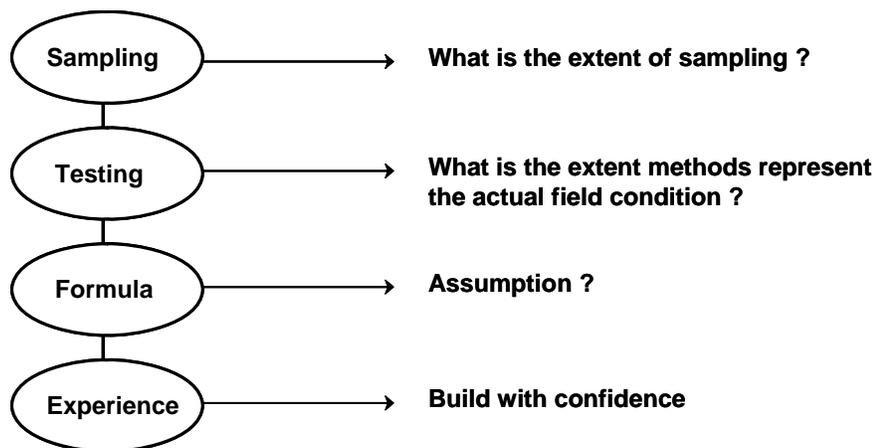


1. 1. Introduction

The trend in civil engineering today more than ever before is to provide

1. economical or robust design at certain levels of safety,
2. use new materials in construction. When newer materials are being used in civil engineering design, there is a need to understand to what extent the structure is safe, and
3. consider uncertainties in design. One has to recognize that there are many processes such as data collection, analysis and design in civil engineering systems which are random in nature. Design of many facilities such as: buildings, foundations, bridges, dams, highways, airports, seaports, offshore structures, tunnels, sanitary landfills, excavation etc. need to address the design issues rationally.

The loading in civil engineering systems are completely unknown. Only some of the features of the loading are known. Some of the examples of loading are frequency and occurrence of earthquakes, movement of ground water, rainfall pattern, wind and ice loadings etc. All these loading are random in nature, and at times they create overloading situation. What we have been doing so far can be schematically shown as follows:



At all stages indicated above, there is an element of uncertainty with regard to the suitability of the site in terms of soils, construction materials, which we transfer to a different level using a set of expressions to obtain the desired quantities such as the floor capacity, allowable loads in buildings etc.

1.2. Probability of failure and reliability

The failure of civil engineering systems is a consequence of decisions making under uncertain conditions and different type of failures such as temporary failures, maintenance failures, failures in design, failure due to natural hazards need to be addressed. For example, a bridge collapses which is a permanent failure, if there is a traffic jam on the bridge, it is a temporary failure. If there is overflow in a filter or a pipe due to heavy rainfall, it is a temporary failure. *Thus definition of failure is important. It is expressed in terms of probability of failure and is assessed by its inability to perform its intended function adequately on demand for a period of time under specific conditions.* The converse of probability of failure is called reliability and is defined in terms of the success of a system or reliability of a system *is the probability of a system performing its required function adequately for specified period of time under stated conditions.*

1. Reliability is expressed as a probability
2. A quality of performance is expected
3. It is expected over a period of time
4. It s expected to perform under specified conditions

1.2.1 Uncertainties in Civil engineering

In dealing with design, uncertainties are unavoidable. Uncertainties are classified into two broad types. Those associated with natural randomness and those associated with inaccuracies in our prediction and estimation of reality. The former type is called aleatory type where as the latter is called epistemic type. Irrespective of the classification understanding the nature of randomness is necessary. The nature of the first type arising out of nature (for example, earthquake and rainfall effects) needs to be handled rationally in design as it can not altered and the second one needs to be reduced using appropriate prediction models and sampling techniques.

The response of materials such as concrete, soil and rock to loading and unloading is of primary concern to the civil engineer. In all types of problems, the engineer is often dealing with incomplete information or uncertain conditions. It is necessary for the

engineer to be aware of many assumptions and idealizations on which methods of analysis and design are based. The use of analytical tools must be combined with sound engineering judgment based on experience and observation.

