Chapter 33

Traffic stream models

33.1 Overview

To figure out the exact relationship between the traffic parameters, a great deal of research has been done over the past several decades. The results of these researches yielded many mathematical models. Some important models among them will be discussed in this chapter.

33.2 Greenshield's macroscopic stream model

Macroscopic stream models represent how the behaviour of one parameter of traffic flow changes with respect to another. Most important among them is the relation between speed and density. The first and most simple relation between them is proposed by Greenshield. Greenshield assumed a linear speed-density relationship as illustrated in figure 33:1 to derive the model. The equation for this relationship is shown below.

$$v = v_f - \left[\frac{v_f}{k_j}\right].k \tag{33.1}$$

where v is the mean speed at density k, v_f is the free speed and k_j is the jam density. This equation (33.1) is often referred to as the Greenshields' model. It indicates that when density becomes zero, speed approaches free flow speed (ie. $v \rightarrow v_f$ when $k \rightarrow 0$).

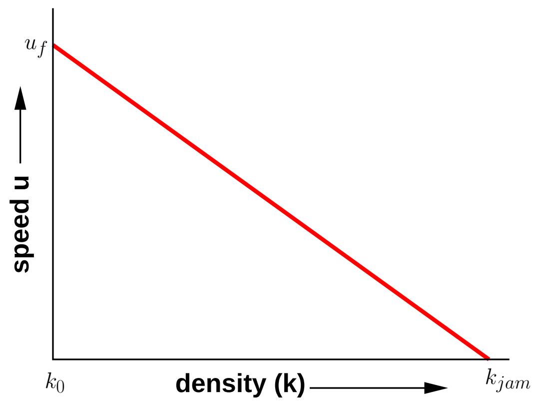

Figure 33:1: Relation between speed and density

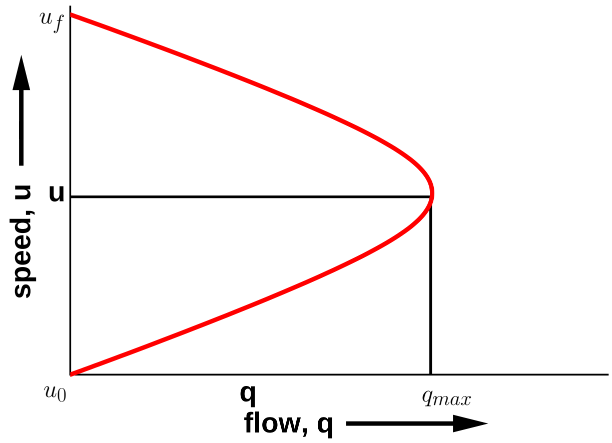

Figure 33:2: Relation between speed and flow

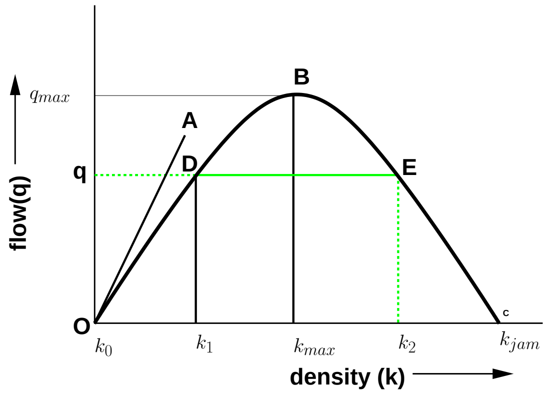

Figure 33:3: Relation between flow and density

Once the relation between speed and flow is established, the relation with flow can be derived. This relation between flow and density is parabolic in shape and is shown in figure 33:3. Also, we know that

$$q = k.v \tag{33.2}$$

Now substituting equation 33.1 in equation 33.2, we get

$$q = v_f.k - \left[\frac{v_f}{k_j}\right] k^2 \tag{33.3}$$

Similarly we can find the relation between speed and flow. For this, put $k = \frac{q}{v}$ in equation 33.1 and solving, we get

