# Chapter 7

# Non-linear Seismic Response of Structures

## 7.1 Introduction

As per the conventional earthquake-resistant design philosophy, the structures are designed for forces, which are much less than the expected design earthquake forces. Hence, when a structure is struck with severe earthquake ground motion, it undergoes inelastic deformations. Even though the structure may not collapse but the damages can be beyond repairs. In reinforced cement concrete (RCC) structures, a structural system can be made ductile, by providing reinforcing steel according to the IS:13920-1993 code. A sufficiently ductile structural system undergoes large deformations in the inelastic region. In order to understand the complete behaviour of structures, time history analysis of different Single Degree of Freedom (SDOF) and Multi Degree of Freedom (MDOF) structures having non-linear characteristics is required to be performed. The results of time history analysis, i.e. non-linear analysis of these structures will help in understanding their true behavior. From the results, it can be predicted, whether the structure will not collapse / partially collapse or totally collapse.

In this chapter, the modeling of SDOF and MDOF structures having non-linear characteristics for seismic response analysis is carried out. The push over analysis of the RCC building is also presented.

## 7.2 Non-linear Force-Deformation Behavior

The structural systems which have linear inertia, damping and restoring forces, are analysed by linear methods. Whenever, the structural system has any or all of the three reactive forces (i.e. inertia, damping and stiffness) having non-linear variation with the response parameters, namely displacement, velocity, and acceleration; a set of non-linear differential equations is evolved. To obtain the response, these equations need be solved. The most common non-linearity is the stiffness and the damping non-linearity. The stiffness non-linearity comprises of two types namely the geometric non-linearity and the material non-linearity.

For the material non-linearity, restoring action shows a hysteretic behavior under cyclic loading. For the geometric non-linearity, no such hysteretic behavior is exhibited. During unloading, the load deformation path follows that of the loading. Figure 7.1(a) shows the case of load deformation behavior of the non-hysteretic type. Figure 7.2(b) shows the hysteretic behavior of a non-linear restoring force under cyclic loading (material non-linearity).

Damping non-linearity may be encountered in dynamic problems associated with structural control, offshore structures, and aerodynamics of structures. Most of the damping non-linearities are of a non-hysteretic type. Most structures under earthquake excitation undergo yielding. Hence, it is necessary to discuss material non-linearity exhibiting hysteretic behavior.

For structural systems having linear behaviour (when subjected to weak ground motions) of inertial forces, spring elastic forces and linear damping characteristics, linear methods of analysis can be employed. Displacement, velocity and acceleration are important response parameters of any structural system. When any or all of the reactive forces, viz. inertia force / spring force or damping force has nonlinear variation with the response parameters, the analysis involves non-linear differential equations. Solution of these equations will give the response of the system. The popular method to obtain the response is Newmark's Beta method.

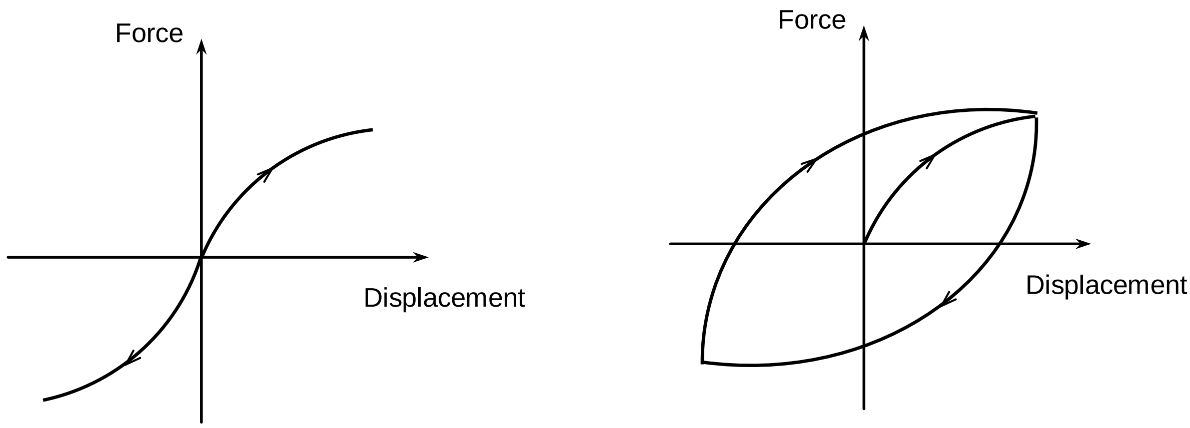



Figure 7.1 (a) Non-linear restoring force for geometrical non-linearity (non-hysteretic type) and (b) Non-linear restoring forces (hysteretic type)

## 7.3 Non-Linear Analysis of SDOF system

Consider a SDOF system having non-linear damping and stiffness characteristics as shown in the Figure 7.2.

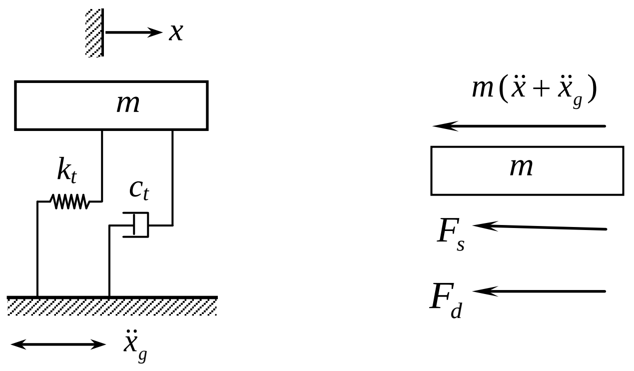



Figure 7.2 Non-linear SDOF system and its free body diagram.

For the SDOF system, the equation of motion in the incremental form is expressed as

$$m\Delta\ddot{x}_i + c_t\Delta\dot{x}_i + k_t\Delta x_i = -m\Delta\ddot{x}_{\boldsymbol{g}}^i \tag{7.1}$$

where $m$ is the mass of the SDOF system, $c_t$ is the initial tangent damping coefficient and $k_t$ is the initial tangent stiffness at the beginning of the time step, respectively.

The solution of the equation of motion for the SDOF system is obtained using the numerical integration technique. The incremental quantities in the equation (7.1) are the change in the responses from time $t_i$ to $t_{i+1}$ given by

$$\left.\begin{aligned} \Delta\ddot{x}_i &= \ddot{x}_{i+1} - \ddot{x}_i \\ \Delta\dot{x}_i &= \dot{x}_{i+1} - \dot{x}_i \\ \Delta x_i &= x_{i+1} - x_i \\ \Delta t_i &= t_{i+1} - t_i \\ \Delta\ddot{x}_g^{\,i} &= \ddot{x}_g^{\,i+1} - \ddot{x}_g^{\,i} \end{aligned}\right\} \tag{7.2}$$

Assuming the linear variation of acceleration over a small time interval, $\Delta t_i$ the incremental acceleration and velocity (refer Section 3.3.1.1.1 Newmark's Beta Method) are expressed as

$$\Delta\ddot{x}_i = \frac{6}{\Delta t_i^2}\Delta x_i - \frac{6}{\Delta t_i}\dot{x}_i - 3\ddot{x}_i \tag{7.3}$$

$$\Delta\dot{x}_i = \frac{3}{\Delta t_i}\Delta x_i - 3\dot{x}_i - \frac{\Delta t_i}{2}\ddot{x}_i \tag{7.4}$$

Substituting $\Delta\ddot{x}_i$ and $\Delta\dot{x}_i$ in equation (7.1) and solving for the $\Delta x_i$ will give

$$\Delta x_i = \frac{p_{eff}}{k_{eff}} \tag{7.5}$$

where $p_{eff}$ and $k_{eff}$ are the incremental force and incremental stiffness during the $i^{th}$ time step respectively expressed as.

