Encoding LDPC Codes in the 5G Standard

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Protograph example

$$B = \begin{bmatrix} 1 & -1 & 3 & 1 & 0 & -1 \\ 2 & 0 & -1 & 0 & 0 & 0 \\ -1 & 4 & 2 & 1 & -1 & 0 \end{bmatrix}$$

Expansion factor: 5

$H =$

0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0
0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0
1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0
0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1
0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0
1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0
0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1
0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1
0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1

Example: Base matrix entries

- 10 rows and 20 columns of BG2
  - iLS = 3, j = 4, Zc = 48

42 × 52

Expansion: 48

message part: 10×48 bits

double-diagonal structure

										p_1	p_2	p_3	p_4	p_5	p_6				
24	14	23	37	−1	−1	47	−1	−1	8	1	0	−1	−1	−1	−1	−1	−1	−1	−1
5	−1	−1	12	19	12	19	8	29	31	−1	0	0	−1	−1	−1	−1	−1	−1	−1
8	35	−1	46	47	−1	−1	−1	43	−1	0	−1	0	0	−1	−1	−1	−1	−1	−1
−1	41	6	−1	36	28	28	14	12	37	1	−1	−1	0	−1	−1	−1	−1	−1	−1
8	16	−1	−1	−1	−1	−1	−1	−1	−1	−1	5	−1	−1	0	−1	−1	−1	−1	−1
41	42	−1	−1	−1	26	−1	27	−1	−1	−1	1	−1	−1	−1	0	−1	−1	−1	−1
27	−1	−1	−1	−1	7	−1	31	−1	30	−1	17	−1	−1	−1	−1	0	−1	−1	−1
−1	7	−1	−1	−1	13	−1	9	−1	−1	−1	6	−1	37	−1	−1	−1	0	−1	−1
3	43	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	8	−1	−1	−1	−1	−1	0	−1
−1	2	−1	−1	−1	−1	−1	−1	30	−1	40	35	−1	−1	−1	−1	−1	−1	−1	0

identity

$\underline{m} = [m_1, m_2 \ldots m_{10}]$ $\quad m_i$: 48 bits $\quad p_i$: 48 bits

Example: Double-diagonal

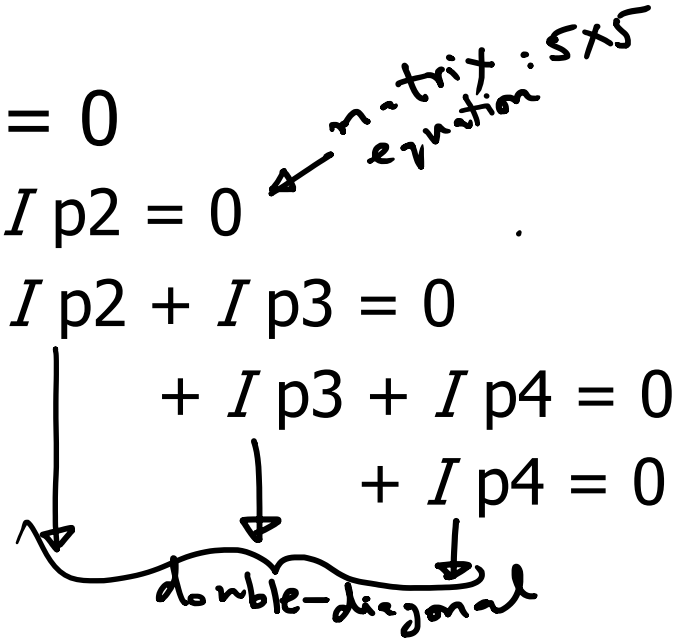
$$H = \left[\begin{array}{cccc|cccc} I_1 & 0 & I_3 & I_1 & I_2 & I & 0 & 0 \\ I_2 & I & 0 & I_3 & 0 & I & I & 0 \\ 0 & I_4 & I_2 & I & I_1 & 0 & I & I \\ I_4 & I_1 & I & 0 & I_2 & 0 & 0 & I \end{array}\right]$$

Expansion: 5

- I_k: identity matrix column-shifted k times
- Message: [m1 m2 m3 m4]
  - m1, m2, m3, m4: 5 bits each
- Codeword: [m1 m2 m3 m4 p1 p2 p3 p4]
  - p1, p2, p3, p4: 5 bits each

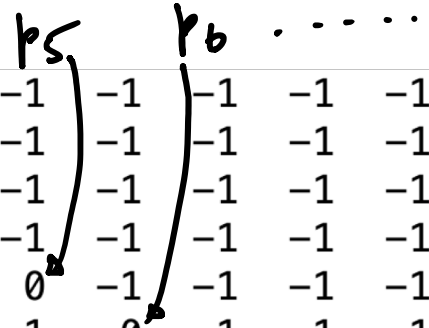
Double-diagonal encoding

- H [m1 m2 m3 m4 p1 p2 p3 p4]^T = 0
  - 1: I_1 m1 + I_3m3 + I_1 m4 + I_2p1 + I p2 = 0
  - 2: I_2m1 + I m2 + I_3 m3 + I p2 + I p3 = 0
  - 3: I_4m2 + I_2m3 + I m4 + I_1p1 + I p3 + I p4 = 0
  - 4: I_4m1 + I_1m2 + I m3 + I_2p1 + I p4 = 0
- Adding all 4
  - I_1p1 = I_1 m1 + I_3m3 + I_1 m4 + I_2m1 + I m2 + I_3 m3 + I_4m2 + I_2m3 + I m4 + I_4m1 + I_1m2 + I m3
  - Find p1 from above
- p2: use p1 in 1, p3: use p2 in 2, p4: use p3 in 3



Example: 5G base matrix

24	14	23	37	-1	-1	47	-1	-1	8	1	0	-1	-1	-1	-1	-1	-1	-1	-1
5	-1	-1	12	19	12	19	8	29	31	-1	0	0	-1	-1	-1	-1	-1	-1	-1
8	35	-1	46	47	-1	-1	-1	43	-1	0	-1	0	0	-1	-1	-1	-1	-1	-1
-1	41	6	-1	36	28	28	14	12	37	1	-1	-1	0	-1	-1	-1	-1	-1	-1
8	16	-1	-1	-1	-1	-1	-1	-1	-1	-1	5	-1	-1	0	-1	-1	-1	-1	-1
41	42	-1	-1	-1	26	-1	27	-1	-1	-1	1	-1	-1	-1	0	-1	-1	-1	-1
27	-1	-1	-1	-1	7	-1	31	-1	30	-1	17	-1	-1	-1	-1	0	-1	-1	-1
-1	7	-1	-1	-1	13	-1	9	-1	-1	-1	6	-1	37	-1	-1	-1	0	-1	-1
3	43	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	8	-1	-1	-1	-1	-1	0	-1
-1	2	-1	-1	-1	-1	-1	-1	30	-1	40	35	-1	-1	-1	-1	-1	-1	-1	0



- Message: [m1 m2 … m10], each 48 bits
- Parity: [p1 p2 p3 p4 p5 p6 …]
- First four rows: use double-diagonal encoding to find p1, p2, p3, p4
- Row 5: p5, Row 6: p6, and so on