Chapter 4

Response Spectrum Method

4.1 Introduction

In order to perform the seismic analysis and design of a structure to be built at a particular location, the actual time history record is required. However, it is not possible to have such records at each and every location. Further, the seismic analysis of structures cannot be carried out simply based on the peak value of the ground acceleration as the response of the structure depend upon the frequency content of ground motion and its own dynamic properties. To overcome the above difficulties, earthquake response spectrum is the most popular tool in the seismic analysis of structures. There are computational advantages in using the response spectrum method of seismic analysis for prediction of displacements and member forces in structural systems. The method involves the calculation of only the maximum values of the displacements and member forces in each mode of vibration using smooth design spectra that are the average of several earthquake motions.

This chapter deals with response spectrum method and its application to various types of the structures. The codal provisions as per IS:1893 (Part 1)-2002 code for response spectrum analysis of multi-story building is also summarized.

4.2 Response Spectra

Response spectra are curves plotted between maximum response of SDOF system subjected to specified earthquake ground motion and its time period (or frequency). Response spectrum can be interpreted as the locus of maximum response of a SDOF system for given damping ratio. Response spectra thus helps in obtaining the peak structural responses under linear range, which can be used for obtaining lateral forces developed in structure due to earthquake thus facilitates in earthquake-resistant design of structures.

Usually response of a SDOF system is determined by time domain or frequency domain analysis, and for a given time period of system, maximum response is picked. This process is continued for all range of possible time periods of SDOF system. Final plot with system time period on x-axis and response quantity on y-axis is the required response spectra pertaining to specified damping ratio and input ground motion. Same process is carried out with different damping ratios to obtain overall response spectra.

Consider a SDOF system subjected to earthquake acceleration, $\ddot{x}_g(t)$ the equation of motion is given by

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = -m\ddot{x}_g(t) \tag{4.1}$$

Substitute $\omega_0 = \sqrt{k/m}$ and $\xi = \dfrac{c}{2m\omega_0}$ and $\omega_d = \omega_0\sqrt{1-\xi^2}$

The equation (4.1) can be re-written as

$$\ddot{x}(t) + 2\xi\omega_0\dot{x}(t) + \omega_0^2 x(t) = -\ddot{x}_g(t) \tag{4.2}$$

Using Duhamel's integral, the solution of SDOF system initially at rest is given by (Agrawal and Shrikhande, 2006)

$$x(t) = -\int_0^t \ddot{x}_g(\tau)\,\frac{e^{-\xi\omega_0(t-\tau)}}{\omega_d}\,\mathit{ins}\,\omega_d(t-\tau)\,d\tau \tag{4.3}$$

The maximum displacement of the SDOF system having parameters of ξ and ω_0 and subjected to specified earthquake motion, $\ddot{x}_g(t)$ is expressed by

$$\left|x(t)\right|_{\max} = \left|\int_0^t \ddot{x}_g(\tau)\,\frac{e^{-\xi\omega_0(t-\tau)}}{\omega_d}\,\mathit{ins}\,\omega_d(t-\tau)\,d\tau\right|_{\max} \tag{4.4}$$

The relative displacement spectrum is defined as,

$$S_d(\xi, \omega_0) = \left|x(t)\right|_{\max} \tag{4.5}$$

where $S_d(\xi, \omega_0)$ is the relative displacement spectra of the earthquake ground motion for the parameters of ξ and ω_0.

Similarly, the relative velocity spectrum, S_v and absolute acceleration response spectrum, S_a are expressed as,

$$S_v(\xi, \omega_0) = \left|\dot{x}(t)\right|_{\max} \tag{4.6}$$

$$S_a(\xi, \omega_0) = \left|\ddot{x}_a(t)\right|_{\max} = \left|\ddot{x}(t) + \ddot{x}_g(t)\right|_{\max} \tag{4.7}$$

The pseudo velocity response spectrum, S_{pv} for the system is defined as

$$S_{pv}(\xi, \omega_0) = \omega_0 \; S_d(\xi, \omega_0) \tag{4.8}$$

Similarly, the pseudo acceleration response, S_{pa} is obtained by multiplying the S_d to $\omega_0^{\;2}$, thus

$$S_{pa}(\xi, \omega_0) = \omega_0^2 \; S_d(\xi, \omega_0) \tag{4.9}$$

Consider a case where $\xi = 0$ *i.e.* $\ddot{x}(t) + \omega_0^2 x(t) = -\ddot{x}_g(t)$

$$\begin{aligned}
S_a &= |\ddot{x}(t) + \ddot{x}_g(t)|_{\max} \\
&= |-\omega_0^2 x(t)|_{\max} \\
&= \omega_0^2 \, |x_{\max}| \\
&= \omega_0^2 S_d \\
&= S_{pa}
\end{aligned} \tag{4.10}$$

The above equation implies that for an undamped system, $S_a = S_{pa}$.

The quantity S_{pv} is used to calculate the maximum strain energy stored in the structure expressed as

$$E_{\max} = \frac{1}{2} k \, x_{\max}^2 = \frac{1}{2} m \, \omega_0^{\;2} \, S_d^2 = \frac{1}{2} m \, S_{pv}^2 \tag{4.11}$$

The quantity S_{pa} is related to the maximum value of base shear as

$$V_{\max} = k \, x_{\max} = m \, \omega_0^2 \, S_d = m S_{pa} \tag{4.12}$$

The relations between different response spectrum quantities is shown in the Table 4.1.

As limiting case consider a rigid system i.e. $\omega_0 \to \infty$ or $T_0 \to 0$, the values of various response spectra are

$$\lim_{\omega_0 \to \infty} S_d \to 0 \tag{4.13}$$

$$\lim_{\omega_0 \to \infty} S_v \to 0 \tag{4.14}$$

$$\lim_{\omega_0 \to \infty} S_a \to \left|\ddot{x}_g(t)\right|_{\max} \tag{4.15}$$

The three spectra i.e. displacement, pseudo velocity and pseudo acceleration provide the same information on the structural response. However, each one of them provides a physically meaningful quantity (refer equations (4.11) and (4.12)) and therefore, all three spectra are useful in understanding the nature of an earthquake and its influence on the design. A combined plot showing all three of the spectral quantities is possible because of the relationship that exists between these three quantities. Taking the log of equations (4.8) and (4.9)

$$\log S_{pv} = \log S_d + \log \omega_0 \tag{4.16}$$

$$\log S_{pv} = \log S_{pa} - \log \omega_0 \tag{4.17}$$

From the Equations (4.16) and (4.17), it is clear that a plot on logarithmic scale with $\log S_{pv}$ as ordinate and $\log\omega_0$ as abscissa, the two equations are straight lines with slopes +45º and -45º for constant values of $\log S_d$ and $\log S_{pa}$, respectively. This implies that the combined spectra of displacement, pseudo velocity and pseudo acceleration can be plotted in a single graph (refer Figure 2.5 for combined Displacement, Velocity and Acceleration Spectrum taken from Datta, 2010).

Table 4.1 Response Spectrum Relationship.

Relative displacement, $\lvert x(t)\rvert_{\max}$	$= S_d$	$\simeq \frac{S_v}{\omega_0}$	$\simeq \frac{S_a}{\omega_0^2}$ *	$= \frac{S_{pv}}{\omega_0}$	$= \frac{S_{pa}}{\omega_0^2}$
Relative velocity, $\lvert \dot{x}(t)\rvert_{\max}$	$\simeq \omega_0 S_d$	$= S_v$	$\simeq \frac{S_a}{\omega_0}$	$\simeq S_{pv}$	$\simeq \frac{S_{pa}}{\omega_0}$
Absolute acceleration, $\lvert \ddot{x}_a(t)\rvert_{\max}$	$\simeq \omega_0^2 S_d$ *	$\simeq \omega_0 S_v$	$= S_a$	$\simeq \omega_0 S_{pv}$	$\simeq S_{pa}$ *

(* If $\xi = 0$ these relations are exact and the sign $\simeq$ is valid up to $0 < \xi < 0.2$)

4.2.1 Factor Influencing Response Spectra

The response spectral values depends upon the following parameters,

I) Energy release mechanism
II) Epicentral distance
III) Focal depth
IV) Soil condition
V) Richter magnitude
VI) Damping in the system
VII) Time period of the system

4.2.2 Errors in Evaluation of Response Spectrum

The following errors are introduced in evaluation of response spectra (Nigam and Jennings, 1969),

1. Straight line Approximation: - In the digital computation of spectra, the actual earthquake record is replaced by linear segments between the points of digitization. This is a minor approximation provided that the length of the time intervals is much shorter than the periods of interest.

2. Truncation Error: - In general, a truncation error exists in numerical methods for integrating differential equations. For example, in third-order Runge-Kutta methods the error is proportional to $(\Delta t_i)^4$.

3. Error Due to Rounding the Time Record: - For earthquake records digitized at irregular time intervals, the integration technique proposed in this report requires rounding of the time record and the attendant error depends on the way the rounding is done. For round-off to 0.005 sec, the average error in spectrum values is expected to be less than 2 percent.

4. Error Due to Discretization: - In any numerical method of computing the spectra, the response is obtained at a set of discrete points. Since spectral values represent maximum values of response parameters which may not occur at these discrete points, discretization introduces an error which gives spectrum values lower than the true values. The error will be a maximum if the maximum response occurs exactly midway between two discrete points as shown in Figure 4.1. An estimate for the upper bound of this error is shown in Table 4.2 by noting that at the time of maximum displacement or velocity, the response of the oscillator is nearly sinusoidal at a frequency equal to its natural frequency. Under this assumption the error can be related to the maximum interval of integration, Δt_i and the period of the oscillator as shown in Figure 4.1.

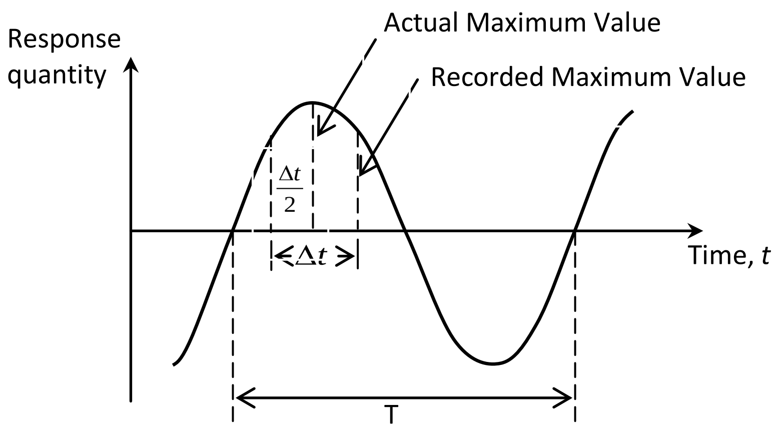


Figure 4.1 Error in response spectra due discretization.

Table 4.2 Variation of Percentage error in response quantity with time step chosen.

Δt_i	Maximum Error (%)
$\leq$ T/10	$\leq$ 4.9
$\leq$ T/20	$\leq$ 1.2
$\leq$ T/40	$\leq$ 0.3

4.2.3 Response Spectra of El-Centro-1940 Earthquake Ground Motion

The response spectra of the El-Centro, 1940 earthquake ground motion are computed using the exact method described in the earlier Chapter (refer Appendix-I, for digitized values of the earthquake). The spectra are plotted for the three damping ratios i.e. ξ=0.02, 0.05 and 0.1. The displacement, velocity and acceleration spectra are shown in the Figures 4.2, 4.3 and 4.4, respectively.

Further, comparison of the real and pseudo spectra for velocity and acceleration response is shown in the Figure 4.5. As expected, there is no difference between real and pseudo absolute acceleration response spectra. However, the velocity response spectra may have some difference.

The digitized values of the response spectra S_d, S_v and S_a of the El-Centro, 1940 earthquake is given in the Appendix – II at an interval of 0.01 sec time period for damping ratio of 2% and 5%.