In the last two decades the need for solving complex problems has led to the development and use of advanced quantitative methods of modeling and analysis. For example, the versatile finite element method has proved to be valuable in problems of stability, deformation, earthquake response analysis etc. The rapid development of computers and computing methods has facilitated the use of such methods. However, it is well known that the information derived from sophisticated methods of analysis will be useful only if comprehensive inputs data are available and only if the data are reliable. Thus, the question of uncertainty and randomness of data is central to design and analysis in civil engineering.

Decisions have to be made on the basis of information which is limited or incomplete. It is, therefore, desirable to use methods and concepts in engineering planning and design which facilitate the evaluation and analysis of uncertainty. Traditional deterministic methods of analysis must be supplemented by methods which use the principles of statistics and probability. These latter methods, often called probabilistic methods, enable a logical analysis of uncertainty to be made and provide a quantitative basis for assessing the reliability of foundations and structures. Consequently, these methods provide a sound basis for the development and exercise of engineering judgment. Practical experience is always important and the observational approach can prove to be valuable; yet, the capacity to benefit from these is greatly enhanced by rational analysis of uncertainty.

1.2.2 Types of uncertainty

There are many uncertainties in civil geotechnical engineering and these may be classified into three main groups as follows:

(a) The first group consists of uncertainties in material parameters such as modulus of concrete, steel stability of concrete and steel in different condition such as tension and

flexure, soil unit weight, cohesion, angle of internal friction, pore water pressure, compressibility and permeability.

For example in a so-called homogeneous soil, each parameter may vary significantly. Moreover, natural media, i.e. earth masses are often heterogeneous and an isotropic and the soil profile is complex due to discontinuities and minor geological details.

(b) The second group consists of uncertainties in loads. Under static loading conditions, one is concerned with dead and live load and there are usually more uncertainties in relation to live loads. Structures and soil masses may also be subjected to dynamic loads from earthquakes, wind and waves. Significant uncertainties are associated with such random loads. Often the uncertainties associated with static loads may be negligible in comparison to those associated with material parameters. On the other hand, uncertainties associated with dynamic loads may be of the same order of magnitude or even greater than those associated with material parameters. It should also be noted that under dynamic loads, the magnitude of material parameters may change significantly.

For example, the shear strength of a soil decreases during cyclic loading and, as such, there are additional uncertainties concerning geotechnical performance.

(c) The third group consists of uncertainties in mathematical modeling and methods of analysis. Each model of soil behavior is based on some idealization of real situations. Each method of analysis or design is based on simplifying assumptions and arbitrary factors of safety's are often used.

1.3 Deterministic and probabilistic approaches

1.3.1. Deterministic approach

An approach based on the premise that a given problem can be stated in the form of a question or a set of questions to which there is an explicit and unique answer is a deterministic approach. For example, the concept that unique mathematical relationships govern mechanical behavior of soil mass or a soil structure system.

In this method of analysis or design one is concerned with relatively simple cause and effect relationship. For each situation it is assumed that there is a single outcome; for each problem a single and unique solution. Of course, one may not be able to arrive at the exact solution and also unique solution may not exist. In such circumstances a deterministic approach aims at obtaining approximate solution. Empirical and semi-empirical methods have always been used in civil engineering although with varying degrees of success. Finally, in deterministic method of analysis, uncertainty is not formally recognized or accounted for one is not concerned with the probabilistic outcome but with well defined outcomes which may or may not occur, that is, either a 100% probability of occurrence or 0% without intermediate value.

For example, one may arrive at the conclusion that a foundation will be safe on the basis that the safety factor, F , has a magnitude greater than one. On the other hand, one may conclude that a foundation or a slope is not safe on the basis that the magnitude of the factor of safety F is less than one. A given magnitude of F . e.g. $F = 2.5$ represents a unique answer to a problem posed in specific terms with certain unique values of loads and of shear strength parameters. In conventional analysis one is not concerned with the reliability associated with this unique value.

1.3.2. Probabilistic approach

A probabilistic approach is based on the concept that several or varied outcomes of a situation are possible to this approach uncertainty is recognized and yes/no type of answer to a question concerning geotechnical performance is considered to be simplistic. Probabilistic modeling aims at study of a range of outcomes given input data. Accordingly the description of a physical situation or system includes randomness of data and other uncertainties. The selected data for a deterministic approach would, in general not be sufficient for a probabilistic study of the same problem. The raw data would have to be organized in a more logical way. Often additional data would be for meaningful probabilistic analysis.