$$p_{eff} = -m\Delta\ddot{x}_g^{\,i} + \left(\frac{6}{\Delta t_i}m + 2c_t\right)\dot{x}_i + \left(2m + \frac{\Delta t_i}{2}c_t\right)\ddot{x}_i \tag{7.6}$$

$$k_{eff} = \frac{6}{\Delta t_i^{\,2}}m + \frac{3}{\Delta t_i}c_t + k_t \tag{7.7}$$

Knowing the $\Delta x_i$, determine $\Delta\dot{x}_i$ from equation (7.4). At time, $t = t_{i+1}$, the displacement and velocity can be determined as

$$\left.\begin{aligned} x_{i+1} &= x_i + \Delta x_i \\ \dot{x}_{i+1} &= \dot{x}_i + \Delta\dot{x}_i \end{aligned}\right\} \tag{7.8}$$

The acceleration at time, $t = t_{i+1}$ is calculated by considering equilibrium of the system (refer Figure 7.1) to avoid the accumulation of the unbalanced forces i.e.

$$\ddot{x}_{i+1} = \frac{1}{m}\left[-m\ddot{x}_g^{i+1} - \mathrm{F}_d^{i+1} - F_s^{i+1}\right] \tag{7.9}$$

where $\mathrm{F}_d^{i+1}$ and $F_s^{i+1}$ denote the damping and stiffness/restoring force at time, $t_{i+1}$, respectively.

While obtaining the above solution, it is assumed that the damping and restoring force non-linearities follow the specified path during loading and unloading condition.

### 7.3.1 Elasto-plastic Material Behavior

For the elasto-plastic material behavior, $c_t$ is assumed to be constant. The stiffness matrix $k_t$ is taken either as k or zero depending upon whether the system is elastically loaded and unloaded or plastically deformed. When the system enters from elastic to plastic state or vice-versa, the stiffness varies within the time step. Due to variation of stiffness with respect to time, the system no more remains in equilibrium.

If the system is elastic at the beginning of the time step and remains elastic at the end of the time step, then the computation is not changed i.e.

$$\left|F_s^{i+1}\right| < Q \tag{7.10}$$

and the computations for the next time step start. In equation (7.10), $Q$ is the yield force.

The system enters into plastic state from elastic state as soon as

$$\left|F_s^{i+1}\right| = Q \tag{7.11}$$

Normally, it is not possible to have the scenario that at time, $t_{i+1}$, the spring force is just equal to the yield force. However, this can be archived by reducing the time interval of the computation.

Once the system enters into the plastic state, it continues to remain in that state until the incremental displacement and the stiffness force are in the same direction. Thus, the plastic state exists until

$$F_s^{i+1} \times \Delta x_i > 0 \tag{7.12}$$

When the velocity at the end of the time interval changes the sign then the unloading takes place. Thus, the system changes from plastic to elastic state when

$$F_s^{i+1} \times \Delta x_i < 0 \tag{7.13}$$

The system remains in the elastic state until equation (7.10) is satisfied otherwise, it will change from elastic to plastic state.

### *Example 7.1*

Consider an elasto-plastic SDOF system having mass = 1 kg, elastic stiffness = 39.478 N/m and damping constant = 0.251 N.sec/m. Determine the displacement response of the system under the El-Centro, 1940 motion for (i) yield displacement = 0.05m and (ii) yield displacement = 0.025m.

**Solution:**

Based on the method developed in the Section 7.2, a computer program is written in the FORTAN language and the response of the SDOF system with the above parameters and elasto-plastic behavior under El-Centro, 1940 earthquake motion is obtained. The time period of the system based on the elastic stiffness is 1 sec and the damping ratio is 0.02. The time variation of displacement and spring force of the system is plotted in Figures 7.2 and 7.3 for the yield displacement, $q$ = 0.05m and 0.025m, respectively. The salient values of the maximum response of the system are summarized as below:

| Response quantity | $q$ = 0.05m | $q$ = 0.025m |
|---|---|---|
| Maximum displacement (m) | 0.099 | 0.113 |
| Maximum Stiffness force (N) | 1.974 | 0.987 |
| Time of change of first elastic to plastic state | 1.92 sec | 1.84 sec |
| Time of change of first plastic to elastic state | 1.98 sec | 2.02 sec |
| Maximum elastic deflection (refer Example 3.5) | 0.1516 m | |

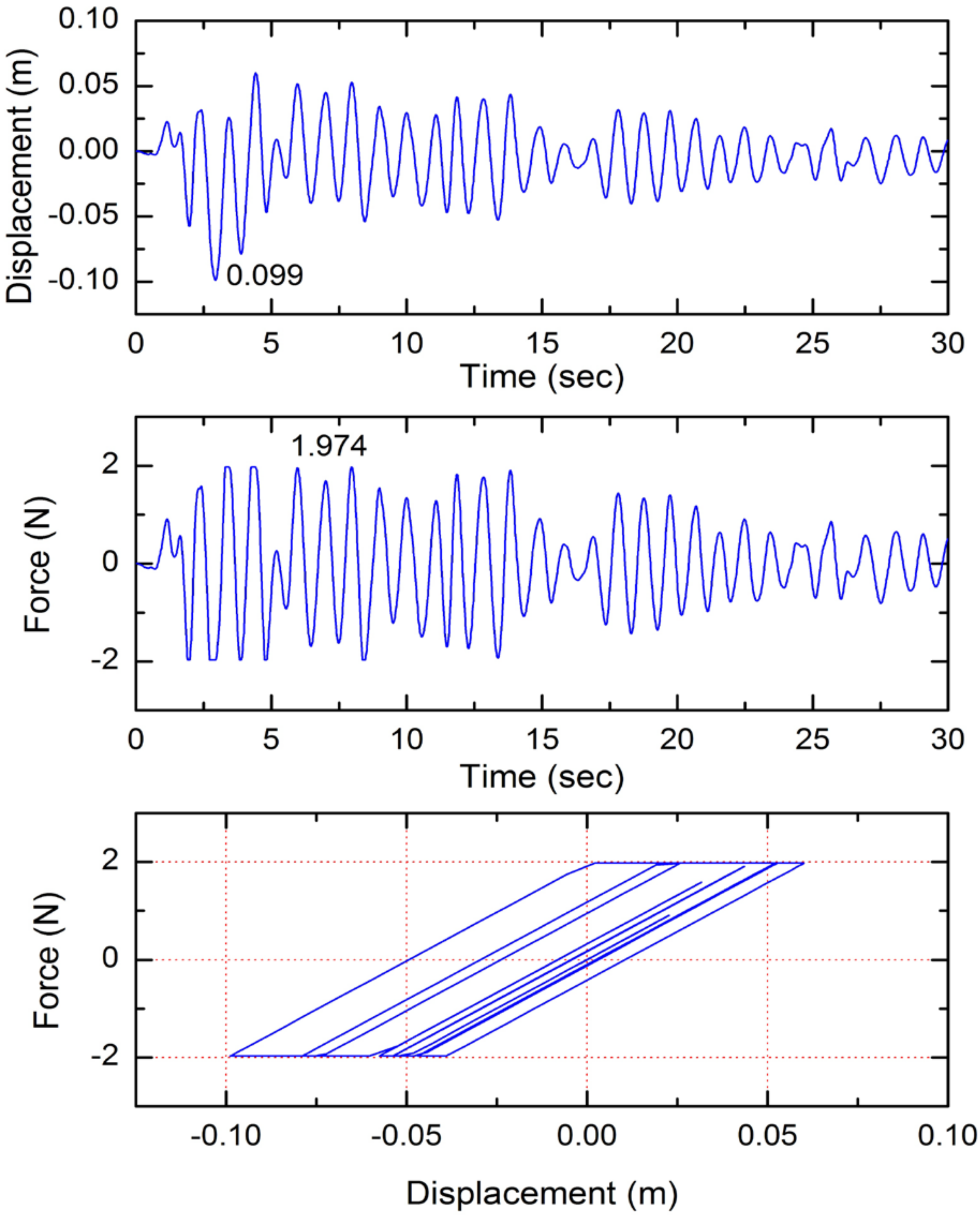



Figure 7.3 Response of elasto-plastic SDOF system with yield displacement of 0.05m.