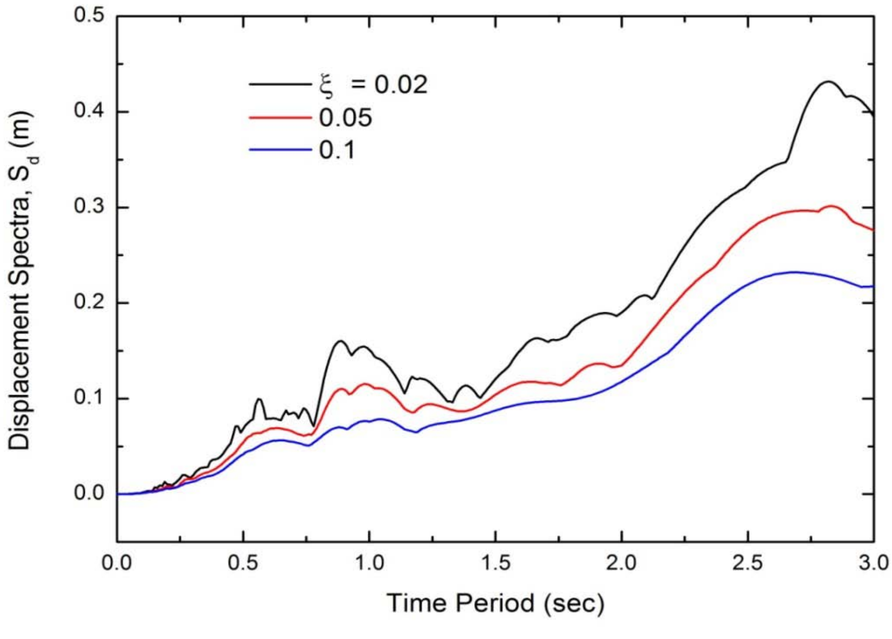


Figure 4.2 Displacement response spectra of El-Centro, 1940 earthquake ground motion.

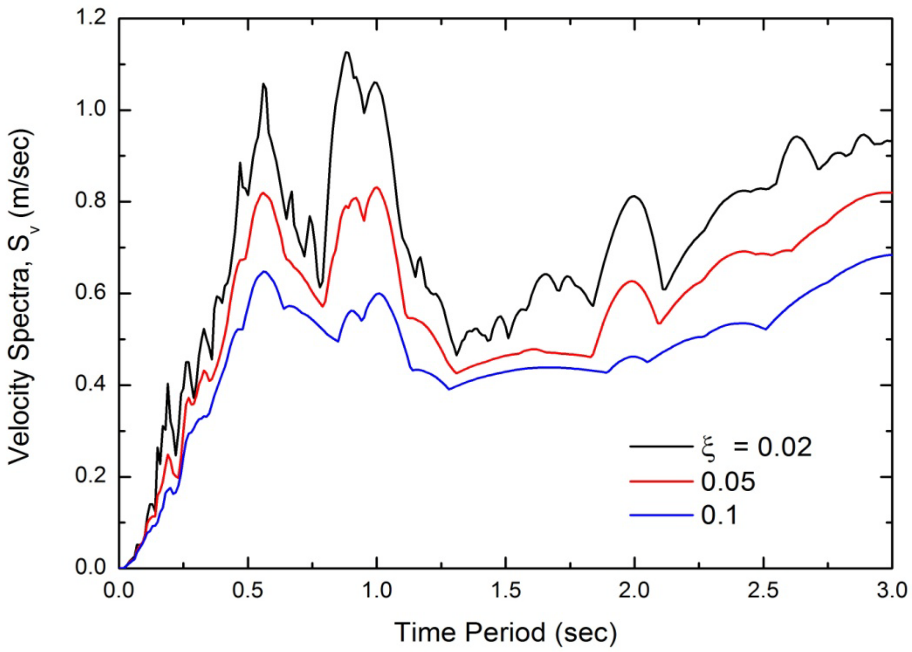


Figure 4.3 Velocity response spectra of El-Centro, 1940 earthquake ground motion.

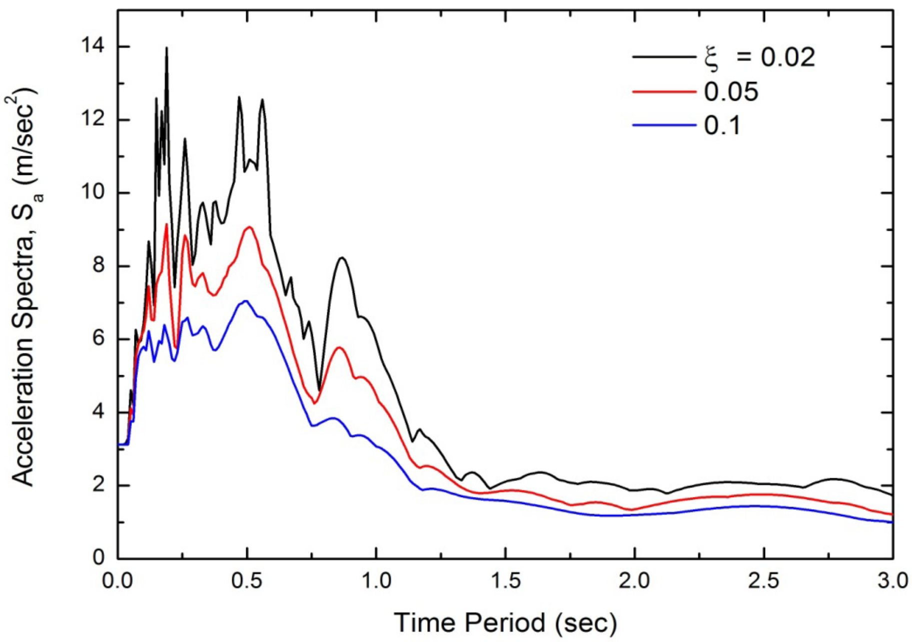


Figure 4.4 Acceleration response spectra of El-Centro, 1940 earthquake ground motion.

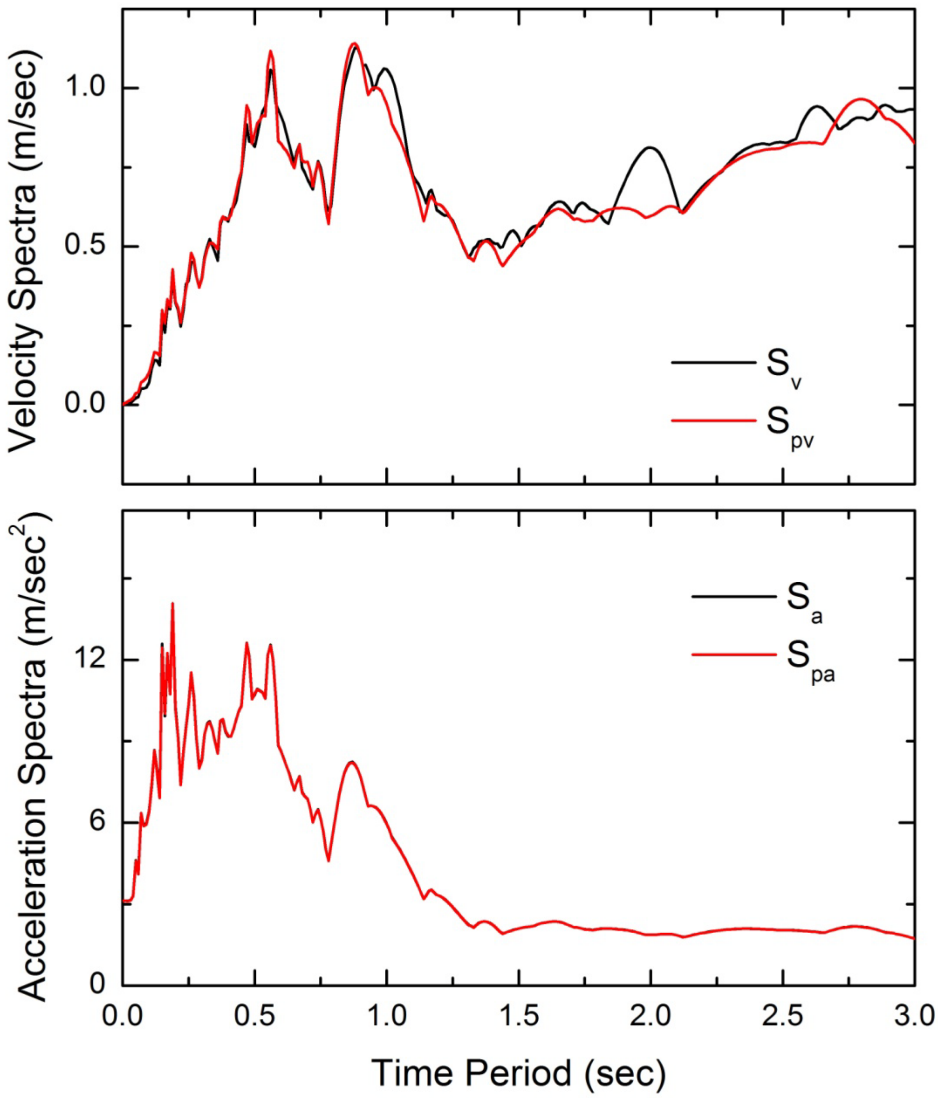


Figure 4.5 Comparison of real and pseudo velocity and acceleration response spectra of El-Centro, 1940 earthquake ground motion (damping ratio=0.02).

4.3 Numerical Examples

Example 4.1

Consider a SDOF system with mass, $m = 2 \times 10^3$ kg, stiffness, $k = 60$ kN/m and damping, $c = 0.44$ kN.sec/m. Using the response spectra of El-Centro, 1940 earthquake, compute (a) Maximum relative displacement, (b) Maximum base shear and (c) Maximum strain energy.

Solution: The natural frequency, time period and damping ratio of the SDOF system are

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{60 \times 10^3}{2 \times 10^3}} = 5.48\ rad/\sec$$

$$T_0 = \frac{2\pi}{\omega_0} = 1.15 \text{ sec}$$

$$\xi = \frac{c}{2m\omega_0} = \frac{0.44 \times 10^3}{2 \times 2 \times 10^3 \times 5.48} = 0.02$$

From the response spectrum curve of El-Centro, 1940 earthquake ground motion for the time period of 1.15 sec and damping ratio of 0.02 (refer Figures 4.2 and 4.4 or Appendix-II)

$S_d = 0.11147$m and $S_a = 3.321$ m/sec^2

(a) The maximum displacement

$$x_{\max} = S_d = 111.47 \text{ mm}$$

Alternatively, $x_{\max} \simeq \dfrac{S_a}{\omega_0^2} = \dfrac{3.321}{5.48^2} = 0.11055 \text{ m} = 110.55 \text{ mm}$

(b) The maximum base shear

$$V_{\max} = m S_a = 2 \times 10^3 \times 3.321 = 6.64 \text{ kN}$$

Alternatively, $V_{\max} = k\, x_{\max} = 60 \times 10^3 \times 0.11147 = 6.688$ kN

(c) The maximum strain energy

$$E_{\max} = \frac{1}{2} k\, x_{\max}^2 = \frac{1}{2} \times 60 \times 10^3 \times (0.11147)^2 = 372.76 \text{ N.m}$$

Example 4.2

Plot the pseudo acceleration response spectra for the earthquake acceleration expressed as

$\ddot{x}_g(t) = \ddot{x}_0 \sin(\overline{\omega} t) = 0.5g \ \sin(10t)$

Solution: Using time domain analysis (Duhamel's integral)

$$x(t) = -\int_0^t h(t-\tau)\, \ddot{x}_g(\tau)\, d\tau$$

where,

$$h(t-\tau) = \frac{e^{-\xi\omega_0(t-\tau)}}{\omega_d} \sin(\omega_d t)$$

$$\ddot{x}_g(t) = \ddot{x}_0 \sin(\overline{\omega} t)$$

On integrating,

$$x(t) = -\frac{\ddot{x}_0}{\omega_0^2} \frac{1}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}} \sin(\overline{\omega} t - \theta)$$

where $\beta = \dfrac{\overline{\omega}}{\omega_0}$ and $\theta = \tan^{-1}\left(\dfrac{2\xi\beta}{1-\beta^2}\right)$

The displacement spectra is given by

$$S_d = |x(t)|_{\max}$$

$$= \frac{\ddot{x}_0}{\omega_0^2} \frac{1}{\sqrt{(1-\beta^2) + (2\xi\beta)^2}}$$

Pseudo acceleration spectra is given by

$$S_{pa} = \omega_0^2 \times S_d$$

$$= \frac{\ddot{x}_0}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}}$$

The required response spectra is plotted in Figure 4.6 for damping ratios of 2%, 5% and 10%.