A probabilistic approach aims determining the probability p , of an outcome, one of many

that may occur, The probability would be any percentage between $p = 0\%$ and $p=100\%$ or any fraction between $p = 0$ and $p=1$. In a specific problem the number of likely outcomes may be limited and it may be possible to consider the probability of each outcome.

1.4 Risk and reliability

In engineering practice, we routinely encounter situations that involve some event that might occur and that, if it did, would bring with it some adverse consequence. We might be able to assign probability to the occurrence of the event and some quantified magnitude or cost to the adversity associated with its occurrence. This combination of uncertain event and adverse consequence is the determinant of risk. In engineering practice to assess risk, three things need to be defined.

1. Scenario,
2. Range of consequences,
3. Probability of the event's leading to the consequences.

Based on the above, the risk analysis attempts to answer three questions:

1. What can happen?
2. How likely is it to happen?
3. Given that it occurs, what are the consequences?

Thus, in engineering, risk is usually defined as comprising:

- A set of scenarios (or events), $E_i, i= 1, \dots, n$;
- Probabilities associated with each element, p_i and
- Consequences associated with each element, c_i .

The quantitative measure of this risk might be defined in a number of ways.

In engineering context, risk is commonly defined as the product of probability of failure and consequence, or expressed another way, risk is taken as the expectation of adverse outcome:

$$\text{Risk} = (\text{probability of failure} \times \text{consequence}) = (pc) \quad \text{-----}(1)$$

The term risk is used, when more than one event may lead to an adverse outcome then above equation is extended to be the expectation of consequence over that set of events:

$$\text{Risk} = \sum_i p_i c_i \quad \text{-----}(2)$$

In which p_i is i^{th} probability and c_i is corresponding consequence.

Another measure is called reliability which is defined as (1-probability of failure) and expresses probability of safety. It is called reliability and is related to reliability index (β) which is a useful way of describing the boundary between the safe and unsafe boundaries.

1.4.1 Acceptable Risks

In engineering as in other aspects, lower risk usually means higher cost. Thus we are faced with question “*how safe is safe enough*” or “*what risk is acceptable?*”. In the United States, the government acting through Congress has not defined acceptable levels of risk for civil infrastructure, or indeed for most regulated activities. The setting of ‘reasonable’ risk levels or at least the prohibition of ‘unreasonable’ risks is left up to regulatory agencies, such as the Environmental Protection Agency, Nuclear Regulatory Commission, or Federal Energy Regulatory Commission. The procedures these regulatory agencies use to separate reasonable from unreasonable risks vary from highly analytical to qualitatively procedural.

People face individual risks to health and safety every day, from the risk of catching a dread disease, to the risk of being seriously injured in a car crash (Table 1). Society faces risks that large numbers of individuals are injured or killed in major catastrophes (Table 2). We face financial risks every day, too, from the calamities mentioned to the risk of losses or gains in investments. Some risks we take on voluntarily, like participating in sports or driving an automobile. Others we are exposed to involuntarily, like a dam failing upstream of our home or disease.

Table 1 – Average risk of death of an individual from various human caused and natural accidents (US Nuclear regulatory Commission 1975)

Accident Type	Total Number	Individual Chance Per Year
Motor Vehicles	55,791	1 in 4,000
Falls	17,827	1 in 10,000
Fires and Hot Substances	7,451	1 in 25,000
Drowning	6,181	1 in 30,000
Firearms	2,309	1 in 100,000
Air Travel	1,778	1 in 100,000
Falling Objects	1,271	1 in 160,000
Electrocution	1,148	1 in 160,000
Lightning	160	1 in 2,500,000
Tornadoes	91	1 in 2,500,000
Hurricanes	93	1 in 2,500,000
All Accidents	111,992	1 in 1,600

Table 2 : Average risk to society of multiple injuries of deaths from various human-caused and natural accidents (US Nuclear Commission 1975)