$$q = k_j.v - \left[\frac{k_j}{v_f}\right] v^2 \tag{33.4}$$

This relationship is again parabolic and is shown in figure 33:2. Once the relationship between the fundamental variables of traffic flow is established, the boundary conditions can be derived. The boundary conditions that are of interest are jam density, freeflow speed, and maximum flow. To find density at maximum flow, differentiate equation 33.3 with respect to k and equate it to zero. ie.,

$$\begin{aligned} \frac{dq}{dk} &= 0 \\ v_f - \frac{v_f}{k_j}.2k &= 0 \\ k &= \frac{k_j}{2} \end{aligned}$$

Denoting the density corresponding to maximum flow as k_0,

$$k_0 = \frac{k_j}{2} \tag{33.5}$$

Therefore, density corresponding to maximum flow is half the jam density Once we get k_0, we can derive for maximum flow, q_{max}. Substituting equation 33.5 in equation 33.3

$$\begin{aligned} q_{max} &= v_f.\frac{k_j}{2} - \frac{v_f}{k_j}.\left[\frac{k_j}{2}\right]^2 \\ &= v_f.\frac{k_j}{2} - v_f.\frac{k_j}{4} \\ &= \frac{v_f.k_j}{4} \end{aligned}$$

Thus the maximum flow is one fourth the product of free flow and jam density. Finally to get the speed at maximum flow, v_0, substitute equation 33.5 in equation 33.1 and solving we get,

$$v_0 = v_f - \frac{v_f}{k_j}.\frac{k_j}{2}$$

$$v_0 = \frac{v_f}{2} \tag{33.6}$$

Therefore, speed at maximum flow is half of the free speed.

33.3 Calibration of Greenshield's model

Inorder to use this model for any traffic stream, one should get the boundary values, especially free flow speed (v_f) and jam density (k_j). This has to be obtained by field survey and this is called calibration process. Although it is difficult to determine exact free flow speed and jam density directly from the field, approximate values can be obtained from a number of speed and density observations and then fitting a linear equation between them. Let the linear equation be $y = a + bx$ such that y is density k and x denotes the speed v. Using linear regression method, coefficients a and b can be solved as,

$$b = \frac{n.\sum_{i=1}^{n} xy - \sum_{i=1}^{n} x.\sum_{i=1}^{n} y}{n.\sum_{i=1}^{n} x^2 - \sum_{i=1}^{n} x^2} \tag{33.7}$$

$$a = \bar{y} - b\bar{x} \tag{33.8}$$

Alternate method of solving for b is,

$$b = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{33.9}$$

where x_i and y_i are the samples, n is the number of samples, and $\bar{x}$ and $\bar{y}$ are the mean of x_i and y_i respectively.

Problem

For the following data on speed and density, determine the parameters of the Greenshields' model. Also find the maximum flow and density corresponding to a speed of 30 km/hr.

k	v
171	5
129	15
20	40
70	25

Solution Denoting y = v and x = k, solve for a and b using equation 33.8 and equation 33.9. The solution is tabulated as shown below.

x(k)	y(v)	$(x_i - \bar{x})$	$(y_i - \bar{y})$	$(x_i - \bar{x})(y_i - \bar{y})$	$(x_i - \bar{x}^2)$
171	5	73.5	-16.3	-1198.1	5402.3
129	15	31.5	-6.3	-198.5	992.3
20	40	-77.5	18.7	-1449.3	6006.3
70	25	-27.5	3.7	-101.8	756.3
390	85			-2947.7	13157.2