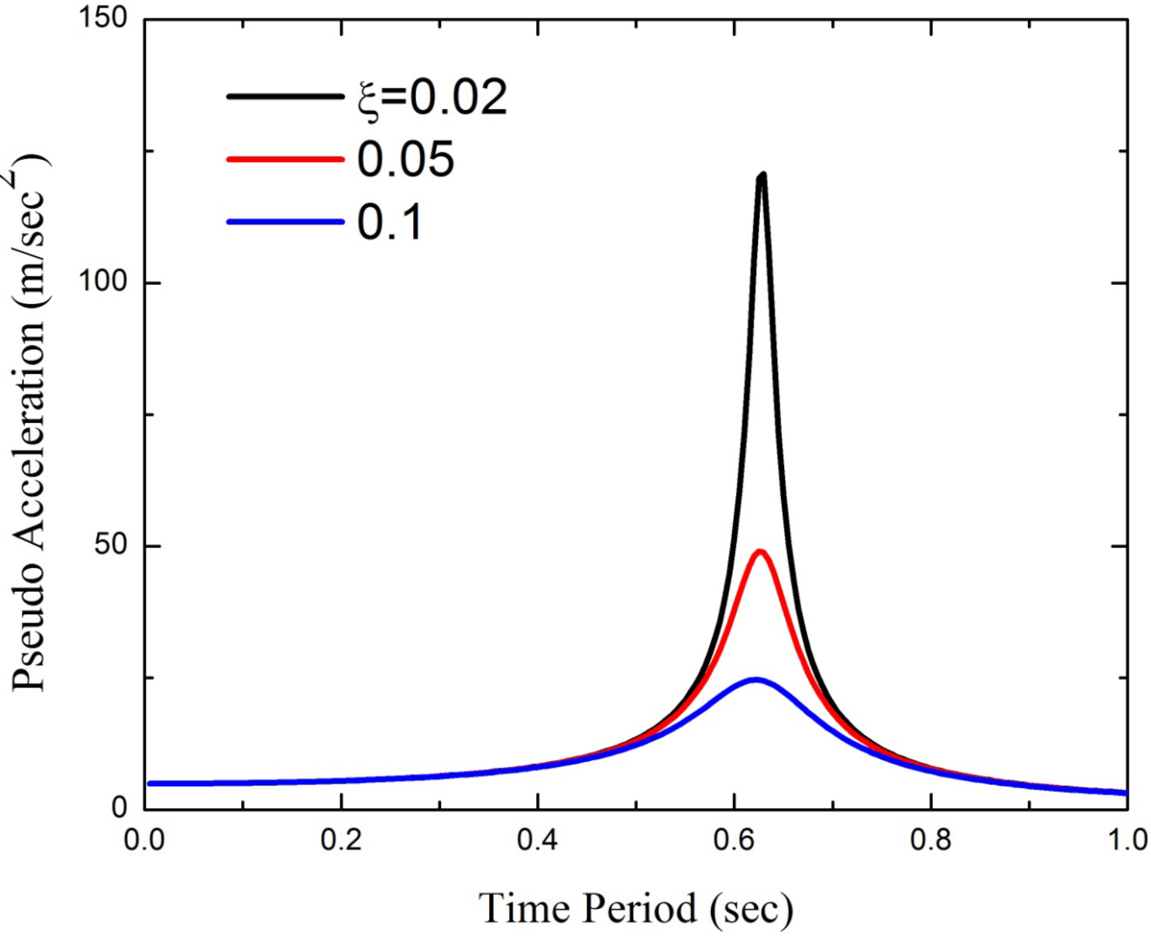


Figure 4.6 Pseudo acceleration response spectra for different damping ratios of Example 4.2.

Example 4.3

Plot the pseudo acceleration response spectra of the ground motion given by

$\ddot{x}_g(t) = c_0\delta(t-2)$

where 'δ' is Dirac delta function. Take duration of acceleration as 30 sec.

Solution: Using Duhamel's integral the displacement of the system is (i.e. equation 4.3)

$$x(t) = -\int_0^t \frac{e^{-\xi\omega_0(t-\tau)}}{\omega_d}\sin\omega_d(t-\tau)c_0\delta(t-2)\,d\tau$$

$$= -c_0\left[\frac{e^{-\xi\omega_0(t-\tau)}}{\omega_d}\sin\omega_d(t-\tau)\right]_{\tau=2}$$

$$= -\frac{c_0}{\omega_d}e^{-\xi\omega_0(t-2)}\sin\omega_d(t-2)$$

For the maximum displacement

$$\frac{dx(t)}{dt} = 0,$$

$$0 = -\frac{c_0}{\omega_d}\left[-\xi\omega_0\; e^{-\xi\omega_0(t-2)}\sin\omega_d(t-2) + e^{-\xi\omega_0(t-2)}\omega_d\cos\omega_d(t-2)\right]$$

$$0 = -c_0\; e^{-\xi\omega_0(t-2)}\left[-\xi\omega_0\sin\omega_d(t-2) + \omega_d\cos\omega_d(t-2)\right]$$

$$-\xi\omega_0\sin(t-2) + \omega_d\cos\omega_d(t-2) = 0$$

$$\tan\omega_d(t-2) = \frac{\omega_d}{\xi\omega_0} = \frac{\sqrt{1-\xi^2}}{\xi}$$

Thus, the time t_m at which the maximum displacement occurs is

$$\omega_d(t_m-2) = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

and implying from the Figure 4.7 that,

$$\sin(\omega_d(t_m-2)) = \sqrt{1-\xi^2}$$

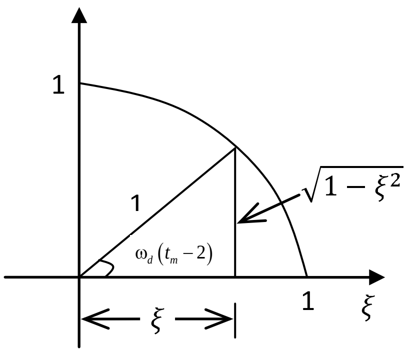


Figure 4.7

The maximum displacement will be given by

$$x_{\max} = -\frac{c_0}{\omega_d} e^{-\xi\omega_0(t_m-2)} \sin \omega_d (t_m - 2)$$

$$= -\frac{c_0}{\omega_d} \exp\left[-\xi\omega_0 \frac{1}{\omega_d} \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right] \times \sqrt{1-\xi^2}$$

Substituting for $\omega_d = \omega_0\sqrt{1-\xi^2}$ and simplifying

$$x_{\max} = -\frac{c_0}{\omega_0} \exp\left[-\frac{\xi}{\sqrt{1-\xi^2}} \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]$$

The displacement spectra is given by

$$S_d = |x_{\max}|$$

$$= \frac{c_0}{\omega_0} \exp\left[-\frac{\xi}{\sqrt{1-\xi^2}} \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]$$

The pseudo acceleration spectra is expressed by

$$S_{pa} = \omega_0^2 \, S_d$$

$$S_{pa} = c_0\omega_0 \exp\left[-\frac{\xi}{\sqrt{1-\xi^2}} \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]$$

4.4 Response Spectra Method for MDOF System

4.4.1 Response Analysis of MDOF System

Multi degree of freedom (MDOF) systems are usually analyzed using Modal Analysis. A typical MDOF system with '*n*' degree of freedom is shown in Figure (4.8). This system when subjected to ground motion undergoes deformations in number of possible ways. These deformed shapes are known as modes of vibration or mode shapes. Each shape is vibrating with a particular natural frequency. Total unique modes for each MDOF system are equal to the possible degree of freedom of system. The equations of motion for MDOF system is given by

$$[m]\{\ddot{x}(t)\} + [c]\{\dot{x}(t)\} + [k]\{x(t)\} = -[m]\{r\}\,\ddot{x}_g(\mathrm{t}) \qquad (4.18)$$

where, $[m]$ = Mass matrix ($n \times n$); $[k]$ = Stiffness matix ($n \times n$); $[c]$ = Damping matrix ($n \times n$); $\{r\}$ = Influence coefficient vector ($n\times1$); $\{x(t)\}$= relative displacement vector; $\{\dot{x}(t)\}$= relative velocity vector, $\{\ddot{x}(t)\}$= relative acceleration vector, and $\ddot{x}_g(t)$= earthquake ground acceleration.

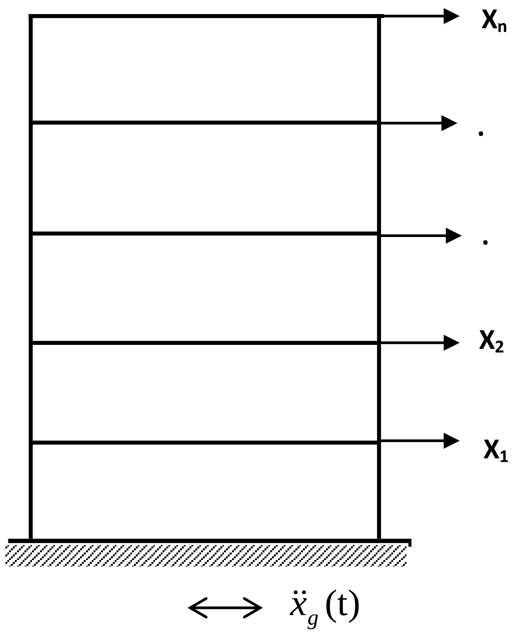


Figure 4.8 MDOF system with '*n*' degrees-of-freedom.

The undamped eigen values and eigen vectors of the MDOF system are found form the characteristic equation

$$\left\{[k]-\omega_i^2[m]\right\}\phi_i = 0 \qquad i = 1,2,3,\ldots,n \tag{4.19}$$

$$\det\left|\left\{[k]-\omega_i^2[m]\right\}\right| = 0 \tag{4.20}$$

where,

ω_i^2 = eigen values of the ith mode

ϕ_i = eigen vector or mode shape of the ith mode

ω_i = natural frequency in the ith mode.

Let the displacement response of the MDOF system is expressed as

$$\{x(t)\} = [\phi]\{y(t)\} \tag{4.21}$$

where $\{y(t)\}$ represents the modal displacement vector, and $[\phi]$ is the mode shape matrix given by

$$[\phi] = [\ \phi_1, \phi_2\ ,\ldots\ldots\ldots\ \phi_n\ ] \tag{4.22}$$

Substituting $\{x\}=[\phi]\{y\}$ in equation (4.18) and pre-multiply by $[\phi]^T$

$$[\phi]^T[m][\phi]\{\ddot{y}(t)\}+[\phi]^T[c][\phi]\{\dot{y}(t)\}+[\phi]^T[k][\phi]\{y(t)\} = -[\phi]^T[m]\{r\}\ \ddot{x}_g(t) \tag{4.23}$$

The above equation reduces to

$$[M_m]\{\ddot{y}(t)\}+[C_d]\{\dot{y}(t)\}+[K_d]\{y(t)\} = -[\phi]^T[m]\{r\}\ \ddot{x}_g(t) \tag{4.24}$$

where,

$[\phi]^T[m][\phi]=[M_m]$ = generalized mass matrix

$[\phi]^T[c][\phi]=[C_d]$ = generalized damping matrix

$[\phi]^T[k][\phi]=[K_d]$ = generalized stiffness matrix

By virtue of the properties of the [ϕ], the matrices [M_m] and [K_d] are diagonal matrices. However, for the classically damped system (i.e. if the [C_d] is also a diagonal matrix), the equation (4.24) reduces to the following equation

$$\ddot{y}_i(t) + 2\xi_i\omega_i\dot{y}_i(t) + \omega_i^2 y_i(t) = -\Gamma_i\ \ddot{x}_g(t) \qquad (i = 1,2,3,\ldots,n) \quad (4.25)$$

where,

$y_i(t)$ = modal displacement response in the i[th] mode,

ξ_i = modal damping ration in the i[th] mode, and

Γ_i = modal participation factor for i[th] mode expressed by

$$\Gamma_i = \frac{\{\phi_i\}^T[m]\{r\}}{\{\phi_i\}^T[m]\{\phi_i\}} \quad (4.26)$$

Equation (4.25) is of the form of equation (4.1), representing vibration of SDOF system, the maximum modal displacement response is found from the response spectrum i.e.

$$y_{i,\max} = |y_i(t)|_{\max} = \Gamma_i\ S_d\ (\xi_i, \omega_i) \quad (4.27)$$

The maximum displacement response of the structure in the i[th] mode is

$$x_{i,\max} = \phi_i\ y_{i,\max} \qquad (i = 1,2,\ldots\ldots, n) \quad (4.28)$$

The maximum acceleration response of the structure in the i[th] mode is

$$\{\ \ddot{x}_a\ \}_{i,\max} = \{\phi_i\}\Gamma_i S_{pa}(\xi_i, \omega_i) \qquad (i = 1,2,\ldots\ldots, n) \quad (4.29)$$

The required response quantity of interest, r_i i.e. (displacement, shear force, bending moment etc.) of the structure can be obtained in each mode of vibration using the maximum response obtained in equations (4.28) and (4.29). However, the final maximum response, r_{max} shall be obtained by combining the response in each mode of vibration using the modal combinations rules. Some of the *modal combinations rules* commonly used are described here.