Type of event	Probabilities of 100 or more fatalities	Probability of 100 or more
Human Caused		
Airplane Crash	1 in 2 yrs	1 in 2000 yrs
Fire	1 in 7 yrs	1 in 200 yrs
Explosion	1 in 16 yrs	1 in 120 yrs
Tonic gas	1 in 100 yrs	1 in 1000 yrs
Natural		
Tornado	1 in 5 yrs	Very small
Hurricane	1 in 5 yrs	1 in 25 yrs
Earthquake	1 in 20 yrs	1 in 50 yrs
Meteorite impact	1 in 100,000 yrs	1 in I million yrs

Four observations made in literature on acceptable risk are:

1. The public is willing to accept ‘voluntary risks roughly 1000 times greater than ‘involuntary’ risks’,
2. Statistical risk of death from disease appears to be a psychological yardstick for establishing the level of acceptability of other risks
3. The acceptability of risk appears to be proportional to the third—power of the benefits,
4. The societal acceptance of risk is influenced by public awareness of the benefits of an activity, as determined by advertising, usefulness and the number of people participating. The exactness of these conclusions has been criticized, but the insight that acceptable risk exhibits regularities is important.

1.4.2. Risk perception

People view risks not only by whether those risks are voluntary or involuntary, or by whether the associated benefits outweigh the dangers but also along other dimensions. Over the past twenty years researchers have attempted to determine how average citizens perceive technological risks. Better understanding of the way people perceive risk may help in planning projects and in communication. The public’s perception of risk is more subtle than the engineers.

Table 3. Risk perception

Separation of risk perception along two factor dimension	
Factor I : Controllable vs. Uncontrollable	
Controllable	Uncontrollable
Not dread	Dread
Local	Global
Consequences not fatal	Consequences fatal
Equitable	Not equitable
Individual	Catastrophic
Low risk to future generation	High risk to future generation
Easily reduced	Not easily reduced
Risk decreasing	Risk increasing

Voluntary	Involuntary
Factor II : Observable vs. Unobservable	
Observable	Unobservable
Known to those exposed	Unknown to those exposed
Effect immediate	Effect delayed
Old risk	New risk
Risk known to science	Risk unknown to science

1.5.F-N Charts

In a simple form, quantitative risk analysis involves identification of risks and damages/fatalities. It is recognized that in many cases, the idea of annual probability of failure, depending on F-N relationships (frequency of fatalities (f), and number of fatalities (N)) is a useful basis. In UK, risk criteria for land use planning made based on F-N curves (frequency - Number of fatalities) on annual basis suggest lower and upper limits of 10^{-4} and 10^{-6} per annum for probability of failure or risk. Guidance on risk assessment is reasonably well developed in many countries such as USA, Canada and Hong Kong. Whitman (1984) based on the collected data pertaining to performance of different engineering systems categorized these systems in terms of annual probability of failure and their associated failure consequences, as shown in Fig.1.

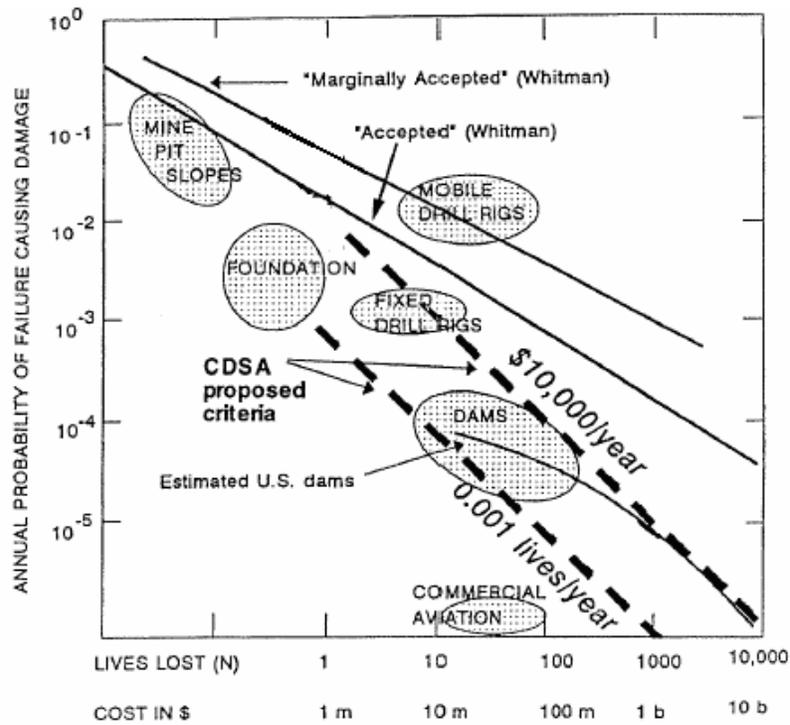


Figure 1. Annual probabilities of failure and consequence of failure for various engineering projects (Whitman 1984)

