LDPC Codes in the 5G Standard

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Definition

Low Density Parity Check

- LDPC Code: Linear block code with a sparse parity-check matrix[1]
- Sparse parity-check matrix also called an LDPC matrix
- H: LDPC matrix $n-k \times n$
  - Number of 1s << n(n-k) total entries
- Definition is very general; the class of LDPC codes is very large

[1]Gallager, R.G., Low-Density Parity-Check Codes, *MIT Press*, **1963**

Tanner Graph

- Graphical representation of a parity-check matrix[1]

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$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

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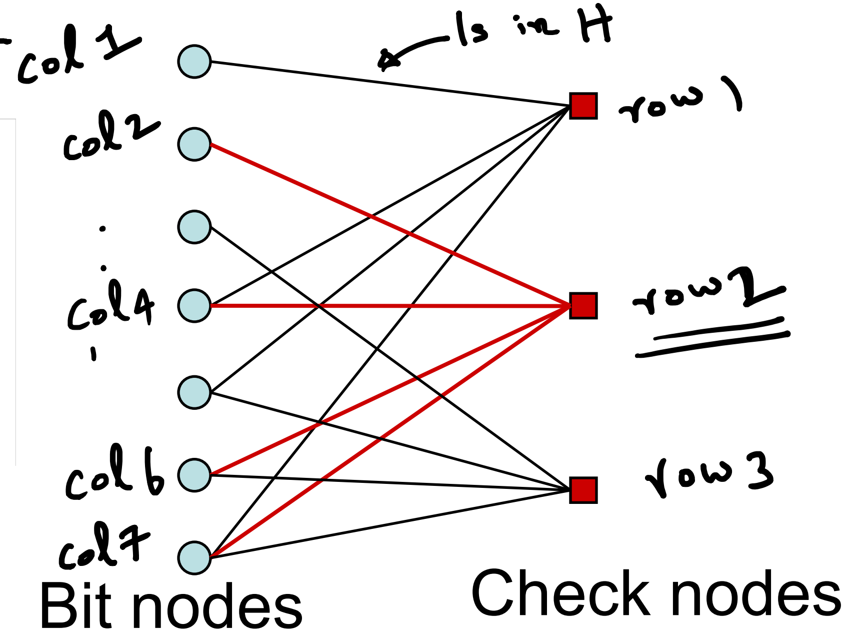


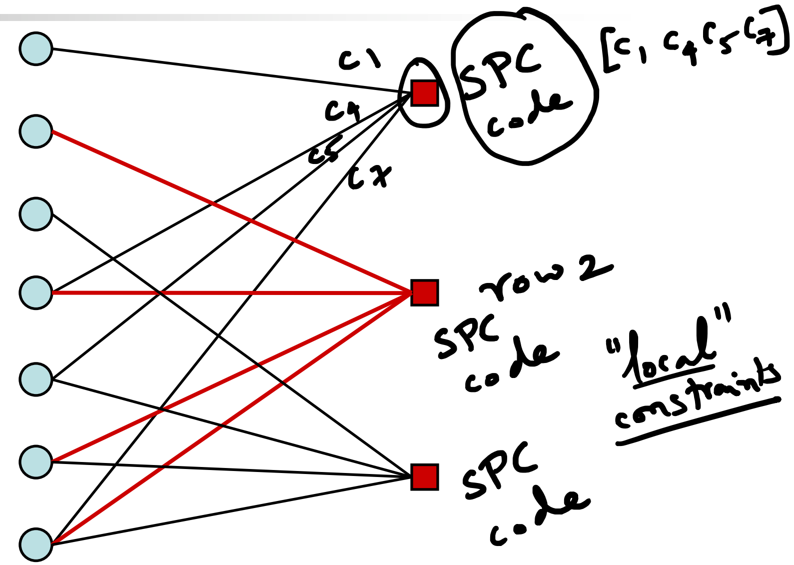
[1] Tanner, R.M., A recursive approach to low complexity codes, *IEEE Trans. on Info. Theory,* **1981** , *27* , 533-547

Each check node: SPC code

c = [c1 c2 c3 c4 c5 c6 c7]

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ \boxed{0} & \boxed{1} & \boxed{0} & \boxed{1} & \boxed{0} & \boxed{1} & \boxed{1} \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix}$$

$Hc^T = \underline{0}$

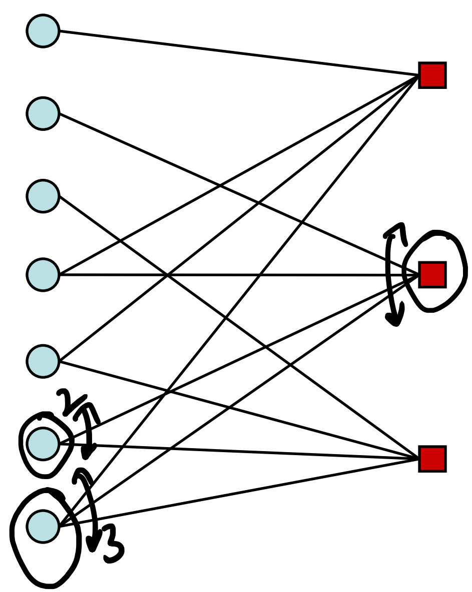


- Check node 2 <-> Row 2 of H
- c2 + c4 + c6 + c7 = 0 (mod 2)
- [c2 c4 c6 c7]: belong to (4,3) SPC code

Tanner Graph

- Bit nodes ⇔ Columns
- Check nodes ⇔ Rows
- Edges ⇔ 1s
- Degree of bit node ⇔ Weight of column
- Degree of check node ⇔ Weight of row

$$H = \begin{array}{l} \left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right] \begin{array}{c} \\ \leftarrow 4 \\ \\ \end{array} \\ \begin{array}{ccccccc} & & & & & \downarrow 2 & \downarrow 3 \end{array} \end{array}$$



LDPC codes in standards

- Protograph construction
  - Base graphs for different rates Base matrix
  - Expanded by right shift permutation matrices
  - Optimised for performance vs complexity
- All standards have such codes
- 5G – NR LDPC codes
  - Two base graphs
  - Several expansions
  - Shortening and puncturing for multiple rates

Protograph example

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$\boxed{-1, 0, 1, 2, 3, 4}$

Base matrix

$$B = \begin{bmatrix} 1 & -1 & 3 & 1 & 0 & -1 \\ 2 & 0 & -1 & 0 & 0 & 0 \\ -1 & 4 & 2 & 1 & -1 & 0 \end{bmatrix}$$

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Expansion factor: 5

30

5×5 identity

5×5 all zero

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$$H = \left[\begin{array}{ccccc|ccccc|ccccc|ccccc|ccccc|ccccc}
0&1&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
\hline
0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0\\
0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0\\
0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0\\
1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0\\
0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1\\
\hline
0&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{array}\right]$$

5G NR base matrices

- Two base matrices
  - BG1: 46 x 68 and BG2: 42 x 52
- Block structure of base matrices

$$\begin{bmatrix} A & E & O \\ B & C & I \end{bmatrix}$$

- BG 1
  - A: 4 x 22, E: 4 x 4, O: 4 x 42 all zero all −1's
  - B: 42 x 22, C: 42 x 4, I: 42 x 42 identity
- BG 2
  - A: 4 x 10, E: 4 x 4, O: 4 x 38 all zero
  - B: 38 x 10, C: 38 x 4, I: 38 x 38 identity