4.4.2 Modal Combination Rules

The commonly used methods for obtaining the peak response quantity of interest for a MDOF system are as follows:

- Absolute Sum (ABSSUM) Method,
- Square root of sum of squares (SRSS) method, and
- Complete quadratic combination (CQC) method

In ABSSUM method, the peak responses of all the modes are added algebraically, assuming that all modal peaks occur at same time. The maximum response is given by

$$r_{\max} = \sum_{i=1}^{n} |r_i| \tag{4.30}$$

The ABSSUM method provides a much conservative estimate of resulting response quantity and thus provides an upper bound to peak value of total response. (Chopra, 2007)

In the SRSS method, the maximum response is obtained by square root of sum of square of response in each mode of vibration and is expressed by

$$r_{\max} = \sqrt{\sum_{i=1}^{n} r_i^2} \tag{4.31}$$

The SRSS method of combining maximum modal responses is fundamentally sound where the modal frequencies are well separated. However, this method yield poor results where frequencies of major contributing modes are very close together.

The alternative procedure is the Complete Quadratic Combination (CQC) method. The maximum response from all the modes is calculated as

$$r_{\max} = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} r_i \, \alpha_{ij} \, r_j} \tag{4.32}$$

where r_i and r_j are maximum responses in the i^{th} and j^{th} modes, respectively and α_{ij} is correlation coefficient given by

$$\alpha_{ij} = \frac{8 \left(\xi_i \, \xi_j\right)^{1/2} \left(\xi_i + \beta \xi_j\right) \beta^{3/2}}{\left(1 - \beta^2\right)^2 + 4 \, \xi_i \xi_j \beta \left(1 + \beta^2\right) + 4 \left(\xi_i^2 + \xi_j^2\right) \beta^2} \tag{4.33}$$

where ξ_i and ξ_j are damping ratio in i^{th} and j^{th} modes of vibration, respectively and

$$\beta = \frac{\omega_i}{\omega_j} \qquad (\omega_j > \omega_i) \tag{4.34}$$

The range of coefficient, α_{ij} is $0 < \alpha_{ij} < 1$ and $\alpha_{ii} = \alpha_{jj} = 1$.

For the system having the same damping ratio in two modes i.e. $\xi_i = \xi_j = \xi$, then

$$\alpha_{ij} = \frac{8\,\xi^2 (1 + \beta)\,\beta^{3/2}}{\left(1 - \beta^2\right)^2 + 4\,\xi^2 \beta\,(1+\beta)^2} \tag{4.35}$$

4.4.3 Numerical Examples

Example 4.4

A two-story building is modeled as 2-DOF system and rigid floors as shown in the Figure 4.9. Determine the top floor maximum displacement and base shear due to El-Centro, 1940 earthquake ground motion using the response spectrum method. Take the inter-story stiffness, k =197.392 × 10^3 N/m and the floor mass, m = 2500 kg and damping ratio as 2%.

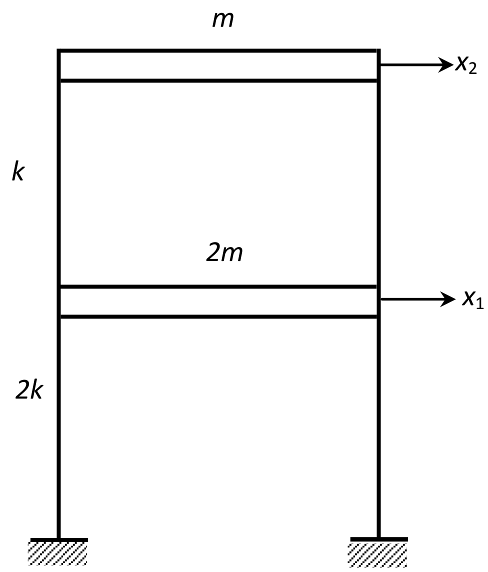


Figure 4.9

Solution:

Mass of each floor, m = 2500 kg and stiffness, k = 197.392 kN/m

thus,

Stiffness matrix = $[k] = \begin{bmatrix} 3k & -k \\ -k & k \end{bmatrix}$

and mass matrix = $[m] = \begin{bmatrix} 5000 & 0 \\ 0 & 2500 \end{bmatrix}$

Using equation (4.19), eigen values and eigen vectors can be obtained as

$\omega_1 = 6.283$ rad/sec and $\omega_2 = 12.566$ rad/sec

$[\phi_1] = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$, and $[\phi_2] = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Modal participation factors are given by

$$\Gamma_i = \frac{\{\phi_i\}^T \{m\}\{r\}}{\{\phi_i\}^T \{m\}\{\phi_i\}}$$

$$\Gamma_1 = \frac{\{\phi_1\}^T \{m\}\{r\}}{\{\phi_1\}^T \{m\}\{\phi_1\}} = \frac{\begin{bmatrix}0.5 & 1\end{bmatrix}\begin{bmatrix}5000 & 0 \\ 0 & 2500\end{bmatrix}\begin{Bmatrix}1 \\ 1\end{Bmatrix}}{\begin{bmatrix}0.5 & 1\end{bmatrix}\begin{bmatrix}5000 & 0 \\ 0 & 2500\end{bmatrix}\begin{bmatrix}0.5 \\ 1\end{bmatrix}} = 1.333$$

Similarly,

$$\Gamma_2 = \frac{\{\phi_2\}^T \{m\}\{r\}}{\{\phi_2\}^T \{m\}\{\phi_2\}} = \frac{\begin{bmatrix}-1 & 1\end{bmatrix}\begin{bmatrix}5000 & 0 \\ 0 & 2500\end{bmatrix}\begin{Bmatrix}1 \\ 1\end{Bmatrix}}{\begin{bmatrix}-1 & 1\end{bmatrix}\begin{bmatrix}5000 & 0 \\ 0 & 2500\end{bmatrix}\begin{bmatrix}-1 \\ 1\end{bmatrix}} = -0.333$$

1st Mode Response

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{6.283} = 1 \text{ sec}$$

$$\xi = 0.02$$

From the response spectra, (refer Figures 4.2 and 4.4 or Appendix-II)

$S_{a1} = 6.17\, m/s^2$ and $S_{d1} = 0.153m$

Top floor displacement $= \Gamma_1 \times \phi_{21} \times S_{d1} = 1.333 \times 1 \times 0.153$

$= 0.204m$

Base shear $= 2k \times \phi_{11} \times \Gamma_1 \times S_{d1} = 2 \times 197.392 \times 10^3 \times 0.5 \times 1.33 \times 0.153$

$= 40.16\ kN$

2nd Mode Response

$$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{12.566} = 0.5 \text{ sec}$$

$$\xi = 0.02$$

From response spectra, (refer Figures 4.2 and 4.4 or Appendix-II)

$S_{a2} = 10.582\, m/s^2$ and $S_{d2} = 0.06445m$

Top floor displacement $= \Gamma_2 \times \phi_{22} \times S_{d2} = -0.333 \times 1 \times 0.06445$

$= -0.0214m$

Base shear $= 2k \times \phi_{12} \times \Gamma_2 \times S_{d2} = 2 \times 197.392 \times 10^3 \times 1 \times -0.333 \times 0.06445$

$= -8.652\ kN$

Mode	Top floor displacement (m)	Base shear (kN)
1	0.204	40.16
2	-0.0214	-8.652
SRSS	0.2052	41.08
Exact Response (from Example 3.6)	0.202	40.72

Example 4.5

An industrial structure is modeled as 2-DOF system as shown in the Figure 4.10. Determine the lateral displacement, base shear and base moment of the structure due to El-Centro, 1940 earthquake ground motion using the response spectrum method. Take EI =80 × 10^3 Nm^2, L= 2 m, m_1= 100 kg; m_2= 200 kg.

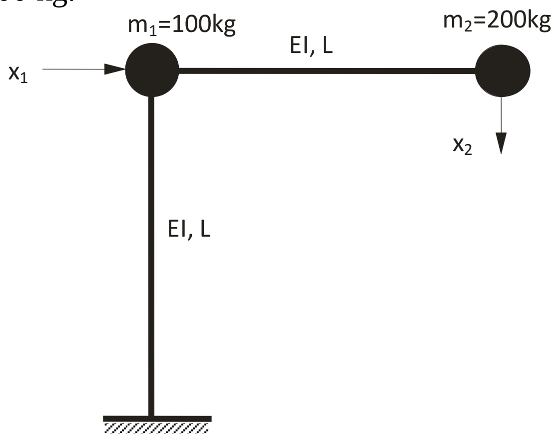


Figure 4.10

Solution: Given, Mass, m_1= 100 kg; m_2= 200 kg; L= 2 m; EI= 80 × 10^3 Nm^2

Stiffness matrix (found by the inverse of flexibility matrix) and mass matrix of above MDOF system is given by,

$$[k] = \frac{6EI}{7L^3}\begin{bmatrix} 8 & -3 \\ -3 & 2 \end{bmatrix}; \ [m] = \begin{bmatrix} 300 & 0 \\ 0 & 200 \end{bmatrix}$$

Finding eigen values and eigen vector using equation (4.19)

$$\{[k] - \omega^2 [m]\}\{\phi\} = 0$$

$$\det\left\|[k] - \omega^2 [m]\right\| = 0$$

ω_1 = 5.4925 rad/sec; ω_2 = 16.856 rad/sec

$$\{\phi_1\} = \begin{Bmatrix} 2.7 \\ 6.25 \end{Bmatrix}; \ \{\phi_2\} = \begin{Bmatrix} 5.103 \\ -3.307 \end{Bmatrix}$$

Modal participation factor,

$$\Gamma_i = \frac{\{\phi_i\}^T \{m\}\{r\}}{\{\phi_i\}^T \{m\}\{\phi_i\}}$$

$$\Gamma_1 = \frac{\begin{bmatrix} 2.7 & 6.25 \end{bmatrix}\begin{bmatrix} 300 & 0 \\ 0 & 200 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}{\begin{bmatrix} 2.7 & 6.25 \end{bmatrix}\begin{bmatrix} 300 & 0 \\ 0 & 200 \end{bmatrix}\begin{bmatrix} 2.7 \\ 6.25 \end{bmatrix}} = 0.081;$$

$$\Gamma_2 = \frac{\begin{bmatrix} 5.103 & -3.307 \end{bmatrix}\begin{bmatrix} 300 & 0 \\ 0 & 200 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}{\begin{bmatrix} 5.103 & -3.307 \end{bmatrix}\begin{bmatrix} 300 & 0 \\ 0 & 200 \end{bmatrix}\begin{bmatrix} 5.103 \\ -3.307 \end{bmatrix}} = 0.153$$

1st Mode Response

$T_1 = \frac{2\pi}{\omega_1}$ =1.1439 sec; ξ_1 = 2%

From response spectra, (refer Figures 4.2 and 4.4 or Appendix-II)

S_{a_1} = 3.203 m/s^2; $S_{d_1} = 0.1053$ m

Lateral displacement

$$x_{1,1} = \Gamma_1 \times \phi_{11} \times S_{d1} = 0.081 \times 2.7 \times 0.1053$$

$$= 0.023 \text{ m}$$

Force vector

$$\{f\}_1 = [m]\{\phi_1\} \times \Gamma_1 \times S_{a1} = \begin{bmatrix} 300 & 0 \\ 0 & 200 \end{bmatrix}\begin{Bmatrix} 2.7 \\ 6.25 \end{Bmatrix} \times 0.081 \times 3.203$$

$$= \begin{Bmatrix} 210.15 \\ 324.3 \end{Bmatrix}$$

Base shear= 210.15 N

Base moment = 2 × (210.15+324.3) = 1068.9 N.m

2^nd^ Mode Response

$T_2 = \dfrac{2\pi}{\omega_2}$ =0.375 sec; ξ_2 = 2%

From response spectra, (refer Figures 4.2 and 4.4 or Appendix-II)

S_{a_2} = 9.7 m/s^2 and S_{d_2} =0.034 m

Lateral displacement

$x_{1,2} = \Gamma_2 \times \phi_{12} \times S_{d2} = 0.153 \times 5.103 \times 0.034$

$= 0.0269$ m

Force vector

$$\{f\}_2 = [m]\{\phi_2\} \times \Gamma_2 \times S_{a2} = \begin{bmatrix} 300 & 0 \\ 0 & 200 \end{bmatrix} \begin{Bmatrix} 5.103 \\ -3.307 \end{Bmatrix} \times 0.153 \times 9.7$$

$$= \begin{Bmatrix} 2272 \\ -981.6 \end{Bmatrix}$$

Base shear = 2272 N

Base moment = 2 × (2272 - 981.6) = 2580.8 N.m

The peak responses using the modal combination rules SRSS and CQC method are summarized below

	Lateral Displacement (mm)	**Base Shear (N)**	**Base Moment (Nm)**
Mode 1	23	210.15	1068.9
Mode 2	26.9	2272	2580.8
SRSS	35.4	2281.7	2793.4
CQC	35.5	2290	2798

Example 4.6

A 2-degree-of-freedom system is subjected to horizontal earthquake excitation (Figure 4.11) with its response spectra as given below. Take the flexural rigidity, EI = 10^6 Nm^2 and length, L = 2m. Each lumped mass is 100 kg. Determine the maximum top mass floor displacement and base shear. Take 2% damping in each mode of vibration.

