

## Chapter 17

# Vertical alignment-I

### 17.1 Overview

The vertical alignment of a road consists of gradients (straight lines in a vertical plane) and vertical curves. The vertical alignment is usually drawn as a profile, which is a graph with elevation as vertical axis and the horizontal distance along the centre line of the road as the horizontal axis. Just as a circular curve is used to connect horizontal straight stretches of road, vertical curves connect two gradients. When these two curves meet, they form either convex or concave. The former is called a summit curve, while the latter is called a valley curve. This section covers a discussion on gradient and summit curves.

### 17.2 Gradient

Gradient is the rate of rise or fall along the length of the road with respect to the horizontal. While aligning a highway, the gradient is decided for designing the vertical curve. Before finalizing the gradients, the construction cost, vehicular operation cost and the practical problems in the site also has to be considered. Usually steep gradients are avoided as far as possible because of the difficulty to climb and increase in the construction cost. More about gradients are discussed below.

#### 17.2.1 Effect of gradient

The effect of long steep gradient on the vehicular speed is considerable. This is particularly important in roads where the proportion of heavy vehicles is significant. Due to restrictive sight distance at uphill gradients the speed of traffic is often controlled by these heavy vehicles. As a result, not only the operating costs of the vehicles are increased, but also capacity of the roads will have to be reduced. Further, due to high differential speed between heavy and light vehicles, and between uphill and downhill gradients, accidents abound in gradients.

#### 17.2.2 Representation of gradient

The positive gradient or the ascending gradient is denoted as  $+n$  and the negative gradient as  $-n$ . The deviation angle  $N$  is: when two grades meet, the angle which measures the change of direction and is given by the algebraic difference between the two grades ( $n_1 - (-n_2)$ ) =  $n_1 + n_2 = \alpha_1 + \alpha_2$ . Example: 1 in 30 = 3.33%  $\approx 2^\circ$  is a steep gradient, while 1 in 50 = 2%  $\approx 1^\circ 10'$  is a flatter gradient. The gradient representation is illustrated in the figure 17:1.

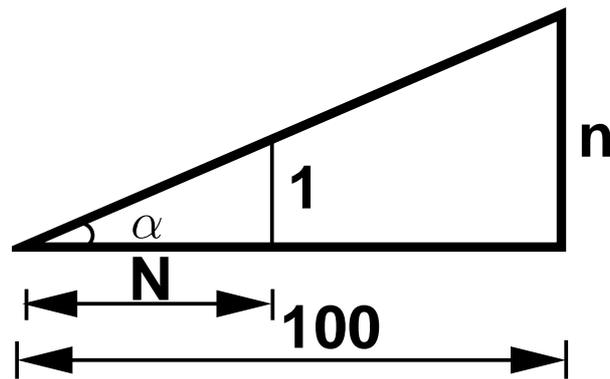


Figure 17:1: Representation of gradient

Table 17:1: IRC Specifications for gradients for different roads

Terrain	Ruling	Limitings	Exceptional
Plain/Rolling	3.3	5.0	6.7
Hilly	5.0	6.0	7.0
Steep	6.0	7.0	8.0

### 17.2.3 Types of gradient

Many studies have shown that gradient upto seven percent can have considerable effect on the speeds of the passenger cars. On the contrary, the speeds of the heavy vehicles are considerably reduced when long gradients as flat as two percent is adopted. Although, flatter gradients are desirable, it is evident that the cost of construction will also be very high. Therefore, IRC has specified the desirable gradients for each terrain. However, it may not be economically viable to adopt such gradients in certain locations, steeper gradients are permitted for short duration. Different types of grades are discussed below and the recommended type of gradients for each type of terrain and type of gradient is given in table 17:1.

Ruling gradient, limiting gradient, exceptional gradient and minimum gradient are some types of gradients which are discussed below.

#### Ruling gradient

The ruling gradient or the design gradient is the maximum gradient with which the designer attempts to design the vertical profile of the road. This depends on the terrain, length of the grade, speed, pulling power of the vehicle and the presence of the horizontal curve. In flatter terrain, it may be possible to provide flat gradients, but in hilly terrain it is not economical and sometimes not possible also. The ruling gradient is adopted by the designer by considering a particular speed as the design speed and for a design vehicle with standard dimensions. But our country has a heterogeneous traffic and hence it is not possible to lay down precise standards for the country as a whole. Hence IRC has recommended some values for ruling gradient for different types of terrain.