Some guidelines on tolerable risk criteria are formulated by a number of researchers and engineers involved in risk assessment. They indicate that the incremental risk should not be significant compared to other risks and that the risks should be reduced to "As Low As Reasonably Practicable" (ALARP) as indicated in Fig.2. Figure 2 shows a typical f-N diagram adopted by Hong Kong Planning Department (Hong Kong government planning department 1994).

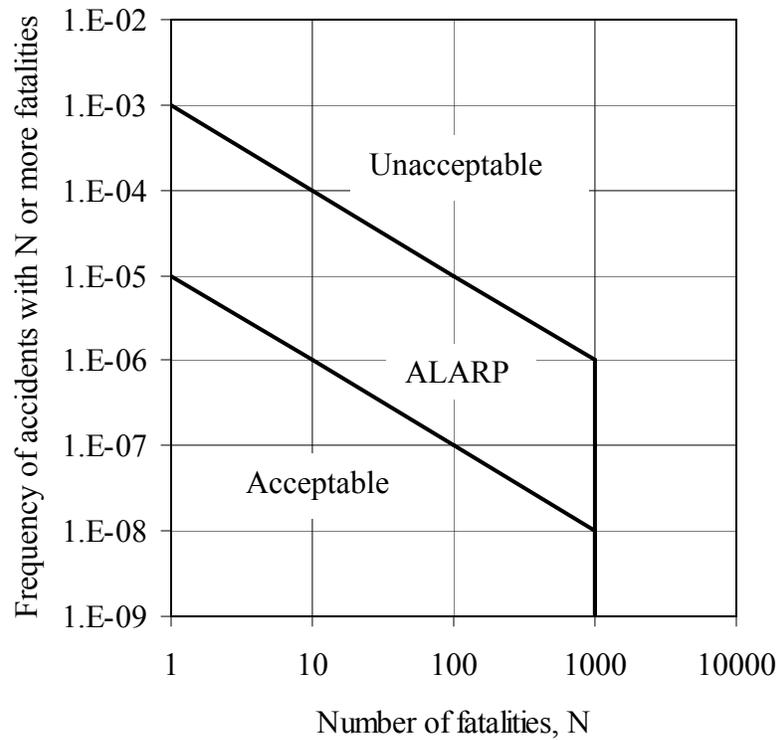


Figure 2. F–N diagram adopted by Hong Kong Planning Department for planning purposes.

The slope of the lines dividing regions of acceptability expresses a policy decision between the relative acceptability of low probability/high consequence risks and high probability/low consequence risks. The steeper the boundary lines, the more averse are the policy to the former. The boundary lines in the Hong Kong guidelines are twice as steep (in log-log space) as the slopes in the Dutch case. Also that, in the Hong Kong case there is an absolute upper bound of 1000 on the number of deaths, no matter how low the corresponding probability.

The role of probabilistic considerations is recognized in Corps of Engineers USA and guidelines on reliability based design of structures is suggested. Fig.3 presents the classification. The annual probability of failure corresponds to an expected factor of safety $E(F)$, which is variable and the variability is expressed in terms of standard deviation of factor of safety σ_F . If factor of safety is assumed to be normally distributed, reliability index (β) is expressed by

$$\beta = \frac{(E(F) - 1.0)}{\sigma_F} \quad (3)$$

The guidelines present the recommendations in terms of probability of failure p_f , or reliability index (β).