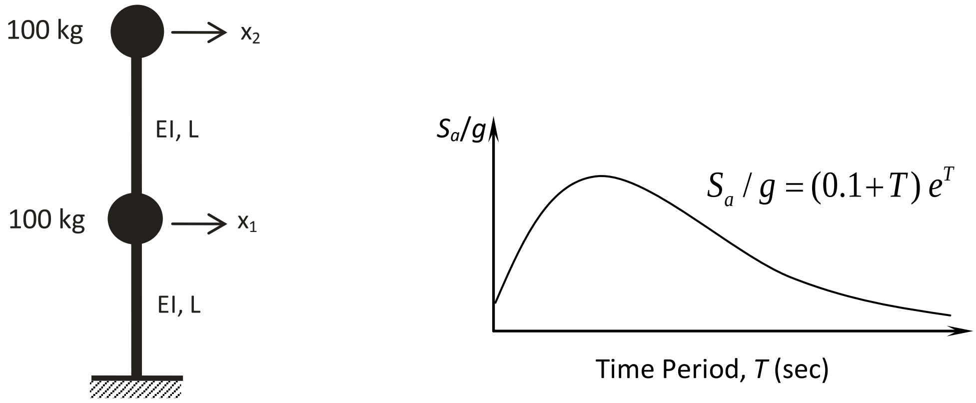


Figure 4.11

Solution: Given, EI = 10^6 N.m^2, m = 100 kg and L = 2 m.

The flexibility matrix of the structure is

$$[f] = \frac{L^3}{6EI}\begin{bmatrix} 2 & 5 \\ 5 & 16 \end{bmatrix}$$

Thus, the stiffness matrix,

$$[k] = [f]^{-1} = \frac{6EI}{7L^3}\begin{bmatrix} 16 & -5 \\ -5 & 2 \end{bmatrix}$$

Eigen values and eigen vector using equation (4.19) are

$$\{[k] - \omega^2 [m]\}\{\phi\} = 0$$

$$\det\left|[k] - \omega^2 [m]\right| = 0$$

$$\left| \frac{6EI}{7L^3}\begin{bmatrix} 16 & -5 \\ -5 & 2 \end{bmatrix} - \omega^2 \begin{bmatrix} m & o \\ 0 & m \end{bmatrix} \right| = 0$$

Let, $\lambda = \dfrac{7mL^3}{6EI}\omega^2$

Thus,

$$\begin{vmatrix} 16-\lambda & -5 \\ -5 & 2-\lambda \end{vmatrix} = 0$$

$$(16-\lambda)(2-\lambda) - 25 = 0$$

$$\lambda^2 - 18\lambda + 7 = 0$$

$$\lambda = 9 \pm \sqrt{74} = 9 \pm 8.6023$$

$\lambda_1 = 0.39767$ and $\lambda_2 = 17.6023$

On substituting λ, the natural frequency of the system will be

$\omega_1 = 20.64\ rad/\sec$; $T_1 = 0.3044 \sec$

$\omega_2 = 137.33\ rad/\sec$; $T_2 = 0.04575 \sec$

On substituting, ω^2 in characteristic equation (4.19), eigen vectors are obtained as

$$\{\phi_1\} = \begin{Bmatrix} 1 \\ 3.1204 \end{Bmatrix} \text{ and } \{\phi_2\} = \begin{Bmatrix} 1 \\ -0.32 \end{Bmatrix}$$

Modal Participation factors

$$\Gamma_i = \frac{\{\phi_i\}^T \{m\}\{r\}}{\{\phi_i\}^T \{m\}\{\phi_i\}}$$

$$\Gamma_1 = \frac{\begin{bmatrix} 1 & 3.1204 \end{bmatrix} \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}}{\begin{bmatrix} 1 & 3.1204 \end{bmatrix} \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \begin{Bmatrix} 1 \\ 3.1204 \end{Bmatrix}} = \frac{412.04}{1073.68} = 0.383$$

$$\Gamma_2 = \frac{\begin{bmatrix} 1 & -0.32 \end{bmatrix} \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}}{\begin{bmatrix} 1 & -0.32 \end{bmatrix} \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \begin{Bmatrix} 1 \\ -0.32 \end{Bmatrix}} = \frac{68}{110.24} = .616$$

1^st^ Mode Response

$T_1 = 0.3044 \sec$

From the response spectra curve,

$$\frac{S_{a_1}}{g} = \frac{(0.1 + 0.3044)}{e^{0.3044}} = 0.298 \quad \text{and} \quad S_{d_1} = \frac{S_{a_1}}{\omega_1^2} = \frac{0.298 \times 9.81}{20.64^2} = 6.86 \times 10^{-3} m$$

Top mass displacement = $\Gamma_1 \times \phi_{21} \times S_{d1} = 3.1204 \times .383 \times 6.86 \times 10^{-3}$

$$= 8.2 \times 10^{-3} m$$

$$f_{s1} = [m]\{\phi_1\} \times \Gamma_1 \times S_{a1} = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \begin{Bmatrix} 1 \\ 3.1204 \end{Bmatrix} 0.383 \times 0.298 \times 9.81$$

$$= \begin{Bmatrix} 100 \\ 312.04 \end{Bmatrix} 0.383 \times 0.298 \times 9.81$$

Base Shear $= (100 + 312.04)\ 0.383 \times 0.298 \times 9.81 = 461.34 N$

2nd Mode Response

$T_2 = 0.04575 \sec$

From response spectra, $\frac{S_a}{g} = 0.138$ and $S_{d_1} = .072 \times 10^{-3} m$

Top mass displacement $= \Gamma_2 \times \phi_{22} \times S_{d2} = -0.32 \times 0.616 \times 0.072 \times 10^{-3} = -0.014 \times 10^{-3} m$

$$f_s = [m]\{\phi_2\} \times \Gamma_2 \times S_{a2} = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \begin{Bmatrix} 1 \\ -0.32 \end{Bmatrix} 0.616 \times 0.138 \times 9.81$$

Base Shear $= 100 \times .68 \times 0.616 \times 0.138 \times 9.81 = 56.7 N$

Final Response by SRSS Method

Top mass displacement $= 8.2 \times 10^{-3} m$

Base Shear = 464.8 N

Example 4.7

A three-story building is modeled as 3-DOF system and rigid floors as shown in Figure 4.12. Determine the top floor maximum displacement and base shear due to El-Centro, 1940 earthquake ground motion using the response spectrum method. Take the inter-story lateral stiffness of floors i.e. k_1 = k_2= k_3=16357.5 x 10^3 N/m and the floor mass m_1 = m_2= 10000 kg and m_3=5000 kg.

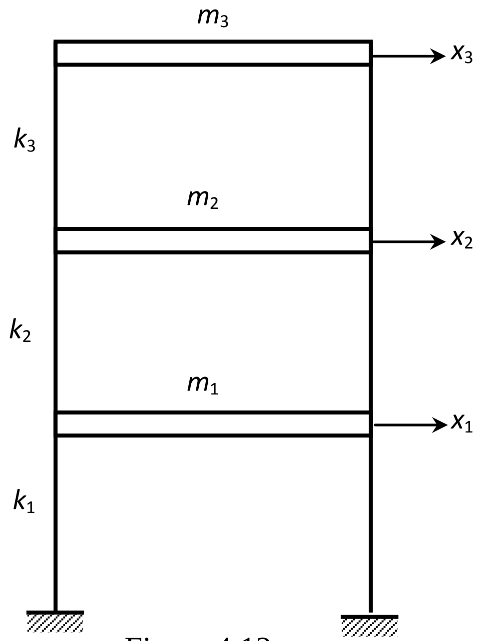


Figure 4.12

Solution: The mass matrix of the structure

$$[m]=\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}=\begin{bmatrix} 10000 & 0 & 0 \\ 0 & 10000 & 0 \\ 0 & 0 & 5000 \end{bmatrix}$$

and the stiffness matrix,

$$[k]=\begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}=\begin{bmatrix} 32715 & -16357.5 & 0 \\ -16357.5 & 32715 & -16357.5 \\ 0 & -16357.5 & 16357.5 \end{bmatrix}$$

Finding eigen values and eigen vectors using the equation (4.19)

$$\{[k]-\omega^2[m]\}\{\phi\}=0$$

$$\det\left|[k]-\omega^2[m]\right|=0$$

$$\det\left|16357.5\times10^3\begin{bmatrix}2 & -1 & 0\\ -2 & 2-2\lambda & 0\\ 0 & -1 & 1-\lambda\end{bmatrix}-\omega^2\times5000\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 1\end{bmatrix}\right|=0$$

$$\lambda=\frac{\omega^2\times5000}{16357.5\times10^3}$$

$$\det\begin{vmatrix}2-2\lambda & -1 & 0\\ -1 & 2-2\lambda & 0\\ 0 & -1 & 1-\lambda\end{vmatrix}=0$$

$$(2-2\lambda)\left[(2-2\lambda)(1-\lambda)-1\right]+\left[-1+\lambda\right]=0$$

$$(2-2\lambda)\left[2-2\lambda-2\lambda+2\lambda^2-1\right]-1+\lambda=0$$

$$2-2\lambda\left[2\lambda^2-4\lambda+1\right]+\lambda-1=0$$

$$4\lambda^2-8\lambda+2-4\lambda^3+2\lambda^2-2\lambda+\lambda-1=0$$

$$-4\lambda^3+12\lambda^2-9\lambda+1=0$$

$\lambda_1=0.134,\quad \lambda_2=1\ ,\quad \lambda_3=1.866$

Implying that

$\omega_1=20.937\,rad/\sec\quad \omega_2=57.2\,rad/\sec\qquad \omega_3=78.13\,rad/\sec$

On substituting ω^2 in the characteristic equation,

For Mode 1

$$\begin{bmatrix}2-2\times0.134 & -1 & 0\\ -1 & 2-2\times0.134 & -1\\ 0 & -1 & 1-0.134\end{bmatrix}\begin{bmatrix}\phi_{11}\\ \phi_{21}\\ \phi_{31}\end{bmatrix}=0$$