**Limiting gradient**

This gradient is adopted when the ruling gradient results in enormous increase in cost of construction. On rolling terrain and hilly terrain it may be frequently necessary to adopt limiting gradient. But the length of the limiting gradient stretches should be limited and must be sandwiched by either straight roads or easier grades.

**Exceptional gradient**

Exceptional gradient are very steeper gradients given at unavoidable situations. They should be limited for short stretches not exceeding about 100 metres at a stretch. In mountainous and steep terrain, successive exceptional gradients must be separated by a minimum 100 metre length gentler gradient. At hairpin bends, the gradient is restricted to 2.5%.

**Critical length of the grade**

The maximum length of the ascending gradient which a loaded truck can operate without undue reduction in speed is called critical length of the grade. A speed of 25 kmph is a reasonable value. This value depends on the size, power, load, grad-ability of the truck, initial speed, final desirable minimum speed etc.

**Minimum gradient**

This is important only at locations where surface drainage is important. Camber will take care of the lateral drainage. But the longitudinal drainage along the side drains require some slope for smooth flow of water. Therefore minimum gradient is provided for drainage purpose and it depends on the rain fall, type of soil and other site conditions. A minimum of 1 in 500 may be sufficient for concrete drain and 1 in 200 for open soil drains are found to give satisfactory performance..

**17.2.4 Creeper lane**

When the uphill climb is extremely long, it may be desirable to introduce an additional lane so as to allow slow ascending vehicles to be removed from the main stream so that the fast moving vehicles are not affected. Such a newly introduced lane is called *creeper lane*. There are no hard and fast rules as when to introduce a creeper lane. But generally, it can be said that it is desirable to provide a creeper lane when the speed of the vehicle gets reduced to half the design speed. When there is no restrictive sight distance to reduce the speed of the approaching vehicle, the additional lane may be initiated at some distance uphill from the beginning of the slope. But when the restrictions are responsible for the lowering of speeds, obviously the lane should be initiated at a point closer to the bottom of the hill. Also the creeper lane should end at a point well beyond the hill crest, so that the slow moving vehicles can return back to the normal lane without any danger. In addition, the creeper lane should not end suddenly, but only in a tapered manner for efficient as well as safer transition of vehicles to the normal lane.

**17.2.5 Grade compensation**

While a vehicle is negotiating a horizontal curve, if there is a gradient also, then there will be increased resistance to traction due to both curve and the gradient. In such cases, the total resistance should not exceed the resistance due to gradient specified. For the design, in some cases this maximum value is limited to the ruling gradient

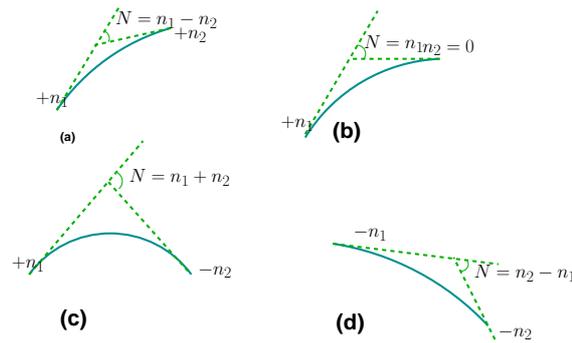


Figure 17:2: Types of summit curves

and in some cases as limiting gradient. So if a curve need to be introduced in a portion which has got the maximum permissible gradient, then some compensation should be provided so as to decrease the gradient for overcoming the tractive loss due to curve. Thus grade compensation can be defined as the reduction in gradient at the horizontal curve because of the additional tractive force required due to curve resistance ( $T - T \cos \alpha$ ), which is intended to offset the extra tractive force involved at the curve. IRC gave the following specification for the grade compensation.

1. Grade compensation is not required for grades flatter than 4% because the loss of tractive force is negligible.
2. Grade compensation is  $\frac{30+R}{R}\%$ , where  $R$  is the radius of the horizontal curve in meters.
3. The maximum grade compensation is limited to  $\frac{75}{R}\%$ .

### 17.3 Summit curve

Summit curves are vertical curves with gradient upwards. They are formed when two gradients meet as illustrated in figure 17:2 in any of the following four ways:

1. when a positive gradient meets another positive gradient [figure 17:2a].
2. when positive gradient meets a flat gradient [figure 17:2b]. .
3. when an ascending gradient meets a descending gradient [figure 17:2c]. .
4. when a descending gradient meets another descending gradient [figure 17:2d]. .