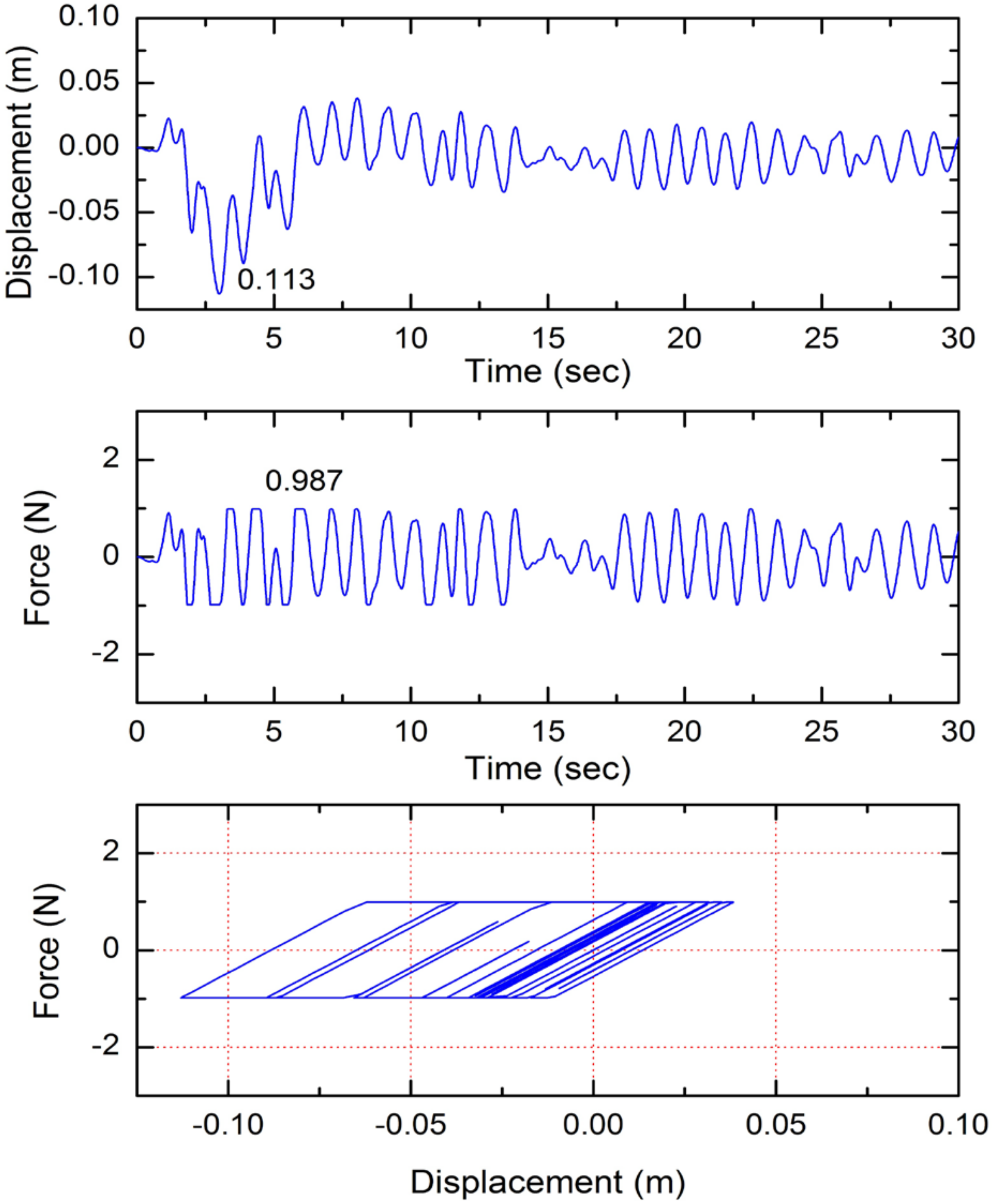



Figure 7.4 Response of elasto-plastic SDOF system with yield displacement of 0.025m.

## 7.4 Non-linear Force-Deformation Behaviour using Wen's Equation

Wen (1976) proposed the equation for modeling non-linear hysteretic force-deformation behavior, in which the force, $F_s$ is given by

$$F_s = \alpha k_0 x + (1-\alpha) Q Z \tag{7.14}$$

where $k_0$ is the initial stiffness; $\alpha$ is an index, which represents the ratio of post to pre-yielding stiffness; $x$ is the relative displacement; $Q$ is the yield strength; and $Z$ is a non-dimensional hysteretic component satisfying the following non-linear first order differential equation expressed as

$$q \frac{dZ}{dt} = \beta |\dot{x}| Z |Z|^{n-1} - \tau \dot{x} |Z|^n + A\dot{x} = g(\dot{x}, Z) \tag{7.15}$$

where $\beta$, $\tau$, $A$ and $n$ are the dimensionless parameters which control the shape of the hysteresis loop; $q$ is the yield displacement; and $\dot{x}$ is the relative velocity.

The parameter $n$ is an integer constant, which controls the smoothness of transition from elastic to plastic state. Various parameters of the Wen's equation are selected in such a way that predicted response from the model closely matches with the experimental results. In order to solve, the equation of motion of a system in the incremental form using the Newmark's step-by-step method will require the incremental force (refer equation (7.1)) which is given by

$$\Delta F_s = \alpha k_0 \Delta x + (1-\alpha) Q \Delta Z \tag{7.16}$$

The above equation involves the incremental displacement component, $\Delta Z$ which can be obtained by solving the differential equation (7.15) using the fourth order Runge-Kutta method by

$$\Delta Z = \frac{K_0 + 2K_1 + 2K_2 + K_3}{6} \tag{7.17}$$

$$K_0 = \Delta t \; g(\dot{x}, Z^t) / q \tag{7.18}$$

$$K_1 = \Delta t \; g(\dot{x}^{t+\Delta t/2}, Z^t + K_0 / 2) / q \tag{7.19}$$

$$K_2 = \Delta t \; g(\dot{x}^{t+\Delta t/2}, Z^t + K_1 / 2) / q \tag{7.20}$$

$$K_3 = \Delta t \; g(\dot{x}^{t+\Delta t}, Z^t + K_2) / q \tag{7.21}$$

It is to be noted that the above solution requires the velocity of the system at time, $t+\Delta t/2$ and $t+\Delta t$ which are not known initially. To start with, it is assumed the same velocity at time, $t$ and $t+\Delta t$ and the value of the incremental hysteretic displacement component is obtained to find the velocity at time, $t+\Delta t$. This is to be iterated until the following convergence criterion is satisfied for incremental hysteretic displacement component i.e.

$$\frac{\left|(\delta Z)^{j+1}\right| - \left|(\delta Z)^{j}\right|}{\left|(\delta Z)^{j}\right|} \le \varepsilon \tag{7.22}$$

where $\varepsilon$ is a small threshold parameter. The superscript to the $\Delta Z$ denotes the iteration number.

The typical force-deformation hysteresis loops generated using the equations (7.14) and (7.15) are shown in the Figure 7.5 under a sinusoidal motion (amplitude of 7.5 cm and frequency of unit Hz). The other parameters of the model considered are $q$ = 2.5 cm, $\beta = \tau$ = 0.5 and $A$ = 1. Thus, by changing the different parameters of the Wen's model one can achieve the desired hysteretic behavior such as elasto-plastic and bi-linear type. The comparison of the loop for two values of the $n$ indicates that for $n$=1, there is smooth transition from elastic to plastic state and vice versa where as for $n$=15 the change in the states takes place immediately.

