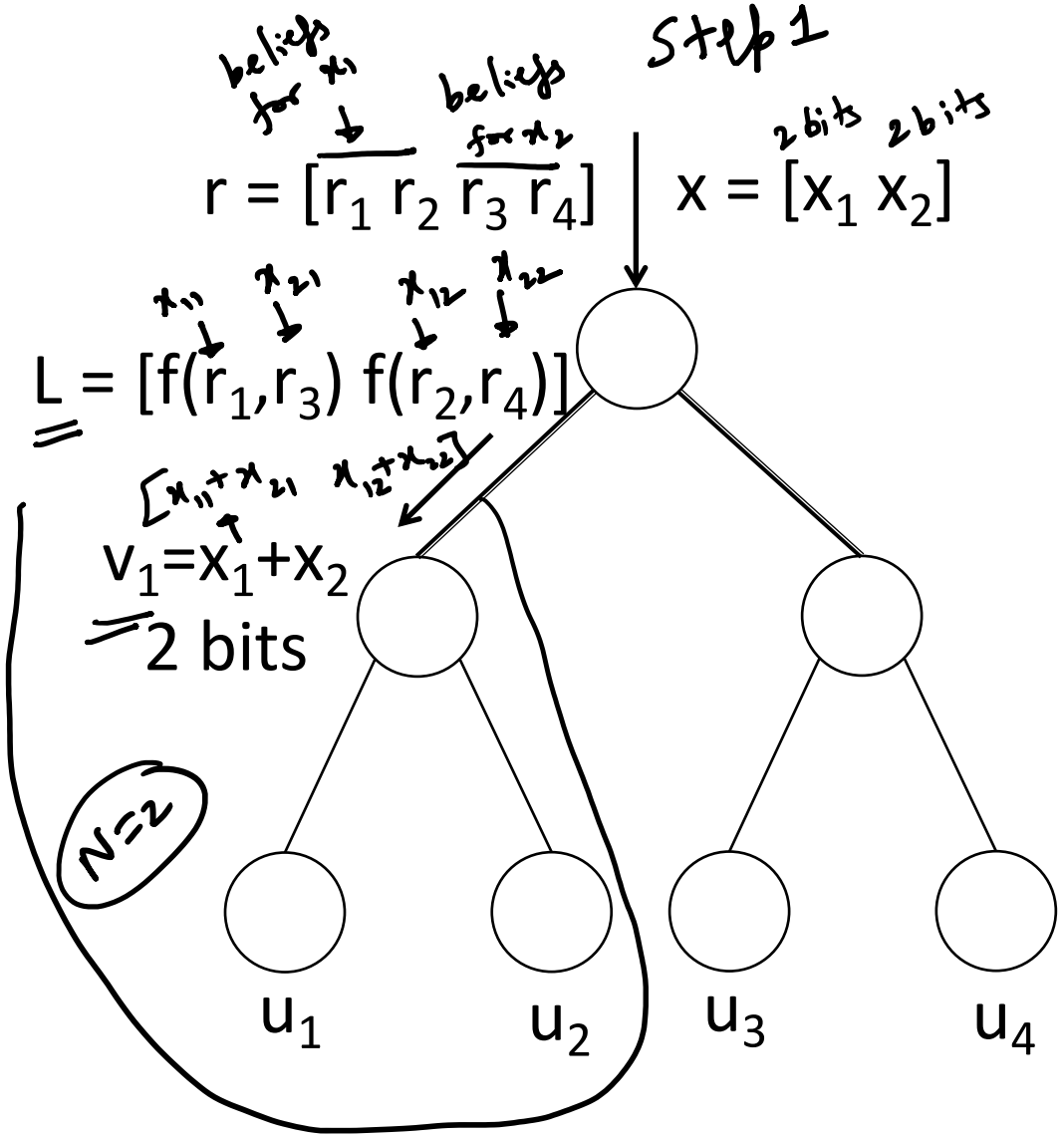
b=0: $r_2 + r_1$

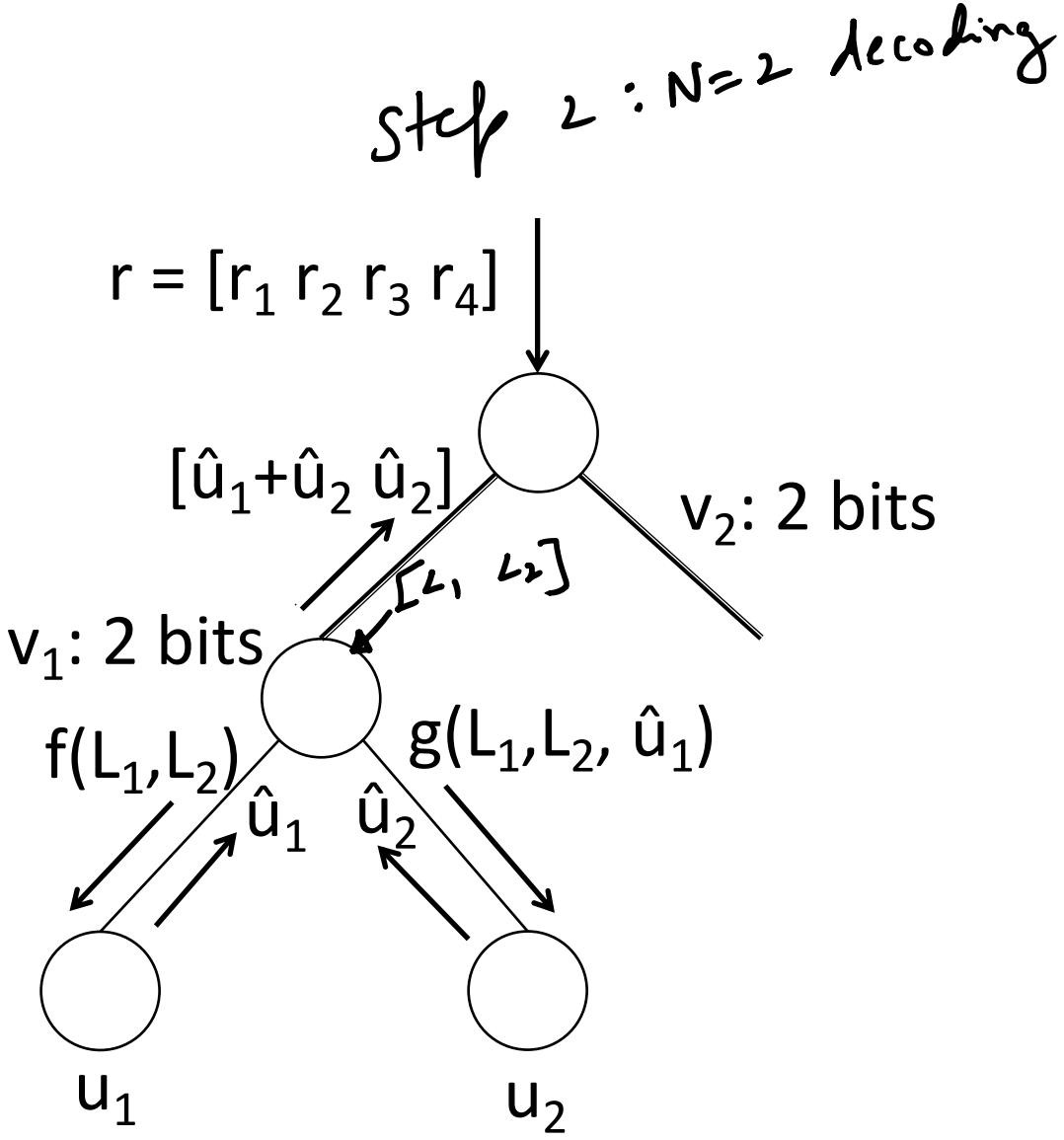
b=1: $r_2 - r_1$

SC decoder: N = 4



SC decoder: N = 4





SC decoder: N = 4