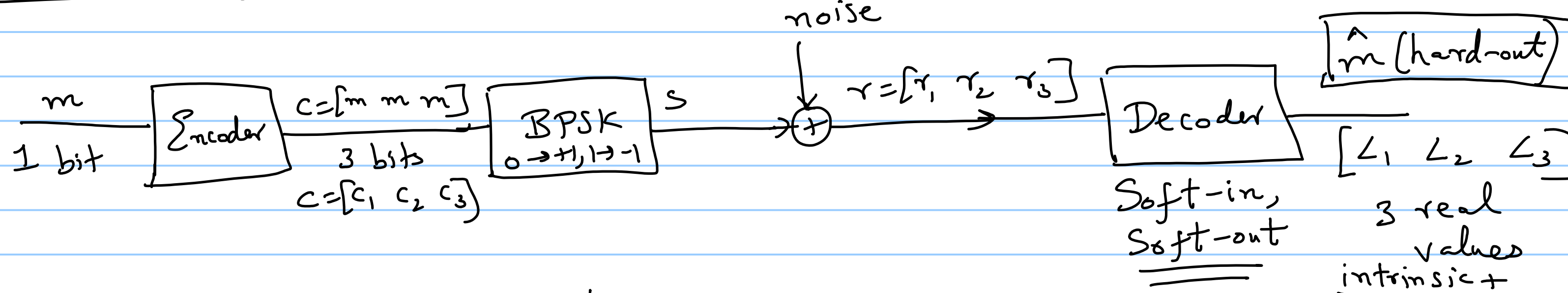
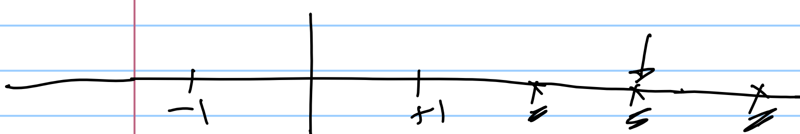
Repetition code (3,1)

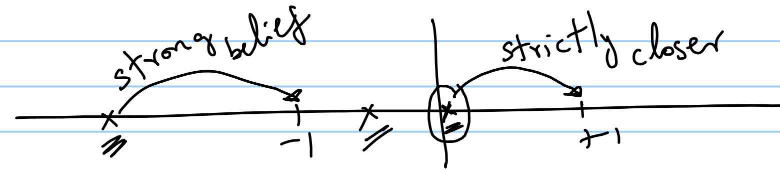


L_i : "belief" that bit c_i is 0.

"belief" about first bit based on r_1 alone

Ex: $r = [3.1 \quad 2.4 \quad 4.3]$ $\qquad r = [0.01 \quad -2.2 \quad -0.5]$





L_1 : computed using r_1, r_2 & r_3

$$L_3 = L_2 = L_1 = (\underbrace{r_1}_{\text{intrinsic}} + \underbrace{r_2 + r_3}_{\text{extrinsic}}) \cancel{\times \text{+ve factor}}$$

ignored in practice

$$L_2 = r_1 + r_2 + r_3$$

intrinsic extrinsic

Log-likelihood ratio:

prior prob $= 1/2$

$$\Pr(c_1=0 \mid r_1) = \frac{f(r_1 \mid c_1=0)\cdot \Pr(c_1=0)}{f(r_1)}$$

$$\Pr(c_1=1 \mid r_1) = \frac{f(r_1 \mid c_1=1)\, \Pr(c_1=1)}{f(r_1)}$$

likelihood ratio

$$\frac{\Pr(c_1=0 \mid r_1)}{\Pr(c_1=1 \mid r_1)} = \frac{f(r_1 \mid c_1=0)}{f(r_1 \mid c_1=1)}$$

$$= \frac{\frac{1}{\sqrt{2\pi}\,\sigma} e^{-(r_1-1)^2/2\sigma^2}}{\frac{1}{\sqrt{2\pi}\,\sigma} e^{-(r_1+1)^2/2\sigma^2}}$$

$c_1 = 0 \Rightarrow \text{symbol} = +1$
$\Downarrow$
$r_1 = 1 + N(0, \sigma^2)$

$c_1 = 1 \Rightarrow \text{symbol} = -1$
$\Downarrow$
$r_1 = -1 + N(0, \sigma^2)$

Channel LLR (or ~~Input LLR~~)

$$\frac{\Pr(c_1=0 \mid r_1)}{\Pr(c_1=1 \mid r_1)} = e^{2r_1/\sigma^2}$$

intrinsic LLR
↓
"belief"

$$l_1 = \log \frac{\Pr(c_1=0 \mid r_1)}{\Pr(c_1=1 \mid r_1)} = \frac{2}{\sigma^2}\cdot r_1 = r_1 \left(\times \begin{array}{c}+\text{ve}\\ \text{factor}\end{array}\right)$$

ignore

$$l_2 = \log \frac{\Pr(c_2=0 \mid r_2)}{\Pr(c_2=1 \mid r_2)} = \frac{2}{\sigma^2}\cdot r_2$$

Output LLR (SISO decoder)

$$L_i = \log \frac{P_r(c_i=0 \mid r_1, r_2, r_3)}{P_r(c_i=1 \mid r_1, r_2, r_3)}$$

$1/2$

L_1:

$$P_r(c_1=0 \mid r_1, r_2, r_3) = \frac{f(r_1, r_2, r_3 \mid c_1=0) \cdot P_r(c_1=0)}{f(r_1, r_2, r_3)}$$

$$P_r(c_1=1 \mid r_1, r_2, r_3) = \frac{f(r_1, r_2, r_3 \mid c_1=1)\, P_r(c_1=1)}{f(r_1, r_2, r_3)}$$

$$\frac{P_r(c_1=0 \mid r_1, r_2, r_3)}{P_r(c_1=1 \mid r_1, r_2, r_3)} = \frac{f(r_1, r_2, r_3 \mid c_1=0)}{f(r_1, r_2, r_3 \mid c_1=1)}$$

$c_1=0 \Rightarrow$ symbol vector $[+1 \quad +1 \quad +1]$

$$r_1 = 1 + N_1(0, \sigma^2)$$
$$r_2 = 1 + N_2(0, \sigma^2)$$
$$r_3 = 1 + N_3(0, \sigma^2)$$

N_1, N_2, N_3: independent

$$\frac{P_r(c_1=0 \mid r_1, r_2, r_3)}{P_r(c_1=1 \mid r_1, r_2, r_3)} = \frac{e^{-\frac{(r_1-1)^2}{2\sigma^2}} \cdot e^{-\frac{(r_2-1)^2}{2\sigma^2}} \cdot e^{-\frac{(r_3-1)^2}{2\sigma^2}}}{e^{-\frac{(r_1+1)^2}{2\sigma^2}} \cdot e^{-\frac{(r_2+1)^2}{2\sigma^2}} \cdot e^{-\frac{(r_2+1)^2}{2\sigma^2}}}$$

$$= e^{\frac{2}{\sigma^2}(r_1+r_2+r_3)}$$

$$\mathcal{L}_1 = \Big(\underset{\text{intrinsic}}{r_1} + \underbrace{r_2 + r_3}_{\text{extrinsic}}\Big) \times \frac{2}{\sigma^2}$$

$$\mathcal{L}_i = (r_1 + r_2 + r_3) \times \left(\begin{matrix}\text{+ve} \\ \text{factor}\end{matrix}\right)$$

set it ≈ 1