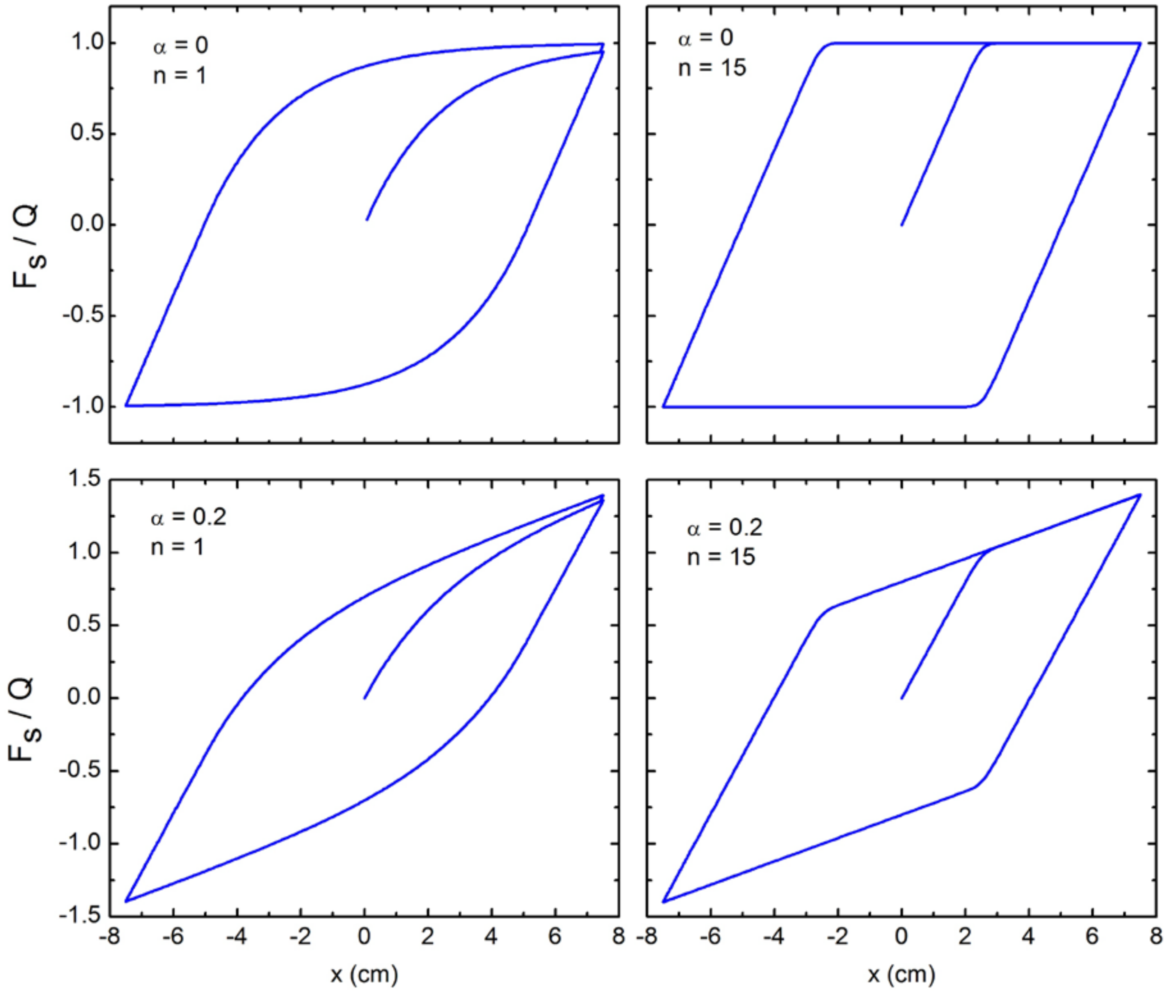



Figure 7.5 Different hysteresis loops from the Wen's equation.

### *Example 7.2*

Consider an elasto-plastic SDOF system having mass = 1kg, elastic stiffness = 39.478 N/m and damping constant = 0.251 N.sec/m. The yield displacement = 0.05m. Determine the displacement response of the system under the El-Centro, 1940 motion using the Wen's equation.

**Solution:**

Based on the given input, the values of the various parameters of the Wen's equation will be

$k_0$ = 39.478 N/m

$q$ = 0.05m

$Q = 39.478 \times 0.05 = 1.974$ N

$\beta = \tau = 0.5$

$A = 1$

$n = 15$

Based on the method developed in the Section 7.3, a computer program is written in the FORTAN language and the response of the SDOF system with above parameters subjected to the El-Centro, 1940 earthquake motion is obtained. The time variation of displacement and spring force response is plotted in Figure 7.6. As expected, the response of the system is similar to that shown in the Figure 7.3 using the conventional approach. A comparison of the displacement response of the elasto-plastic system with two approaches is shown in the Figure 7.7. The maximum response is the same for the two methods; however, there is difference in the permanent drift of the system by two approaches.

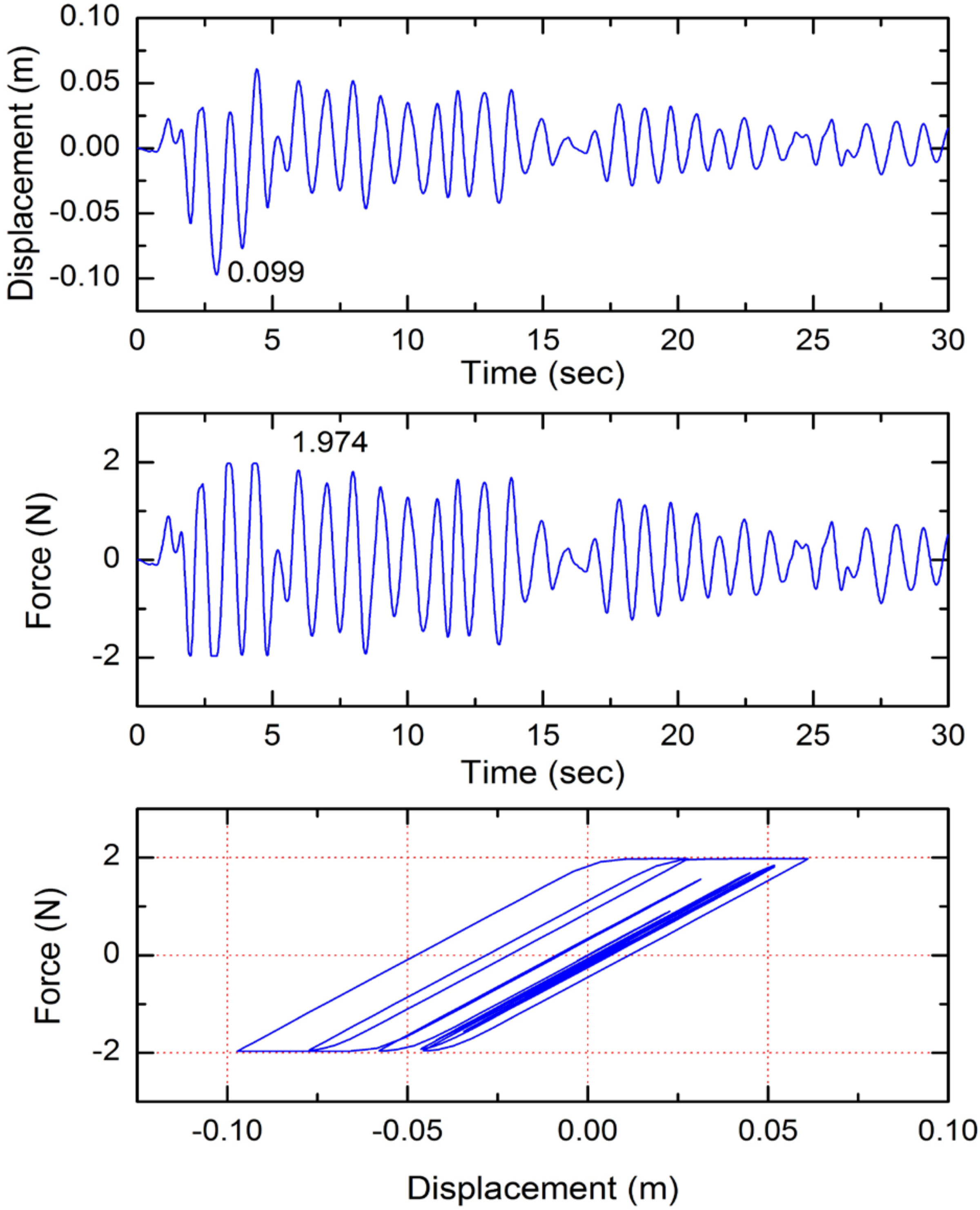



Figure 7.6 Response of the elasto-plastic SDOF system of Example 7.2.