Assuming Φ_{11} = 1,

$2-2\times0.134=\phi_{21}$

$\phi_{21}=1.732$ and $\phi_{31}=2.0$

$$\{\phi_1\}=\begin{Bmatrix}1\\ 1.732\\ 2.0\end{Bmatrix}$$

For Mode 2

$$\begin{bmatrix} 2-2 & -1 & 0 \\ -1 & 2-2 & -1 \\ 0 & -1 & 1-1 \end{bmatrix} \begin{bmatrix} \phi_{12} \\ \phi_{22} \\ \phi_{32} \end{bmatrix} = 0$$

$$\{\phi_2\} = \begin{Bmatrix} 1 \\ 0 \\ 1 \end{Bmatrix}$$

For Mode 3

$$\begin{bmatrix} 2-2\times1.866 & -1 & 0 \\ -1 & 2-2\times1.866 & -1 \\ 0 & -1 & 1-1.866 \end{bmatrix} \begin{bmatrix} \phi_{13} \\ \phi_{23} \\ \phi_{33} \end{bmatrix} = 0$$

$$\{\phi_3\} = \begin{Bmatrix} 1 \\ -1.733 \\ 2.0 \end{Bmatrix}$$

The influence coefficient vector is given by

$$\{r\} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

Modal Participation Factors

$$\Gamma_1 = \frac{\{\phi_1\}^T \{m\}\{r\}}{\{\phi_1\}^T \{m\}\{\phi\}} = \frac{\begin{bmatrix} 1 & 1.733 & 2 \end{bmatrix} \times 5000 \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}{\begin{bmatrix} 1 & 1.733 & 2 \end{bmatrix} \times 5000 \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1.733 \\ 2 \end{bmatrix}}$$

$$\Gamma_1 = 0.622$$

Similarly,

$$\Gamma_2 = \frac{\{\phi_2\}^T \{m\}\{r\}}{\{\phi_2\}^T \{m\}\{\phi_2\}} = 0.333$$

$$\Gamma_3 = \frac{\{\phi_2\}^T \{m\}\{r\}}{\{\phi_3\}^T \{m\}\{\phi\}} = 0.045$$

1st Mode Response

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{20.937} = 0.30 \text{ sec}$$

$$\xi_1 = 0.02$$

From the response spectra (refer Figures 4.2 and 4.4 or Appendix-II),

$$S_{d1} = 0.01902m$$

Top floor displacement $= \Gamma_1 \times \phi_{31} \times S_{d1} = 2 \times 0.622 \times 0.01902$

$= 0.0236m$

Base shear $= k \times \phi_{11} \times \Gamma_1 \times S_{d1} = 16.357 \times 10^6 \times 1 \times 0.622 \times 0.01902$

$= 193\ kN$

2nd Mode Response

$$T_2 = \frac{2\pi}{57.2} = 0.11\text{sec}$$

$$\xi_2 = 0.02$$

from response spectra (refer Figures 4.2 and 4.4 or Appendix-II),

$$S_{d2} = 0.00231m$$

Top floor displacement $= \phi_{32} \times \Gamma_2 \times S_{d2} = -1 \times 0.333 \times 0.00231$

$= -7.69 \times 10^{-4} m$

Base shear $= k \times \phi_{12} \times \Gamma_2 \times S_{d2} = 16357 \times 10^3 \times 1 \times 0.333 \times 0.00231 =$ $12.58kN$

3rd Mode Response

$$T_3 = \frac{2\pi}{78.13} = 0.08\text{sec}$$

$$\xi_3 = 0.02$$

from response spectra (refer Figures 4.2 and 4.4 or Appendix-II),

$S_{d3} = 9.77 \times 10^{-4} m$

Top floor displacement $= \phi_{33} \times \Gamma_3 \times S_{d3}$

$= 2 \times 0.045 \times 9.77 \times 10^{-4} m$

$= 8.793 \times 10^{-5} m$

Base shear $= k \times \phi_{13} \times \Gamma_3 \times S_{d3}$

$= 16357.5 \times 10^{3} \times 1 \times 0.045 \times 9.77 \times 10^{-4}$

$= 0.719\ kN$

Peak responses using the SRSS modal combination rule are given below

Mode	Top floor displacement (mm)	Base shear (kN)
1	23.6	193
2	-0.769	12.58
3	0.0879	0.719
SRSS	23.6	193.41
Exact Response (from time history analysis)	23.4	196.4

4.5 Design of Earthquake Resistant Structure Based on Codal Provisions

General principles and design philosophy for design of earthquake-resistant structure are as follows:

a) The characteristics of seismic ground vibrations at any location depends upon the magnitude of earth quake, its depth of focus, distance from epicenter, characteristic of the path through which the waves travel, and the soil strata on which the structure stands. Ground motions are predominant in horizontal direction.
b) Earthquake generated vertical forces, if significant, as in large spans where differential settlement is not allowed, must be considered.
c) The response of a structure to the ground motions is a function of the nature of foundation soil, materials size and mode of construction of structures, and the duration and characteristic of ground motion.
d) The design approach is to ensure that structures possess at least a minimum strength to withstand minor earthquake (DBE), which occur frequently, without damage; resist moderate earthquake without significant damage though some nonstructural damage may occur, and aims that structures withstand major earthquake (MCE) without collapse. Actual forces that appeared on structures are much greater then the design forces specified here, but ductility, arising due to inelastic material behavior and detailing, and over strength, arising from the additional reserve strength in structures over and above the design strength are relied upon to account for this difference in actual and design lateral forces.
e) Reinforced and pre-stressed members shall be suitably designed to ensure that premature failure due to shear or bond does not occur, as per IS:456 and IS:1343.
f) In steel structures, members and their connections should be so proportioned that high ductility is obtained.
g) The soil structure interaction refers to the effect of the supporting foundation medium on the motion of structure. The structure interaction may not be considered in the seismic analysis for structures supporting on the rocks.
h) The design lateral forces shall be considered in two orthogonal horizontal directions of the structures. For structures, which have lateral force resisting elements in two orthogonal directions only, design lateral force must be considered in one direction at a time. Structures having lateral resisting elements in two directions other than

orthogonal shall be analyzed according to *clause 2.3.2 IS 1893 (part 1): 2002*. Where both horizontal and vertical forces are taken into account, load combinations must be according to *clause 2.3.3 IS 1893 (part 1): 2002.*

i) When a change in occupancy results in a structure being re-classified to a higher importance factor (*I*), the structure shall be confirm to the seismic requirements of the new structure with high importance factor.

4.6. Design Criteria

For the purpose of determining the design seismic forces, the country (India) is classified into four seismic zones (II, III, IV, and V). Previously, there were five zones, of which Zone I and II are merged into Zone II in fifth revision of code. The design horizontal seismic forces coefficient A_h for a structure shall be determined by following expression

$$A_h = \frac{ZIS_a}{2Rg} \tag{4.36}$$

Z = zone factor for the maximum considerable earthquake (MCE) and service life of the structure in a zone. Factor 2 in denominator is to reduce the MCE to design basis earthquake (DBE).

I = importance factor , depending on the functional purpose of the building, characterized by hazardous consequences of its failure , post earthquake functional needs, historical value , or economic importance.

R = response reduction factor, depending upon the perceived seismic damage performance of the structure, characterized by ductile or brittle deformations however the ratio I/R shall not be greater than 1.

S_a/g = average response acceleration coefficient (Figure 4.13).

For rocky, or hard soil sites;

$$\frac{S_a}{g} = \begin{cases} 1+15T & (0.00 \le T \le 0.10) \\ 2.50 & (0.10 \le T \le 0.40) \\ 1.00/T & (0.4 \le T \le 4.00) \end{cases} \tag{4.37}$$

For medium soil sites

$$\frac{S_a}{g} = \begin{cases} 1+15T & (0.00 \leq T \leq 0.10) \\ 2.50 & (0.10 \leq T \leq 0.55) \\ 1.36/T & (0.55 \leq T \leq 4.00) \end{cases} \tag{4.38}$$

For soft soil sites

$$\frac{S_a}{g} = \begin{cases} 1+15\mathrm{T} & (0.00 \leq \mathrm{T} \leq 0.10) \\ 2.50 & (0.10 \leq \mathrm{T} \leq 0.67) \\ 1.67/\mathrm{T} & (0.67 \leq \mathrm{T} \leq 4.00) \end{cases} \tag{4.39}$$

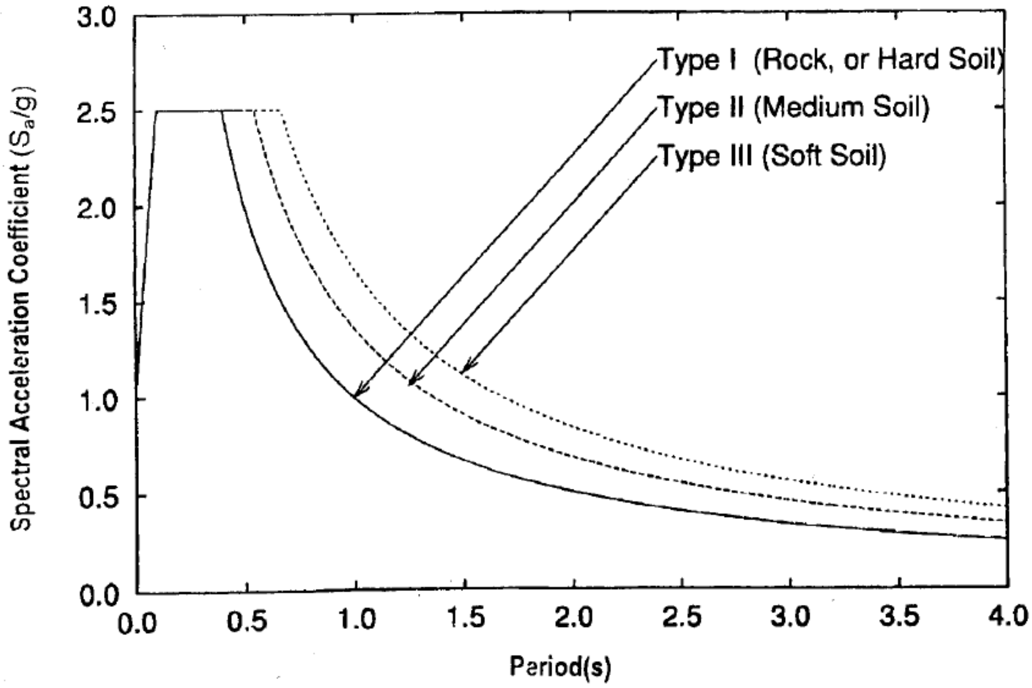


Figure 4.13: Design response spectra curve as per IS:1893-2002 code.

4.7 Design Lateral Force

The total design lateral force or design seismic base shear (V_b) along any principal direction of the building shall be determined by the following expression

$$V_b = A_h W \qquad (4.40)$$

where A_h is the horizontal seismic forces coefficient (refer equation (4.36) and W is the seismic weight of building.

4.7.1 Seismic Weight

The seismic weight of each floor is its full dead load plus appropriate amount of imposed load as specified. While computing the seismic weight of each floor, the weight of columns and walls in any storey shall be equally distributed to the floors above and below the storey.

The seismic weight of the whole building is the sum of the seismic weights of all the floors.

Any weight supported in between the storey shall be distributed to the floors above and below in inverse proportion to its distance from the floors.

4.7.2 Fundamental Natural Period

The fundamental natural time period as mentioned in *clause 7.6 IS 1893 (part 1): 2002* for moment resisting RC frame building without brick infill walls and moment resisting steel frame building without brick infill walls, respectively is given by

$$T_a = 0.075h^{0.75} \qquad (4.41)$$

$$T_a = 0.085h^{0.75} \qquad (4.42)$$

where, h = height of the building in 'm' excluding basement storey, if it is connected with the ground floor decks or fitted in between the building column.

If there is brick filling, then the fundamental natural period of vibration, may be taken as

$$T_a = \frac{0.09h}{\sqrt{d}} \qquad (4.43)$$

where, h = height of the building in m, as defined above, and d = base dimension of the building at the plinth level, in meter, along the considered direction of the lateral force.

4.7.3 Distribution of Design Force

The design base shear, V_b computed above shall be distributed along the height of the building as per the following expression,

$$Q_i = \frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2} \tag{4.44}$$

where,

Q_i = design lateral force at ith floor

W_i = seismic weight of ith floor

h_i = height of ith floor measured from base, and

n = numbers of storey in the building is the number of the levels at which the masses are located

In case of buildings whose floors are capable of providing rigid horizontal diaphragm action, the total shear in any horizontal plane shall be distributed to the various vertical elements of lateral force resisting system, assuming the floors to be infinitely rigid in the horizontal plane.

In case of building whose floor diaphragms cannot be treated infinitely rigid in their own plane , the lateral shear at each floor shall be distributed to the vertical elements resisting the lateral forces, considering the in plane flexibility of the diaphragms.