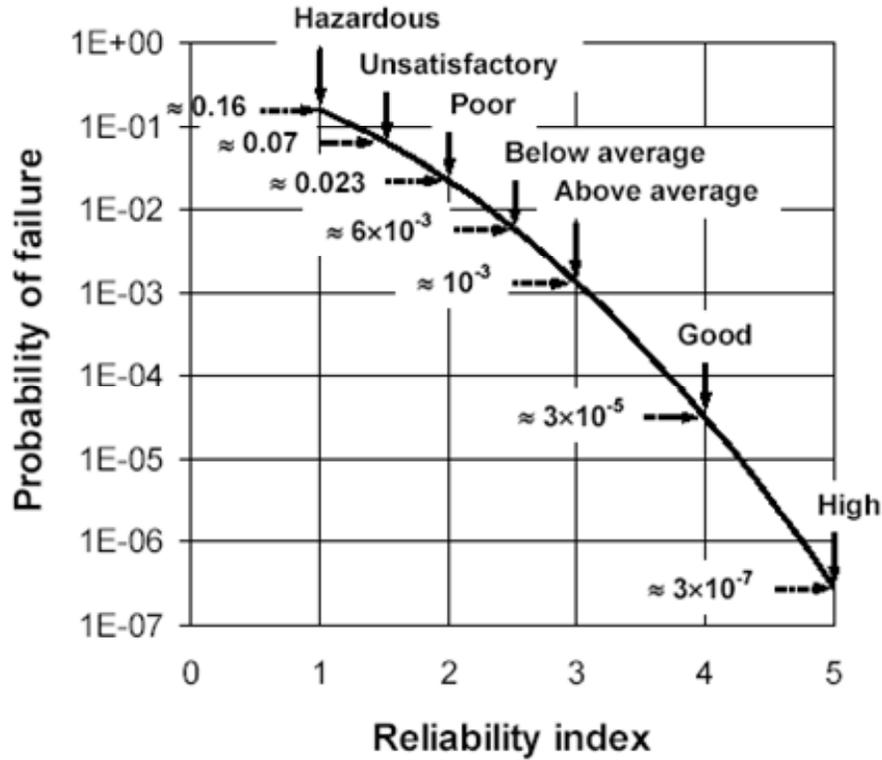


Fig. 3 Relationship between reliability index (β) and probability of failure (p_f) (Phoon 2002) (adapted from US Army Corps of Engineers 1997).

1.6 Role of consequence cost

The role of consequence costs is realised in recent times and has been receiving considerable attention in the geotechnical profession. Recently, Joint Committee on Structural Safety (JCSS 2000) presented relationships between reliability index (β), importance of structure and consequences of failure. The committee divided

consequences into 3 classes based on risk to life and/or economic loss, and they are presented in Tables 4 and 5 respectively.

From these tables, it can be inferred that if the failure of a structure is of minor consequence (i.e., $C^* \leq 2$), then a lower reliability index may be chosen. On the other hand, if the consequence costs are higher (i.e., $C^* = 5$ to 10) and if the relative cost of safety measures is small, higher reliability index values can be chosen. It can also be noted from the tables that reliability index in the range of 3 to 5 can be considered as acceptable in design practice.

Table 4. Relationship between reliability index (β), importance of structure and consequences (JCSS 2000)

Relative cost of safety measure	Minor consequence of failure	Moderate consequence of failure	Large consequence of failure
Large	$\beta = 3.1$	$\beta = 3.3$	$\beta = 3.7$
Normal	$\beta = 3.7$	$\beta = 4.2$	$\beta = 4.4$
Small	$\beta = 4.2$	$\beta = 4.4$	$\beta = 4.7$

Table 5. Classification of consequences (JCSS 2000)

Class	Consequences	C^*	Risk to life and/or economic consequences
1	Minor	≤ 2	Small to negligible and small to negligible
2	Moderate	$2 < C^* \leq 5$	Medium or considerable
3	Large	$5 < C^* \leq 10$	High or significant

where C^* is the normalized consequence cost (normalized with respect to initial cost).

From Tables 4 and 5 the following aspect points are clear.

1. The targeted reliability indices vary from 3 to 5, depending on the expected level of performance.
2. Consequence costs can also be considered in the analysis. If the consequence costs are not significant compared to initial costs ($C^* \leq 2$) (for example slope design in a remote area), lower reliability index can be allowed, whereas higher reliability index is required where the consequence costs are high (for example slope in an urban locality).