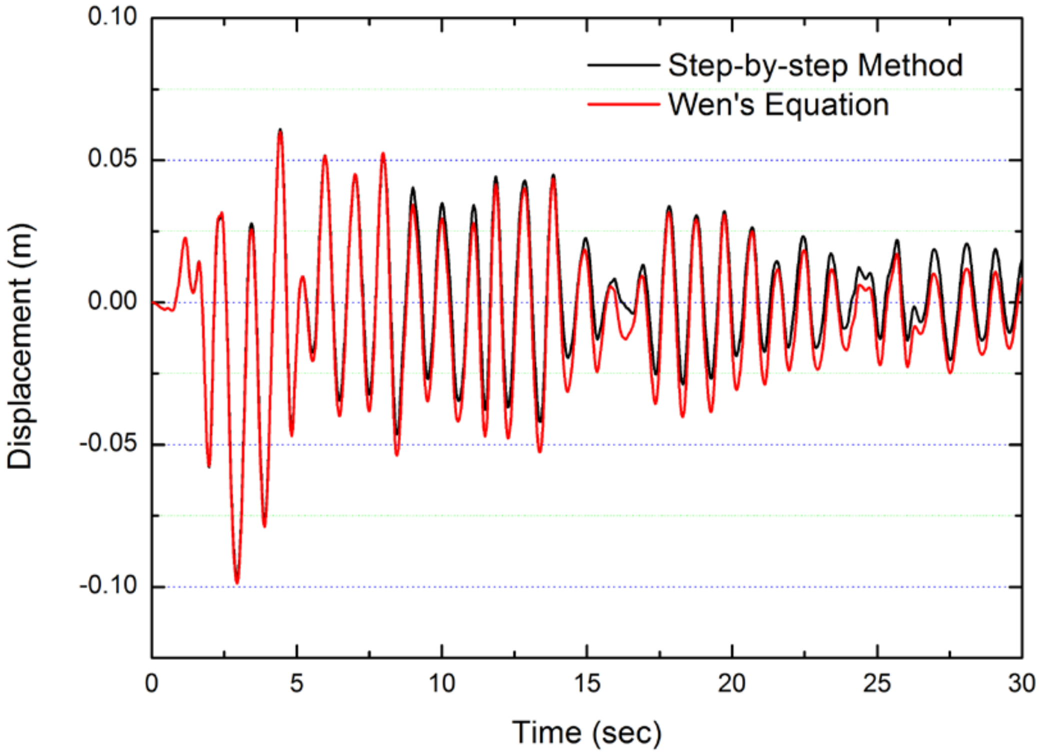



Figure 7.7 Comparison of displacement response of the system by two approaches.

## 7.5 Non-Linear Analysis of Multi-Storey Building Frames

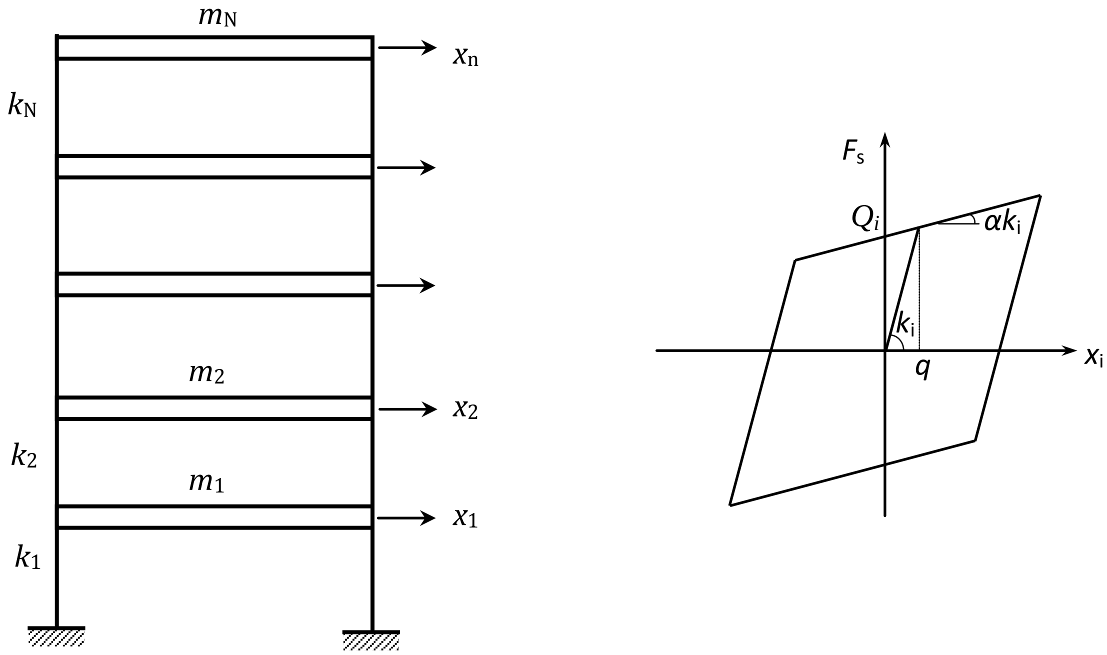



Figure 7.8 Idealized multi-storey building frame with non-linear behavior of columns.

The multi-storey building frames can be idealized as 2D frames as shown in the Figure 7.8. The equations of motion for a MDOF system with bi-linear stiffness are a set of coupled non-linear differential equations. The governing equations of motion are expressed as

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k_t]\{x\} = -[m]\{r\}\,\ddot{x}_g \qquad (7.23)$$

where, $[m]$ = mass matrix ($n \times n$); $[k_t]$ = time dependent stiffness matix ($n \times n$); $[c]$ = damping matrix ($n \times n$); $\{r\}$ = influence coefficient vector ($n$×1); $\{x(t)\}$ = relative displacement vector; $\{\dot{x}(t)\}$ = relative velocity vector, $\{\ddot{x}(t)\}$ = relative acceleration vector, and $\ddot{x}_g(t)$ = earthquake ground acceleration.

The solution can be obtained by using Newmark's Beta iterative technique or any other numerical procedure, by solving the incremental equations of motion. Over the small time interval Δt, the response of the structure is assumed to be linear. The response of the structure in the next time step is obtained from the response in the earlier time step. The incremental equations of motion will be of the form

$$[m]\{\Delta\ddot{x}\} + [c]\{\Delta\dot{x}\} + [k_t]\{\Delta x\} = -[m]\{r\}\,\Delta\ddot{x}_g \qquad (7.24)$$

The solution of the equation of motion for the MDOF system is obtained using the numerical integration technique. The incremental quantities in the equation (7.24) are the change in the responses from time $t_i$ to $t_{i+1}$ given by

$$\left.\begin{array}{rcl} \{\Delta\ddot{x}_i\} & = & \{\ddot{x}_{i+1}\} - \{\ddot{x}_i\} \\ \{\Delta\dot{x}_i\} & = & \{\dot{x}_{i+1}\} - \{\dot{x}_i\} \\ \{\Delta x_i\} & = & \{x_{i+1}\} - \{x_i\} \\ \Delta t_i & = & t_{i+1} - t_i \\ \{\Delta\ddot{x}_g^{\,i}\} & = & \{\ddot{x}_g^{\,i+1}\} - \{\ddot{x}_g^{\,i}\} \end{array}\right\} \tag{7.25}$$