4.8 Response Spectrum Method (Dynamic Analysis)

4.8.1 General Codal Provisions

Dynamic analysis should be performed to obtain the design seismic force, and its distribution to different levels along the height of the building and to various lateral load resisting elements, for the following buildings:

- Regular buildings- Those are greater than 40 m in height in zone IV, V and those are greater than 90 m height in zones II,III, and
- Irregular buildings-All framed buildings higher than 12 m in zone IV and V, and those are greater than 40 m in height in zone II and III.

Dynamic analysis may be performed either by time history method or by the response spectrum method. However in either method, the design base shear V_b shall be compared with a base shear $\overline{V_b}$ calculated using a fundamental period T_a. When V_b is less than $\overline{V_b}$ all the response quantities shall be multiplied by $\overline{V_b}/V_b$

The values of damping for a building may be taken as 2 and 5 percent of the critical, for the purpose of dynamic analysis of steel and reinforced concrete buildings, respectively.

4.8.2 Modes to be Considered

The number of modes to be considered in the analysis should be such that the sum of the total modal masses of all modes considered is at least 90% of the total seismic mass and the missing mass correction beyond 33%.If modes with natural frequency beyond 33 Hz are to be considered, modal combination shall be carried out only for modes up to 33 Hz.

4.8.3 Computation of Dynamic Quantities

Buildings with regular ,or nominally irregular plan configuration may be modeled as a system of masses lumped at the floor levels with each mass having one degree of freedom, that of lateral displacement in the direction of consideration. In such a case, the following expressions shall hold in computations of various quantities.

a) Modal mass

$$M_k = \frac{\left[\sum_{i=1}^{n} W_i \phi_{ik}\right]^2}{g \sum_{i=1}^{n} W_i (\phi_{ik})^2} \tag{4.45}$$

where,

g = acceleration due to gravity

ϕ_{ik} = mode shape coefficient of floor, i in mode, k, and

W_i = seismic weight of floor, i

b) Modal Participation Factor: The factor is given by

$$P_k = \frac{\sum_{i=1}^{n} W_i \varphi_{ik}}{\sum_{i=1}^{n} W_i (\varphi_{ik})^2} \tag{4.46}$$

c) Design lateral force at each floor in each Mode: The peak lateral force at floor i in k^{th} mode is given by

$$Q_{ik} = A_k \phi_{ik} p_k W_i \tag{4.47}$$

where, A_k = Design horizontal acceleration spectrum values using the natural period of vibration

d) Storey shear force in each mode: The storey peak shear force at i^{th} storey in mode k is given by

$$V_{ik} = \sum_{j=i+1}^{n} Q_{ik} \tag{4.48}$$

4.9 Numerical on Seismic Design of Structures

Example 4.7

An eight-story residential RC building is to be constructed in an area of seismic Zone IV having hard soil. The plan dimension of the building is 15m x 20m with storey height of 3.6m. Determine the base shear as per the IS:1893-2002 (Part 1) code. Use both seismic coefficient and response spectrum approach. Take the inter-story lateral stiffness of floors i.e. $k_1=k_2=k_3=k_4=671.52\times10^6$ N/m and $k_5=k_6=k_7=k_8=335.76\times10^6$ N/m. The loading on the floors shall be taken as:

Location	Self Wight + Dead Load (kN/m^2)	Live Load (kN/m^2)
Roof	5	1.5
Floors	10	4

Solution:

Zone factor, Z=0.24 (*Table 2 IS 1893 (part 1): 2002*)

Importance factor, I=1.0 (*Table 6 IS 1893 (part 1): 2002*)

Response reduction factor, R=3 (*Table 7 IS 1893 (part 1): 2002*)

Seismic weight of building (*Clause 7.3.1 IS 1893 (part 1): 2002*)

Seismic weight of roof = 15×20×5 = 1500 kN

Seismic weight of each floor = dead load + a fraction of imposed load

= 15×20×10 + 0.5× (15×20×4) =3600kN

Total seismic weight of building, W= 1500 + 7×3600 = 26700 kN

(A) Analysis by Seismic Coefficient Method

Fundamental natural time period, $T_a = 0.075h^{0.75} = 0.075\times(3.6\times8)^{0.75} = 0.9324$ sec

Spectral acceleration, $S_a/g = 1/T_a = 1/0.9324 = 1.0725$

Design horizontal seismic coefficient, $A_h = \dfrac{ZIS_a}{2Rg}$

$A_h = 0.0429$

Total base shear is given by

$V_b = A_h \times W$

V_b= 0.0429 × 26700 = 1145.42 kN

The lateral forces calculated using the equation (4.44) and are presented below.

Floor/Roof	h_i (m)	W_i (kN)	$W_i h_i^2 / \Sigma W_i h_i^2$	Q_i (kN)	Base Shear (kN)
Roof	28.8	1500	0.16	**183.27**	183.27
7	25.2	3600	0.294	**336.75**	520.02
6	21.6	3600	0.216	**247.41**	767.43
5	18	3600	0.15	**171.81**	939.25
4	14.4	3600	0.096	**109.96**	1049.21
3	10.8	3600	0.054	**61.85**	1111.06
2	7.2	3600	0.024	**27.49**	1138.55
1	3.6	3600	0.006	**6.87**	1145.42

(B) Analysis by Response Spectrum Method

Stiffness Matrix of the building is,

$$[k] = \begin{bmatrix} 1343.04 & -671.52 & 0 & 0 & 0 & 0 & 0 & 0 \\ -671.52 & 1343.04 & -671.52 & 0 & 0 & 0 & 0 & 0 \\ 0 & -671.52 & 1343.04 & -671.52 & 0 & 0 & 0 & 0 \\ 0 & 0 & -671.52 & 1007.28 & -335.76 & 0 & 0 & 0 \\ 0 & 0 & 0 & -335.76 & 671.52 & -335.76 & 0 & 0 \\ 0 & 0 & 0 & 0 & -335.76 & 671.52 & -335.76 & 0 \\ 0 & 0 & 0 & 0 & 0 & -335.76 & 671.52 & -335.76 \\ 0 & 0 & 0 & 0 & 0 & 0 & -335.76 & 335.76 \end{bmatrix} \times 10^6 N/m$$

The mass matrix of the building is,

$$[m]=\begin{bmatrix} 3 & 60 & 00 & 0 & 0 & 0 & 0 & 0 & & \\ 0 & 3 & 60 & 00 & 0 & 0 & 0 & 0 & & \\ 0 & 0 & 3 & 60 & 00 & 0 & 0 & 0 & & \\ 0 & 0 & 0 & 3 & 60 & 00 & 0 & 0 & & \\ 0 & 0 & 0 & 0 & 3 & 60 & 00 & 0 & & \\ 0 & 0 & 0 & 0 & 0 & 3 & 60 & 00 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 60 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 5 & 0 \end{bmatrix}\times 10^3 Kg$$

The mode-shapes and frequencies of the building are obtained using the equation (4.19) and are given below

$$\begin{array}{c c} & \begin{array}{cccccccc} \phi_1 & \phi_2 & \phi_3 & \phi_4 & \phi_5 & \phi_6 & \phi_7 & \phi_8 \end{array} \\ [\phi]= & \begin{pmatrix} .927538E\text{-}03 & .858964E\text{-}03 & .998116E\text{-}03 & .855939E\text{-}03 & .966417E\text{-}03 & .154996E\text{-}02 & .575802E\text{-}04 & .223343E\text{-}05 \\ .901977E\text{-}03 & .698991E\text{-}03 & .477048E\text{-}03 & .433966E\text{-}04 & -.287197E\text{-}03 & -.107689E\text{-}02 & -.630515E\text{-}04 & -.441046E\text{-}05 \\ .816761E\text{-}03 & .226586E\text{-}03 & -.641725E\text{-}03 & -.868018E\text{-}03 & -.646702E\text{-}03 & .676496E\text{-}03 & .133359E\text{-}03 & .204813E\text{-}04 \\ .677526E\text{-}03 & -.347097E\text{-}03 & -.956468E\text{-}03 & .198184E\text{-}03 & .100712E\text{-}02 & -.321754E\text{-}03 & -.340780E\text{-}03 & -.100992E\text{-}03 \\ .493480E\text{-}03 & -.765637E\text{-}03 & -.728334E\text{-}04 & .812858E\text{-}03 & -.474450E\text{-}03 & -.112684E\text{-}04 & .898599E\text{-}03 & .499308E\text{-}03 \\ .385138E\text{-}03 & -.803796E\text{-}03 & .414611E\text{-}03 & .194222E\text{-}03 & -.476702E\text{-}03 & .166894E\text{-}03 & -.740874E\text{-}03 & -.984752E\text{-}03 \\ .264060E\text{-}03 & -.662318E\text{-}03 & .642317E\text{-}03 & -.645664E\text{-}03 & .263085E\text{-}03 & .563832E\text{-}05 & -.517710E\text{-}03 & .105008E\text{-}02 \\ .134250E\text{-}03 & -.372819E\text{-}03 & .467637E\text{-}03 & -.750037E\text{-}03 & .593351E\text{-}03 & -.167083E\text{-}03 & .100703E\text{-}02 & -.667407E\text{-}03 \end{pmatrix} \end{array}$$

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \\ \omega_6 \\ \omega_7 \\ \omega_8 \end{bmatrix} = \begin{pmatrix} 7.85398 \\ 20.4176 \\ 34.1842 \\ 46.0967 \\ 53.8850 \\ 61.5923 \\ 68.4809 \\ 81.6427 \end{pmatrix} \text{rad/sec}$$

The corresponding time periods are given by

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \end{bmatrix} = \begin{pmatrix} 0.8 \\ 0.3076 \\ 0.1837 \\ 0.1362 \\ 0.1165 \\ 0.1020 \\ 0.0917 \\ 0.0769 \end{pmatrix} \text{sec}$$

The modal participation factors using the equation (4.46) are

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} = \begin{pmatrix} 1461.48 \\ -600.55 \\ 268.73 \\ -237.03 \\ 137.22 \\ -29.57 \\ 144.20 \\ -67.23 \end{pmatrix}$$

The horizontal seismic coefficient, A_k in the k^{th} mode is calculated as

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \end{bmatrix} = \begin{pmatrix} 0.5 \\ 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \\ 0.95 \\ 0.86 \end{pmatrix}$$

Design lateral load at i^{th} floor in k^{th} mode (i.e. $Q_{ik} = A_k \phi_{ik} p_k W_i$) is given as

$$
\begin{array}{cc}
 & \begin{array}{cccccccc} Q_{i1} & Q_{i2} & Q_{i3} & Q_{i4} & Q_{i5} & Q_{i6} & Q_{i7} & Q_{i8} \end{array} \\
[Q_{ik}] = & \begin{pmatrix}
101.67 & -77.38 & 40.23 & -30.43 & 19.89 & -6.88 & 1.18 & -0.02 \\
237.28 & -151.12 & 46.15 & -3.70 & -14.19 & 11.47 & -3.11 & 0.09 \\
214.86 & -48.99 & -62.08 & 74.07 & -31.95 & -7.20 & 6.58 & -0.43 \\
178.23 & 75.04 & -92.53 & -16.91 & 49.75 & 3.43 & -16.82 & 2.11 \\
129.82 & 165.53 & -7.05 & -69.36 & -23.44 & 0.12 & 44.34 & -10.41 \\
101.32 & 173.78 & 40.11 & -16.57 & -23.55 & -1.78 & -36.56 & 20.54 \\
69.47 & 143.19 & 62.14 & 55.10 & 13.00 & -0.06 & -25.55 & -21.90 \\
35.32 & 80.60 & 45.24 & 64.00 & 29.31 & 1.78 & 49.69 & 13.92
\end{pmatrix} kN
\end{array}
$$

The storey shear forces in each mode as per equation (4.48) are as follows:

$$
\begin{array}{cc}
 & \begin{array}{cccccccc} V_{i1} & V_{i2} & V_{i3} & V_{i4} & V_{i5} & V_{i6} & V_{i7} & V_{i8} \end{array} \\
[V_{ik}] = & \begin{pmatrix}
101.67 & -77.38 & 40.23 & -30.43 & 19.89 & -6.88 & 1.18 & -0.02 \\
338.95 & -228.50 & 86.38 & -34.14 & 5.70 & 4.59 & -1.93 & 0.07 \\
553.81 & -277.48 & 24.30 & 39.93 & -26.24 & -2.61 & 4.65 & -0.35 \\
732.05 & -202.44 & -68.23 & 23.02 & 23.51 & 0.81 & -12.16 & 1.75 \\
861.86 & -36.91 & -75.27 & -46.34 & 0.07 & 0.93 & 32.18 & -8.66 \\
963.18 & 136.86 & -35.16 & -62.91 & -23.48 & -0.84 & -4.38 & 11.88 \\
1032.65 & 280.06 & 26.97 & -7.82 & -10.48 & -0.90 & -29.93 & -10.03 \\
1067.96 & 360.66 & 72.22 & 56.18 & 18.83 & 0.87 & 19.77 & 3.90
\end{pmatrix} kN
\end{array}
$$