Axioms of probability

A popular definition of probability is in terms of relative frequency of an outcome A occurs T times in N equally likely trials,

$$P[A] = T / N$$

It implies that if large number of trials were conducted this probability is likely. As the concept of repeated trials does not exist in civil engineering, subjective interpretation is considered, which it implies that it is a measure of information as to the likelihood of an occurrence of an outcome. The three axioms of probability are given by

Axiom I: $0 \leq P[A] \leq 1$

Axiom II: The certainty of outcome is unity i.e. $P[A] = 1$

Axiom III: This axiom requires the concept of mutually exclusive outcomes. Two outcomes are mutually exclusive, if they cannot occur simultaneously. The axiom states that

$$P[A_1 + A_2 + \dots + A_N] = P[A_1] + P[A_2] + P[A_3] + \dots + P[A_N] \quad (4)$$

As an example consider the design of structure. After construction only two outcomes are possible either success or failure. Both are mutually exclusive, they are also called exhaustive and no other outcome is also possible. As per axiom III,

$$P[\text{Success}] + P[\text{Failure}] = 1$$

The probability of success of the structure is reliability is given by

$$R + P[\text{Failure}] = 1 \quad \text{or} \quad R = 1 - P[\text{Failure}] \quad (5)$$

Basic Probability Concept

By probability we are referring to a number of possibilities in a given situation and identify an event relative to other events. Probability can be considered as a numerical measure of likelihood of occurrence of an event, relative to a set of alternatives. First requirement is to

1. Identify all possibilities on a set
2. Identify the event of interest

In this context elements of set theory are very useful.

Elements of set theory

Many Characteristics of probability can be understood more clearly from notion of sets and sample spaces.

Sample space

Sample space is a set of all possibilities in a probabilistic problem. This can be further classified as continuous sample space and discrete sample space. Again discrete sample space can be further classified as finite and infinite cases.

Discrete Sample Space

Example for finite case:

1. The winner in a competitive bidding
2. The number of raining days in a year

Example for Infinite case:

1. Number of flaws in a road
2. Number of cars crossing a bridge

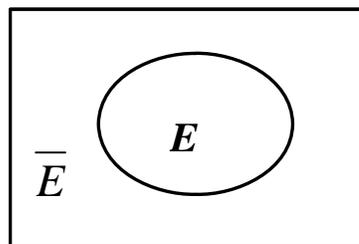
Sample point is a term used to denote each of the individual possibilities is a sample point

Continuous Sample Space

If number of sample points is effectively infinite, then it can be called as continuous sample space. For example, the bearing capacity of clay deposit varies from 150 to 400 kPa and any value between them is a sample point.

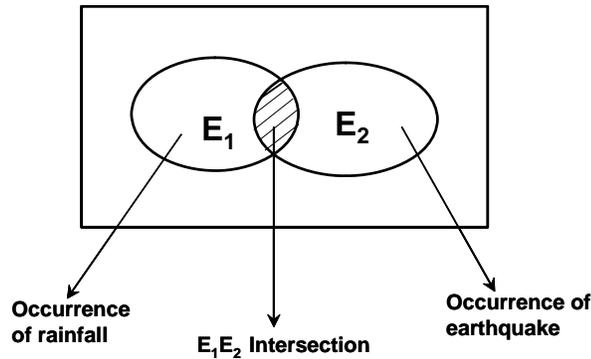
Venn diagram

A sample space is represented by a rectangle, an event (E) is represented by a closed region. The part outside is complimentary event \bar{E}



Combinations of events

Example



If an event E_1 occurs n_1 times out of n times, it does not occur n_2 times. i.e. $n_2 = n - n_1$ for which the probability of non-occurrence being $\frac{n_2}{n}$

$$P[E_1 \cup E_2] = \frac{n_1}{n} + \frac{n_2}{n} = P[E_1] + P[E_2] \quad (6)$$

Multiplication rule and Statistical Independence

The occurrence (or non-occurrence) of one event does not affect the probability of other event, the two events are statistically independent

If they are dependent then

$$\begin{aligned} P[E_1E_2] &= P\left[\frac{E_1}{E_2}\right]P[E_2] \\ &= P\left[\frac{E_2}{E_1}\right]P[E_1] \end{aligned}$$