Assuming the linear variation of acceleration over a small time interval, $\Delta t_i$ the incremental acceleration and velocity (refer Section 3.3.1.1.1 Newmark's Beta Method**)** are expressed as

$$\{\Delta\ddot{x}_i\} = \frac{6}{\Delta t_i^2}\{\Delta x_i\} - \frac{6}{\Delta t_i}\{\dot{x}_i\} - 3\{\ddot{x}_i\} \tag{7.26}$$

$$\{\Delta\dot{x}_i\} = \frac{3}{\Delta t_i}\{\Delta x_i\} - 3\{\dot{x}_i\} - \frac{\Delta t_i}{2}\{\ddot{x}_i\} \tag{7.27}$$

Substituting $\{\Delta\ddot{x}_i\}$ and $\{\Delta\dot{x}_i\}$ in equation (7.19) and solving for the $\Delta x_i$ will give

$$\{\Delta x_i\} = [k_{eff}]^{-1}\{p_{eff}\} \tag{7.28}$$

where,

$$\{p_{eff}\} = -[m]\{r\}\Delta\ddot{x}_g^{\,i} + \left(\frac{6}{\Delta t_i}[m] + 2[c]\right)\{\dot{x}_i\} + \left(2[m] + \frac{\Delta t_i}{2}[c]\right)\{\ddot{x}_i\} \tag{7.29}$$

$$[k_{eff}] = \frac{6}{\Delta t_i^{\,2}}[m] + \frac{3}{\Delta t_i}[c] + k_t[] \tag{7.30}$$

Knowing the $\{\Delta x_i\}$, determine $\{\Delta\dot{x}_i\}$ from equation (7.4). At $t = t_{i+1}$, the displacement and velocity can be determined as

$$\left.\begin{array}{l} \{x_{i+1}\} = \{x_i\} + \{\Delta x_i\} \\ \{\dot{x}_{i+1}\} = \{\dot{x}_i\} + \{\Delta\dot{x}_i\} \end{array}\right\} \tag{7.31}$$

The acceleration at time, $t = t_{i+1}$ is calculated by considering equilibrium of the system (refer Figure 7.8) to avoid the accumulation of the unbalanced forces i.e.

$$\{\ddot{x}_{i+1}\} = [m]^{-1}\left[ -[m]\{r\}\{\ddot{x}_g^{i+1}\} - [\mathrm{c}]\{\dot{x}^{i+1}\} - \{F_s^{i+1}\}\right] \tag{7.32}$$

where $\{F_s^{i+1}\}$ denotes the stiffness/restoring force at time, $t_{i+1}$.

### *Example 7.3*

A two-story building is modeled as 2-DOF system and rigid floors as shown in the Figure 7.9. Determine the floor displacement responses due to El-Centro, 1940 earthquake ground motion. Take the inter-story stiffness, $k$ =197.392 × $10^3$ N/m and the floor mass, $m$ = 2500 kg. The columns of the building are having elasto-plastic behavior with yield displacement of 0.05m.

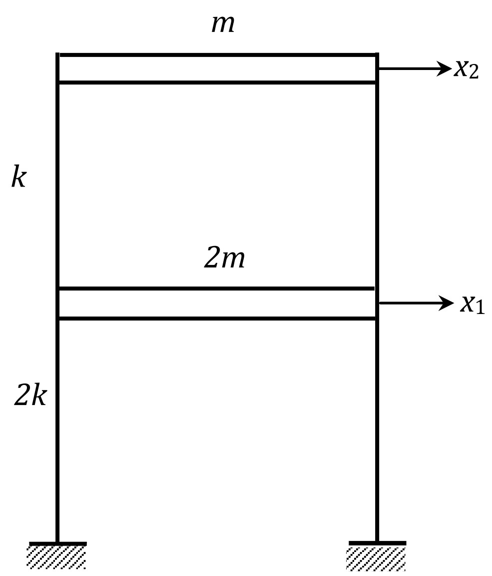



Figure 7.9

**Solution:**

Based on the method developed in the Section 7.5, a computer program is written in the FORTAN language and the response of the system with the above parameters under El-Centro, 1940 earthquake motion is obtained. The time variation of displacement response of the frame is plotted in Figures 7.10 and 7.11. The peak displacement of the top and bottom floor is observed as 0.1483 m and 0.068 m, respectively. The corresponding elasto-plastic force-deformation loops of the two floors are also plotted in the above figures.

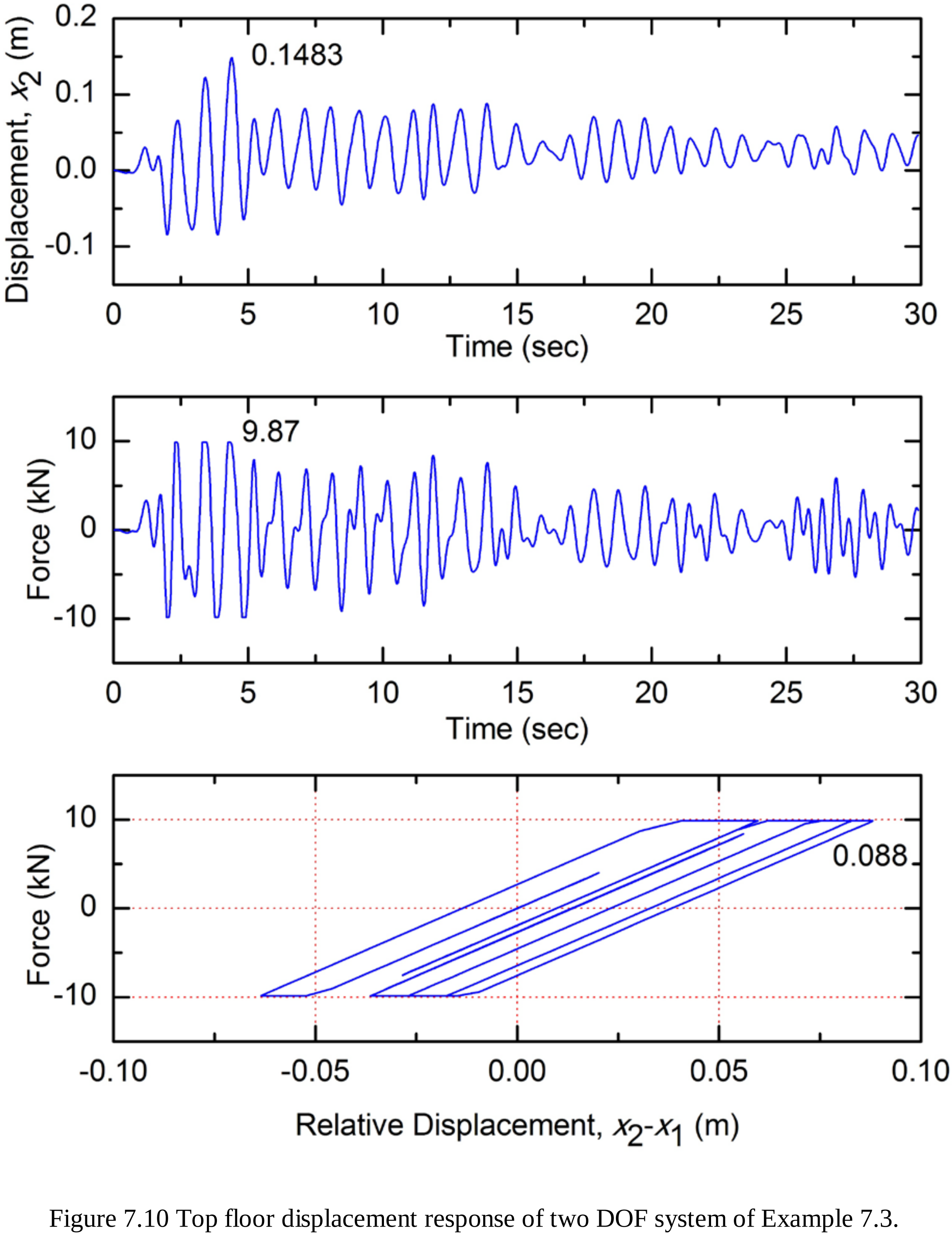



Figure 7.10 Top floor displacement response of two DOF system of Example 7.3.