#### 17.3.1 Type of Summit Curve

Many curve forms can be used with satisfactory results, the common practice has been to use parabolic curves in summit curves. This is primarily because of the ease with it can be laid out as well as allowing a comfortable transition from one gradient to another. Although a circular curve offers equal sight distance at every point on the curve, for very small deviation angles a circular curve and parabolic curves are almost congruent. Furthermore, the use of parabolic curves were found to give excellent riding comfort.



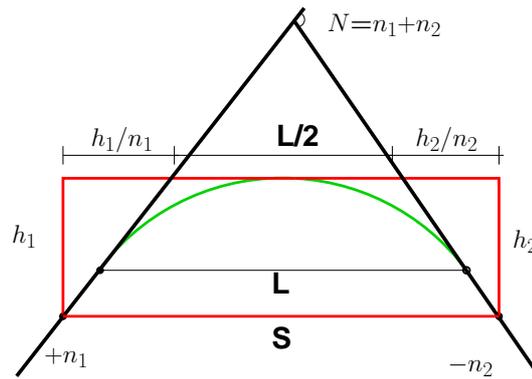


Figure 17:4: Length of summit curve ( $L < S$ )

**Case a. Length of summit curve greater than sight distance ( $L > S$ )**

The situation when the sight distance is less than the length of the curve is shown in figure 17:3.

$$\begin{aligned}
 y &= ax^2 \\
 a &= \frac{N}{2L} \\
 h_1 &= aS_1^2 \\
 h_2 &= aS_2^2 \\
 S_1 &= \sqrt{\frac{h_1}{a}} \\
 S_2 &= \sqrt{\frac{h_2}{a}} \\
 S_1 + S_2 &= \sqrt{\frac{h_1}{a}} + \sqrt{\frac{h_2}{a}} \\
 S^2 &= \left(\frac{1}{\sqrt{a}}\right)^2 (\sqrt{h_1} + \sqrt{h_2})^2 \\
 S^2 &= \frac{2L}{N} (\sqrt{h_1} + \sqrt{h_2})^2 \\
 L &= \frac{NS^2}{2(\sqrt{h_1} + \sqrt{h_2})^2} \tag{17.1}
 \end{aligned}$$

**Case b. Length of summit curve less than sight distance**

The second case is illustrated in figure 17:4

From the basic geometry, one can write

$$S = \frac{L}{2} + \frac{h_1}{n_1} + \frac{h_2}{n_2} = \frac{L}{2} + \frac{h_1}{n_1} + \frac{h_2}{N - n_1} \tag{17.2}$$

Therefore for a given  $L$ ,  $h_1$  and  $h_2$  to get minimum  $S$ , differentiate the above equation with respect to  $h_1$  and equate it to zero. Therefore,

$$\frac{dS}{dh_1} = \frac{-h_1}{n_1^2} + \frac{h_2}{(N - n_1)^2} = 0 \implies h_1(N - n_1)^2 = h_2n_1^2$$

$$\begin{aligned}
h_1 (N^2 + n_1^2 - 2Nn_1) &= h_2 n_1^2 \\
h_1 N^2 + h_1 n_1^2 - 2Nn_1 h_1 &= h_2 n_1^2 \\
(h_2 - h_1) n_1^2 + 2Nn_1 h_1 - h_1 N^2 &= 0
\end{aligned} \tag{17.3}$$

Solving the quadratic equation for  $n_1$ ,

$$\begin{aligned}
n_1 &= \frac{-2Nn_1 \pm \sqrt{(2Nn_1)^2 - 4(h_2 - h_1)(-h_1 N^2)}}{2(h_2 - h_1)} \\
&= \frac{-2Nn_1 + \sqrt{4N^2 h_1^2 + 4h_1 N^2 h_2 - 4h_1^2 N^2}}{2(h_2 - h_1)} \\
&= \frac{-2Nn_1 + 2N\sqrt{h_1 h_2}}{2(h_2 - h_1)} \\
n_1 &= \frac{N\sqrt{h_1 h_2} - h_1 N}{h_2 - h_1}
\end{aligned} \tag{17.4}$$

Now we can substitute  $n$  back to get the value of minimum value of  $L$  for a given  $n_1$ ,  $n_2$ ,  $h_1$  and  $h_2$ . Therefore,

$$S = \frac{L}{2} + \frac{h_1}{\frac{N\sqrt{h_1 h_2} - Nh_1}{h_2 - h_1}} + \frac{h_2}{N - \frac{N\sqrt{h_1 h_2} - Nh_1}{h_2 - h_1}} \tag{17.5}$$