If they are independent then, $P[E_1E_2] = P[E_1]P[E_2]$ (7)

By multiplication rule

$$P\left[\frac{A}{E_i}\right]P[E_i] = P[A]P\left[\frac{E_i}{A}\right] \quad (8)$$

The above equation gives the probability of occurrence of $P\left[\frac{E_i}{A}\right]$ if $P\left[\frac{A}{E_i}\right]$ is known I.

$$P\left[\frac{E_i}{A}\right] = \frac{P\left[\frac{A}{E_i}\right]P[E_i]}{P[A]} \quad (9)$$

Hence $\frac{P[E_1 \cup E_2]}{P[E_2]} = P_1[E_1]$

→ $P[E_1 \cup E_2] = P[E_1]P[E_2]$ ----- Multiplication Rule

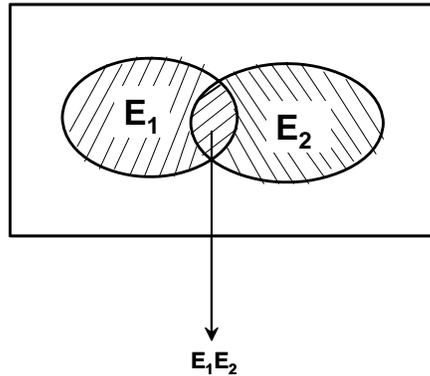
A generalized multiplication rule is

$$P[A_1A_2A_3.....A_N] = P[A_1]P[A_2]P[A_3].....P[A_N] \quad (10)$$

Conditional Probability

The occurrence of an event depends on the occurrence (or non-occurrence) of another event. If this dependence exists, the associated probability is called conditional probability. The conditional probability E_1 assuming E_2 occurred $P\left(\frac{E_1}{E_2}\right)$ means the likelihood of realizing a sample point in E_1 assuming it belongs to E_2 (we are interested in the event E_1 within the new sample space E_2)

$$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 E_2)}{P(E_2)}$$



Total probability theorem and Bayesian Probability

There are N outcomes of an expression A₁, A₂, A₃,....., A_n which are mutually exclusive and collectively exhaustive such that

$$\sum_{i=1}^N P[A_i] = 1$$

For the sample space N=5 and there is an other event B which intersects A₂, A₃, A₄ and A₅ but not A₁. For example, the probability of joint occurrence B and A₂

$$= P[BA_2] = P[A_2]P\left[\frac{B}{A_2}\right]$$

The probability of joint occurrence of B is dependent on the outcome of A₂ having occurred. Since A₂ precipitates that part of B that it overlaps, It is said to be a prior event. The occurrence of the part of B that overlaps A₂ is called posterior. Now considering that we need to determine the occurrence of B as it is a joint event with A₂, A₃, A₄ and A₅, one can write,

$$P[B] = \sum_{i=1}^N P[A_i] P\left[\frac{B}{A_i}\right] \quad \text{Where } i \text{ is from 2 to 5} \quad (11)$$

The above equation is called Total Probability Equation.

We have already examined that

$$P[AB] = P[B] P\left[\frac{A}{B}\right] = P[A] P\left[\frac{B}{A}\right]$$

Hence

$$P[A_i B] = P[A_i] P\left[\frac{B}{A_i}\right] = P[B] P\left[\frac{A_i}{B}\right]$$

$$P\left[\frac{A_i}{B}\right] = \frac{P[A_i] P\left[\frac{B}{A_i}\right]}{P[B]}$$

Using total probability theorem which states that

$$P[B] = P[A_i] P\left[\frac{B}{A_i}\right]$$

$$\text{Hence } P\left[\frac{A_i}{B}\right] = \frac{P[A_i] P\left[\frac{B}{A_i}\right]}{\sum_{i=1}^N P[A_i] P\left[\frac{B}{A_i}\right]} \quad (12)$$

This is called Bayesian theorem. This equation is very useful in civil engineering and science where in based on the initial estimates, estimates of outcome of an event can be made. Once the results of the outcome known, this can be used to determine the revised estimates. In this probability of the event B is estimated knowing that its signatures are available in events A_i .