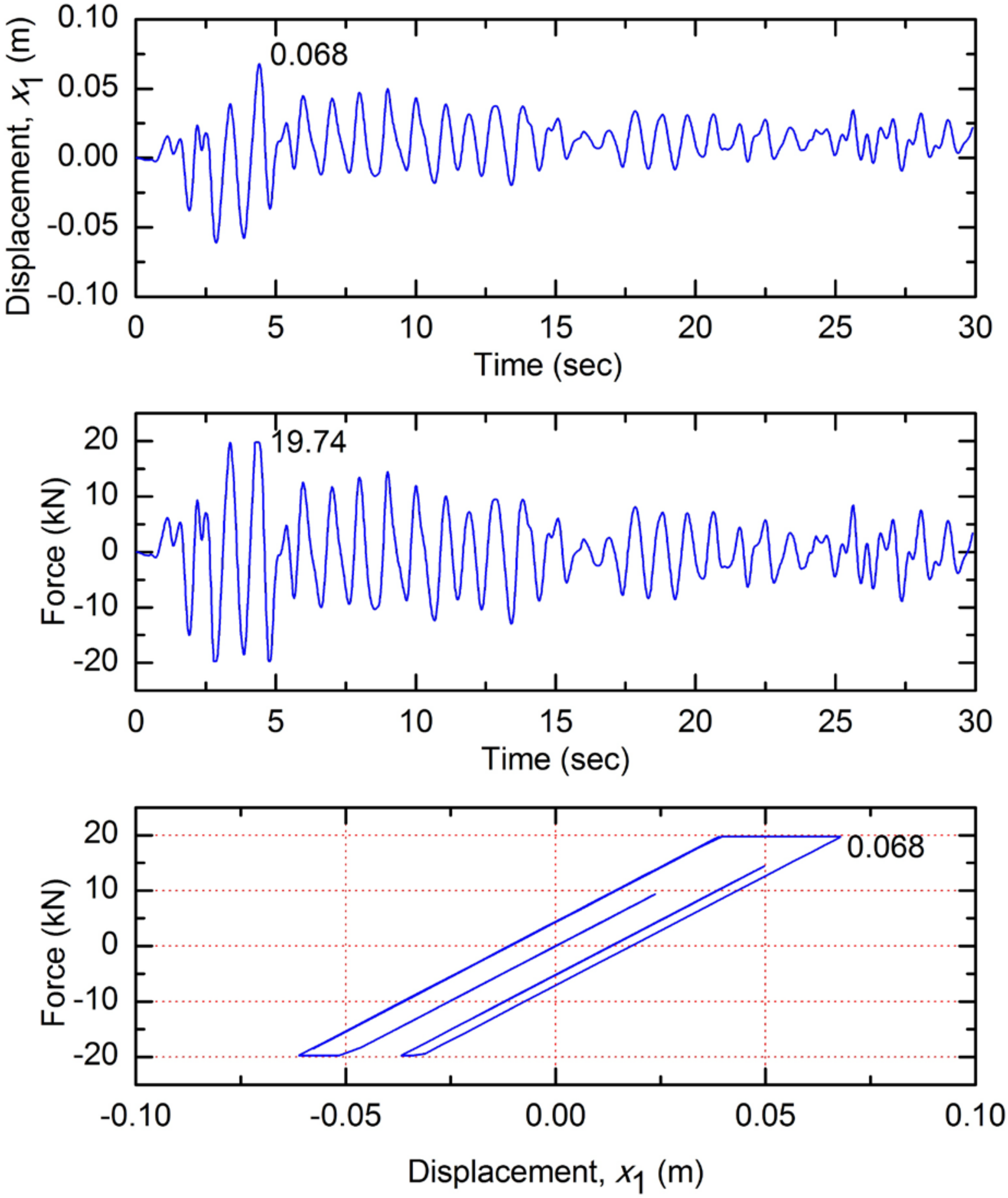



Figure 7.11 Bottom floor displacement response of two DOF system of Example 7.3.

## 7.6 Pushover Analysis

Amongst the natural hazards, earthquakes have the potential for causing the greatest damages. Since earthquake forces are random in nature & unpredictable, the engineering tools need to be sharpened for analyzing structures under the action of these forces. Earthquake loads are to be carefully modeled so as to assess the real behavior of structure with a clear understanding that damage is expected but it should be regulated. In this context pushover analysis which is an iterative procedure is looked upon as an alternative for the conventional analysis procedures. Pushover analysis of multi-story RCC framed buildings subjected to increasing lateral forces is carried out until the preset performance level (target displacement) is reached. The promise of performance-based seismic engineering (PBSE) is to produce structures with predictable seismic performance.

The recent advent of performance based design has brought the non linear static push over analysis procedure to the forefront. Pushover analysis is a static non linear procedure in which the magnitude of the structural loading along the lateral direction of the structure is incrementally increased in accordance with a certain pre-defined pattern. It is generally assumed that the behavior of the structure is controlled by its fundamental mode and the predefined pattern is expressed either in terms of story shear or in terms of fundamental mode shape.

With the increase in magnitude of lateral loading, the progressive non-linear behavior of various structural elements is captured, and weak links and failure modes of the structure are identified. In addition, pushover analysis is also used to ascertain the capability of a structure to withstand a certain level of input motion defined in terms of a response spectrum. Recently, modifications to push over procedures have also been proposed so as to capture contribution of higher modes of vibration of structure, change in distribution of story shear subsequent to yielding of structural members, etc. Push over procedure is gaining popularity during the last few years as appropriate analytical tools are now available (SAP-2000, ETABS).

Pushover analysis is of two types, (i) force controlled or (ii) displacement controlled. In the force control, the total lateral force is applied to the structure in small increments. In the displacement control, the displacement of the top storey of the structure is incremented step by step, such that the required horizontal force pushes the structure laterally. The distance through which the structure is pushed, is proportional to the fundamental horizontal translational mode of the structure. In both types of pushover analysis, for each increment of the load or displacement, the stiffness matrix of the structure may have to be changed, once the structure passes from the elastic state to the inelastic state. The displacement controlled pushover analysis is generally preferred over the force controlled one because the analysis could be carried out up to the desired level of the displacement (refer Figure 7.12).

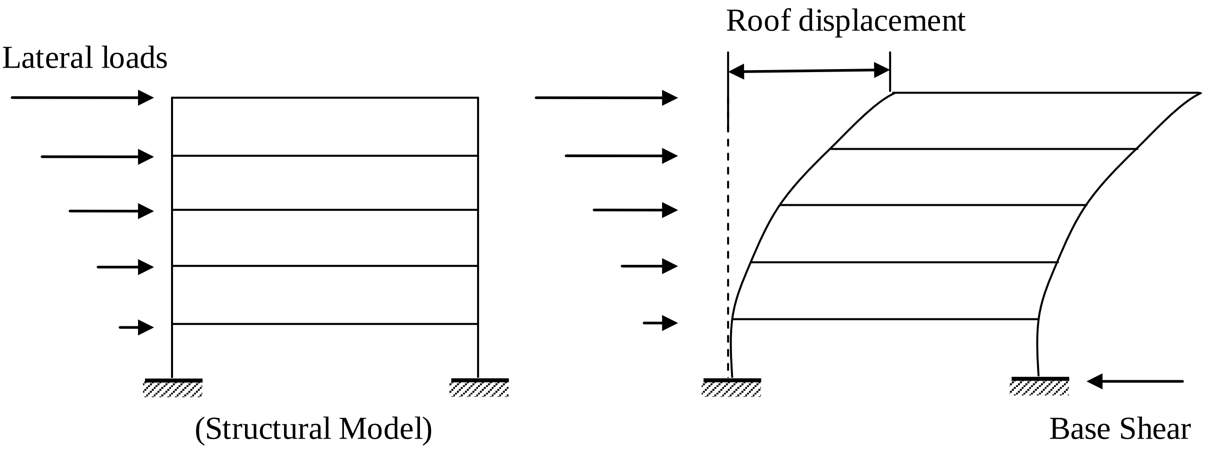



Figure 7.12 Static Approximations in the Pushover Analysis.