$\bar{x} = \frac{\Sigma x}{n} = \frac{390}{4} = 97.5$, $\bar{y} = \frac{\Sigma y}{n} = \frac{85}{4} = 21.3$. From equation 33.9, b = $\frac{-2947.7}{13157.2}$ = -0.2 $a = y - b\bar{x} = 21.3 + 0.2 \times 97.5 = 40.8$ So the linear regression equation will be,

$$v = 40.8 - 0.2k \tag{33.10}$$

Here $v_f = 40.8$ and $\frac{v_f}{k_j} = 0.2$ This implies, $k_j = \frac{40.8}{0.2} = 204$ veh/km The basic parameters of Greenshield's model are free flow speed and jam density and they are obtained as 40.8 kmph and 204 veh/km respectively. To find maximum flow, use equation 33.6, i.e., $q_{max} = \frac{40.8 \times 204}{4} = 2080.8$ veh/hr Density corresponding to the speed 30 km/hr can be found out by substituting $v = 30$ in equation 33.10. i.e, 30 = 40.8 - 0.2 × k Therefore, k = $\frac{40.8-30}{0.2}$ = 54 veh/km

33.4 Other macroscopic stream models

In Greenshield's model, linear relationship between speed and density was assumed. But in field we can hardly find such a relationship between speed and density. Therefore, the validity of Greenshields' model was questioned and many other models came up. Prominent among them are Greenberg's logarithmic model, Underwood's exponential model, Pipe's generalized model, and multiregime models. These are briefly discussed below.

33.4.1 Greenberg's logarithmic model

Greenberg assumed a logarithmic relation between speed and density. He proposed,

$$v = v_0 \ln \frac{k_j}{k} \tag{33.11}$$

This model has gained very good popularity because this model can be derived analytically. (This derivation is beyond the scope of this notes). However, main drawbacks of this model is that as density tends to zero, speed tends to infinity. This shows the inability of the model to predict the speeds at lower densities.

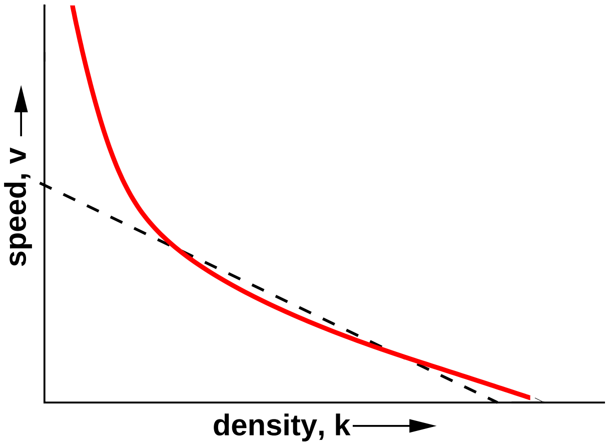

Figure 33:4: Greenberg's logarithmic model

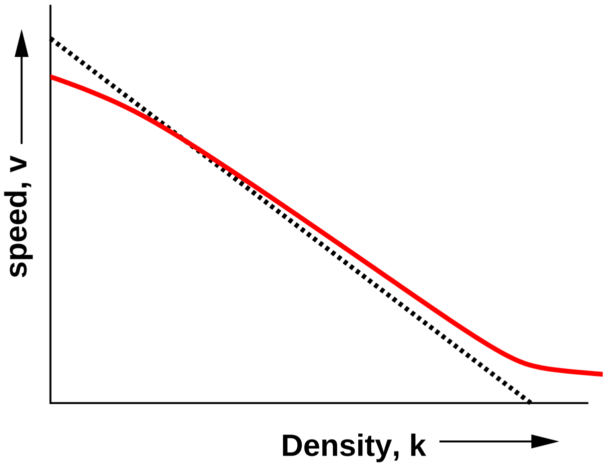

Figure 33:5: Underwood's exponential model

33.4.2 Underwood's exponential model

Trying to overcome the limitation of Greenberg's model, Underwood put forward an exponential model as shown below.

$$v = v_f.e^{\frac{-k}{k_0}} \tag{33.12}$$

where v_fThe model can be graphically expressed as in figure 33:5. is the free flow speedm and k_o is the optimum density, i.e. the densty corresponding to the maximum flow. In this model, speed becomes zero only when density reaches infinity which is the drawback of this model. Hence this cannot be used for predicting speeds at high densities.