The modal combination rule (SRSS method) is applied to obtain storey shear given below,

$$
\begin{bmatrix} V_{roof} \\ V_7 \\ V_6 \\ V_5 \\ V_4 \\ V_3 \\ V_2 \\ V_1 \end{bmatrix} = \begin{pmatrix} 138.97 \\ 419.26 \\ 621.78 \\ 763.39 \\ 867.81 \\ 975.89 \\ 1070.83 \\ 1131.26 \end{pmatrix} kN
$$

Peak Lateral Forces on Each Storey,

Lateral forces are back calculated by storey shear. For example $Q_{Roof} = V_{Roof}$ and $Q_7 = V_7 - V_{Roof}$ similarly $Q_6 = V_6 - V_7$ and so on.

$$\begin{bmatrix} Q_{Roof} \\ Q_7 \\ Q_6 \\ Q_5 \\ Q_4 \\ Q_3 \\ Q_2 \\ Q_1 \end{bmatrix} = \begin{pmatrix} 138.97 \\ 280.39 \\ 202.52 \\ 141.6 \\ 104.4 \\ 108.1 \\ 94.9 \\ 60.4 \end{pmatrix} kN$$

The total base shear obtained is

$$= 138.97+280.39+202.52+141.6+104.4+108.1+94.9+60.4$$

$$= 1131.26 \text{ kN}$$

Since the above calculated base shear is less than that obtained using seismic coefficient method (i.e. 1145.42 kN), therefore, according to C*lause 7.8.2 IS 1893 (part 1): 2002*, the calculated base shear shall multiplied by the factor,

$$\frac{1145.42}{1131.26} = 1.0125$$

The corrected final lateral forces and storey shears are

$$\begin{bmatrix} Q_{roof} \\ Q_7 \\ Q_6 \\ Q_5 \\ Q_4 \\ Q_3 \\ Q_2 \\ Q_1 \end{bmatrix} = \begin{pmatrix} 140.70 \\ 283.89 \\ 205.05 \\ 143.37 \\ 105.705 \\ 109.45 \\ 96.086 \\ 61.16 \end{pmatrix} kN \qquad \begin{bmatrix} V_{roof} \\ V_7 \\ V_6 \\ V_5 \\ V_4 \\ V_3 \\ V_2 \\ V_1 \end{bmatrix} = \begin{pmatrix} 140.70 \\ 424.50 \\ 629.55 \\ 772.93 \\ 878.65 \\ 988.08 \\ 1084.21 \\ 1145.40 \end{pmatrix} kN$$

The values of lateral forces and base shear thus obtained by seismic coefficient method and response spectra method are summarized below

Floor/Roof	**Seismic Coefficient Method**		**Response Spectrum Method**	
	Q_i (kN)	**V_i (kN)**	**Q_i (kN)**	**V_i (kN)**
Roof	183.27	183.27	140.70	140.71
7	336.75	520.02	283.89	424.50
6	247.41	767.43	205.05	629.55
5	171.81	939.25	143.37	772.93
4	109.96	1049.21	105.70	878.66
3	61.85	1111.06	109.45	988.09
2	27.49	1138.55	96.08	1084.22
1	6.87	1145.42	61.16	1145.40

4.10 Tutorial Problems

Q1. Determine the displacement response spectra of SDOF system subjected to earthquake acceleration, $\ddot{x}_g = 0.3g\cos(12t) = \ddot{x}_0\cos(\overline{\omega}t)$.

Q2. A two-story building is modeled as 2-DOF system and rigid floors shown in Figure 4.14. The inter-story lateral stiffness of first and second floor is k_1 and k_2, respectively. Take mass value, m=10000 kg. Determine the k_1 and k_2 so that the time periods in first and second mode of vibration of building are 0.2 sec and 0.1 sec, respectively. Take 2% damping in each mode of vibration. Determine maximum base shear and top floor displacement due to earthquake excitation whose response spectra for 2% damping are given below.

Period (sec)	S_a (m/s^2)	S_d (m)
0.1	6.45	1.65×10^{-3}
0.2	10.29	10.42×10^{-3}

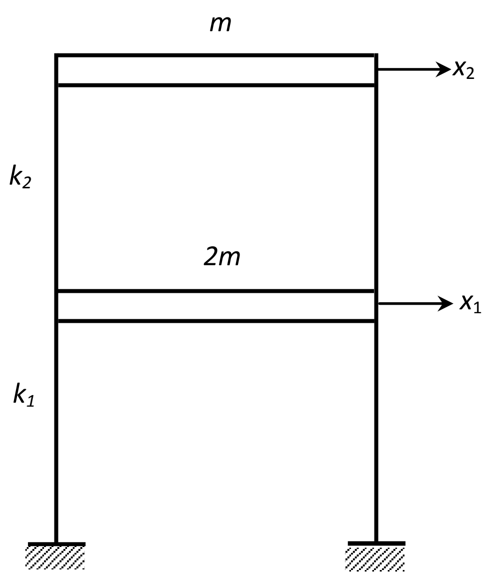
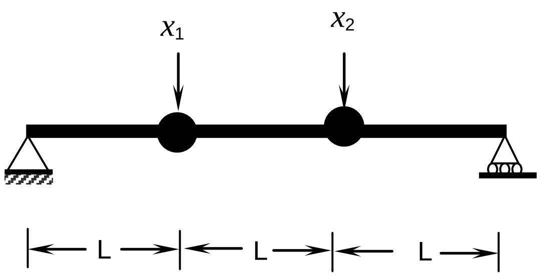
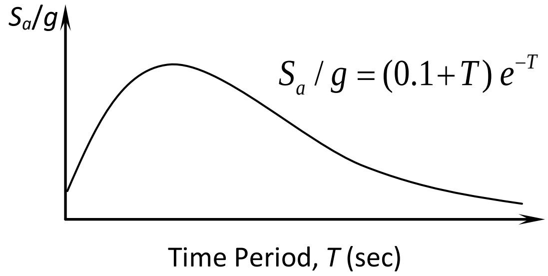


Figure 4.14

Q3. A uniform bridge deck is simply supported as shown in Figure 4.15. The mass of each lumped mass is m and flexural rigidity of deck is EI. The bridge is modeled as a two-degrees-of-freedom discrete system as indicated in the figure. Assuming same earthquake acts simultaneously on both the supports in the vertical direction. Determine the maximum displacement of each mass. Take L = 8m, m = 1000 kg/m and $EI = 8 \times 10^8$ kN.m^2. Use SRSS method for combining the response in two modes. The spectrum of the ground motion is given in the Figure 4.15(b).



(a) Model of Bridge



(b) Response Spectrum

Figure 4.15

Q4. A 5-story building is to be constructed in the area of seismic zone III having medium soil. The dimension of the building is 15m × 20m. The height of each story is 3.5m. The live and dead load on each floor is 2.5 kN/m^2 and 10 kN/m^2, respectively. The live and dead load on the roof is 1.5 kN/m^2 and 5 kN /m^2, respectively. Take importance factor as 1 and response reduction factor as 5. Determine the seismic shear force in each story and overturning moment at the base as per IS: 1893 (Part 1)-2002. Take the value of Z=0.16 for Zone III and spectral acceleration for medium soil from IS: 1893 (Part 1)-2002 as

$$\frac{S_a}{g} = \begin{cases} 1+15T & \text{for} \quad 0 \le T \le 0.1 \\ 2.5 & 0.1 \le T \le 0.55 \\ 1.36/T & 0.55 \le T \le 4 \end{cases}$$

Q5. An 9-story RCC residential building, shown in Figure 4.16 is to be constructed in an area of seismic Zone III having medium soil. The plan dimension of the building is 20m x 30m with storey height of 3.65m. Determine the base shear and lateral forces on each floor as per the IS: 1893-2002 code. Use both seismic coefficient and response spectrum approach. Take inter-story lateral stiffness of floors i.e. $k_1=k_2=k_3=1326\times10^6$ N/m, $k_4=k_5=k_6=994.5\times10^6$ and $k_7=k_8=k_9=663\times10^6$ N/m. The loading on the floors shall be taken as

Location	Floors	Roof
Self Wt + Dead load (kN/m^2)	10	4
Live load (kN/m^2)	5	1.5

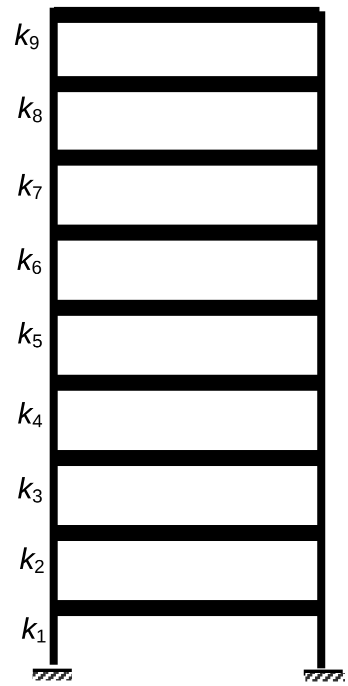


Figure 4.16

Natural Frequencies (rad/sec)

6.98	18.78	30.40	41.96	50.91	59.79	62.21	69.64	79.60

Mode-shapes

1.91	2.01	1.79	2.21	1.91	3.07	2.11	0.17	0.00
1.87	1.69	1.04	0.45	-0.33	-1.90	-1.59	-0.20	-0.01
1.73	0.72	-0.75	-2.17	-1.64	0.50	1.38	0.48	0.03
1.50	-.52	-1.79	-.64	1.67	0.95	-1.46	-1.38	-0.14
1.29	-1.22	-1.29	1.20	0.75	-1.21	0.74	2.22	0.39
1.04	-1.60	0.08	1.51	-1.58	-0.24	0.87	-1.98	-0.88
.75	-1.58	1.39	-0.10	-0.94	1.35	-1.43	0.76	1.88
.51	-1.25	1.68	-1.21	0.86	-.08	-0.15	0.81	-2.52
.26	-.69	1.12	-1.16	1.45	-1.36	1.45	-1.28	1.75

4.11 Answers to Tutorial Problems

Q1. The displacement spectra is given by

$$S_d = \frac{\ddot{x}_0}{\omega_0^2} \frac{1}{\sqrt{\left(1-\beta^2\right)+\left(2\xi\beta\right)^2}}$$

where, $\beta = \frac{\overline{\omega}}{\omega_0} = \frac{12}{\omega_0}$ and $\ddot{x}_0 = 0.3g$

Q2. For first set of values of stiffness (i.e. k_1=59157600 N/m and k_2= 13146133.33 N/m)

Mode	Top floor displacement (mm)	Base shear(kN)
1	13.7544	203.919
2	-0.5445	65
SRSS	**13.765**	**214.03**

For second set of values of stiffness (i.e. k_1=39438400 N/m and k_2= 19719200 N/m)

Mode	Top floor displacement (mm)	Base shear(kN)
1	13.7544	271.225
2	-0.5445	21.474
SRSS	**13.765**	**272.073**

Q3. The maximum displacement of each mass = 36.37mm

Q4.

Q_i (kN)	**V_i (kN)**
4014.1	4014.1
5459.2	9473.2
3070.8	12544.0
1364.8	13908.8
341.2	14250.0

Q5.

Floor/Roof	Seismic Coefficient Method		Response Spectrum Method	
	Q_i (kN)	V_i (kN)	Q_i (kN)	V_i (kN)
Roof	222.8	222.8	160.7	160.7
8	422.5	645.3	217.4	378.1
7	323.5	968.8	236.0	614.1
6	237.7	1206.5	218.8	832.9
5	165.0	1371.5	194.7	1027.7
4	105.6	1477.1	175.3	1203.0
3	59.4	1536.5	137.0	1340.0
2	26.4	1562.9	148.1	1488.1
1	6.6	1569.5	81.4	1569.4