Solving for  $L$ ,

$$\begin{aligned}
&= \frac{L}{2} + \frac{h_1 (h_2 - h_1)}{N (\sqrt{h_1 h_2} - h_1)} + \frac{h_2 (h_2 - h_1)}{Nh_2 - Nh_1 - N\sqrt{h_1 h_2} + Nh_1} \\
&= \frac{L}{2} + \frac{h_1 (h_2 - h_1)}{N (\sqrt{h_1 h_2} - h_1)} + \frac{h_2 (h_2 - h_1)}{N (h_2 - \sqrt{h_1 h_2})} \\
&= \frac{L}{2} + \frac{h_1 (h_2 - h_1) (h_2 - \sqrt{h_1 h_2}) + (h_2 - h_1) h_2 (\sqrt{h_1 h_2} - h_1)}{N (\sqrt{h_1 h_2} - h_1) (h_2 - \sqrt{h_1 h_2})} \\
&= \frac{L}{2} + \frac{(h_2 - h_1) (h_1 h_2 - h_1 \sqrt{h_1 h_2} + h_2 \sqrt{h_1 h_2} - h_1 h_2)}{N (\sqrt{h_1 h_2} - h_1) (h_2 - \sqrt{h_1 h_2})} \\
&= \frac{L}{2} + \frac{(h_2 - h_1) (\sqrt{h_1 h_2} (h_2 - h_1))}{N (h_2 \sqrt{h_1 h_2} - h_1 h_2 + h_1 \sqrt{h_1 h_2} - h_1 h_2)} \\
&= \frac{L}{2} + \frac{(h_2 - h_1) \sqrt{h_1 h_2} (\sqrt{h_2} + \sqrt{h_1}) (\sqrt{h_2} - \sqrt{h_1})}{N \sqrt{h_1 h_2} (h_2 - 2\sqrt{h_1 h_2} + h_2)} \\
&= \frac{L}{2} + \frac{(h_2 - h_1) (\sqrt{h_2} + \sqrt{h_1}) (\sqrt{h_2} - \sqrt{h_1})}{N (\sqrt{h_2} - \sqrt{h_1})^2} \\
&= \frac{L}{2} + \frac{(h_2 - h_1) (\sqrt{h_2} + \sqrt{h_1})}{N (\sqrt{h_2} - \sqrt{h_1})} \\
&= \frac{L}{2} + \frac{(\sqrt{h_2} + \sqrt{h_1}) (\sqrt{h_2} - \sqrt{h_1}) (\sqrt{h_2} + \sqrt{h_1})}{N (\sqrt{h_2} - \sqrt{h_1})} \\
&= \frac{L}{2} + \frac{(\sqrt{h_2} + \sqrt{h_1})^2}{N} \\
L &= 2S - \frac{2 (\sqrt{h_2} + \sqrt{h_1})^2}{N}
\end{aligned} \tag{17.6}$$

$$L = 2S - \frac{(\sqrt{2h_1} + \sqrt{2h_2})^2}{N} \quad (17.7)$$

When stopping sight distance is considered the height of driver's eye above the road surface ( $h_1$ ) is taken as 1.2 metres, and height of object above the pavement surface ( $h_2$ ) is taken as 0.15 metres. If overtaking sight distance is considered, then the value of driver's eye height ( $h_1$ ) and the height of the obstruction ( $h_2$ ) are taken equal as 1.2 metres.

## 17.4 Summary

Different types of gradients and IRC recommendations for their maximum and minimum limit were discussed. At points of combination of horizontal curve and gradient, grade compensation has to be provided. Due to changes in grade in the vertical alignment of the highway, vertical curves become essential. Summit curve, which is a type of vertical curve was discussed in detail in the chapter. One of the application of summit curves that can be seen usually in the urban areas are where fly-overs come.

## 17.5 Problems

1. A vertical summit curve is formed by  $n_1 = +3.0\%$  and  $n_2 = -5.0\%$ . Design the length of the summit curve for  $V=80$  kmph. (Hint:  $SSD=128m$ ). [Ans: 298m]
2.  $n_1 = 1$  in 1,  $n_2 = 1$  in 120. Design summit curve for  $V=80$ kmph,  $OSD=470m$ . [Ans:  $L=417m$ ]
3.  $n_1 = +1/50$  and  $n_2 = -1/80$ ,  $SSD=180m$ ,  $OSD=640m$ . Due to site constraints,  $L$  is limited to 500m. Calculate the length of summit curve to meet  $SSD$ ,  $ISD$  and  $OSD$ . Discuss results. [Ans:  $L$  for  $SSD=240m$ , okay,  $L$  for  $OSD=1387m$ ,  $> 500m$  not ok,  $L$  for  $ISD=439m$  ok.]