In Pushover analysis, a static horizontal force profile, usually proportional to the design force profiles specified in the codes, is applied to the structure. The force profile is then incremented in small steps and the structure is analyzed at each step. As the loads are increased, the building undergoes yielding at a few locations. Every time such yielding takes place, the structural properties are modified approximately to reflect the yielding. The analysis is continued till the structure collapses, or the building reaches certain level of lateral displacement. It provides a load versus deflection curve of the structure starting from the state of rest to the ultimate failure of the structure. The load is representative of the equivalent static load of the fundamental mode of the structure. It is generally taken as the total base shear of the structure and the deflection is selected as the top-storey deflection. The selection of appropriate lateral load distribution is an important step. The first step then is to select a displacement shape and the vector of lateral loads is determined as

$$\{F\} = p[m]\{\Phi\} \tag{7.33}$$

where $\{\Phi\}$ is the assumed displacement shape, and $p$ is the magnitude of the lateral loads. From equation (7.33), it follows that the lateral force at any level is proportional to the assumed displacement shape and story mass. If the assumed displacement shape was exact and remained constant during ground shaking, then distribution of lateral forces would be equal to distribution of effective earthquake forces.

For pushover analysis of any structure, the input required is the assumed collapse mechanism, moment–rotation relationship for the sections that are assumed to yield, the fundamental mode shape, the limiting displacement, and the rotational capacity of the plastic hinges. In addition to data needed for usual elastic analysis, the non-linear force deformation relationship for structural elements under monotonic loading is also required. The most commonly used element is beam element modeled as line element. Seismic demand is traditionally defined in the form of an elastic acceleration spectrum $S_{ae}$, in which spectral accelerations are given as a function of the natural period of structure, *T*.

The structure is modeled as a SDOF system. The displacement shape is assumed to be constant. This is the basic and most critical assumption. The starting point is the equation of motion of planar MDOF model that explicitly includes only lateral translation degrees of freedom.

$$[m]\{\ddot{u}\} + \{\mathrm{R}\} = [m]\{1\}\ddot{x}_g \tag{7.34}$$

where {u} and {R} are the vectors representing displacements and internal forces, {1} is a unit vector, and $\ddot{x}_g$ is ground acceleration as a function of time. The displacement vector, {u} is defined as

$$\{u\} = \{\Phi\}D_t \tag{7.35}$$

where $D_t$ is the time dependent top displacement.

For equilibrium, the internal forces, $\{R\}$ are equal to statically applied external loads $\{F\}$. The equation of motion of equivalent SDOF is written as

$$m^* D''^* + F^* = -m^* \ddot{x}_g \tag{7.36}$$

where $m^*$ is equivalent mass of the SDOF system, $D^*$ and $F^*$ are the displacement and force of the equivalent SDOF system, respectively.

For simplification the force-displacement relationship is assumed to be elastic perfectly plastic for equivalent SDOF as shown in the Figure 7.13.

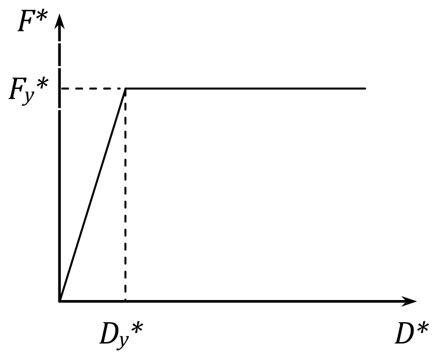



Figure 7.13 Approximate elasto-plastic force-displacement relationships.

Determine the strength, $F_y^*$, yield displacement, $D_y^*$ and period $T^*$. The $T^*$ is given by

$$T^* = 2\pi\sqrt{\frac{m^* D_y^*}{F_y^*}} \tag{7.37}$$

From the acceleration spectrum, the inelastic spectrum in acceleration-displacement format is determined. The capacity diagram in acceleration displacement (AD) format is obtained by dividing the forces in force deformation diagram by $m^*$.

$$S_a = \frac{F^*}{m^*} \tag{7.38}$$

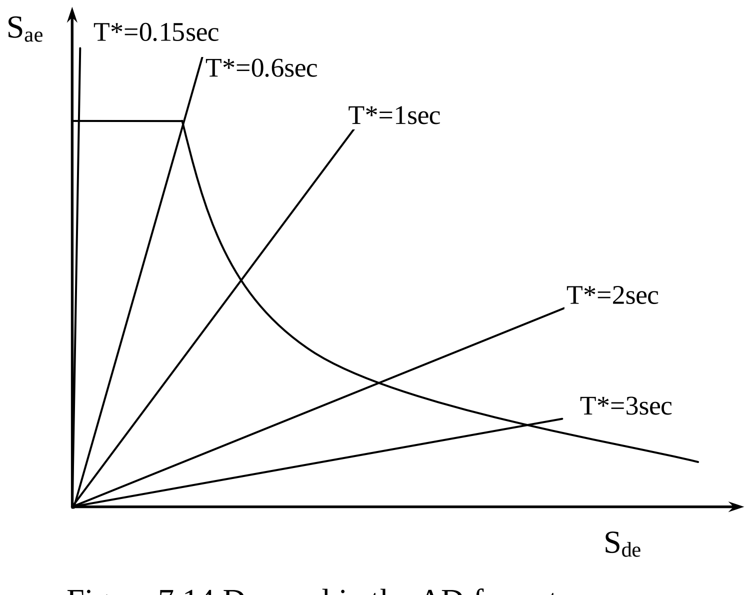



Figure 7.14 Demand in the AD format.

The displacement demand for the SDOF model $S_d$ is transformed into the maximum top displacement $D_t$ of the MDOF system. The local seismic response (e.g. story drifts, joint rotations) can be determined by pushover analysis. Under increasing lateral loads with a fixed pattern the structure is pushed to a target displacement $D_t$. Consequently it is appropriate the likely performance of building under push load up to target displacement.  The expected performance can be assessed by comparing seismic demands with the capacities for the relevant performance level. Global performance can be visualized by comparing displacement capacity and demand.

The seismic performance of a building can be evaluated in terms of pushover curve, performance point, displacement ductility, plastic hinge formation etc. The base shear vs. roof displacement curve (Figure 7.15) is obtained from the pushover analysis from which the maximum base shear capacity of structure can be obtained. This capacity curve is transformed into capacity spectrum by SAP as per ATC40 and demand or response spectrum is also determined for the structure for the required building performance level. The intersection of demand and capacity spectrum gives the performance point of the structure analyzed. This is illustrated in the Figure 7.14.

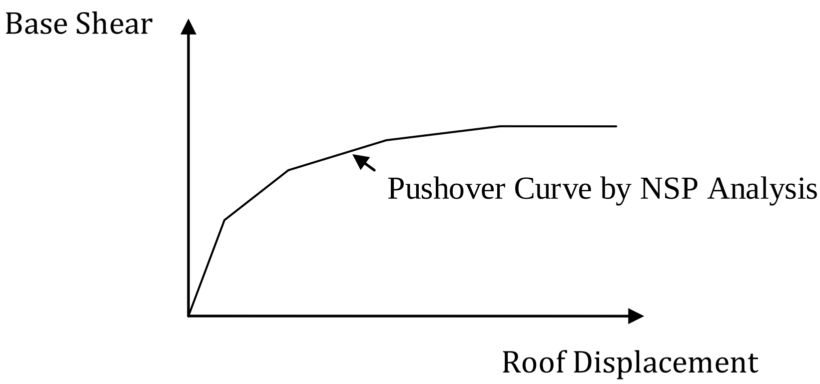



Figure 7.15 Base shear vs roof displacement.

At the performance point, the resulting responses of the building should then be checked using certain acceptability criteria. The Performance Point thus obtained from pushover analysis is then compared with the calculated target displacement.

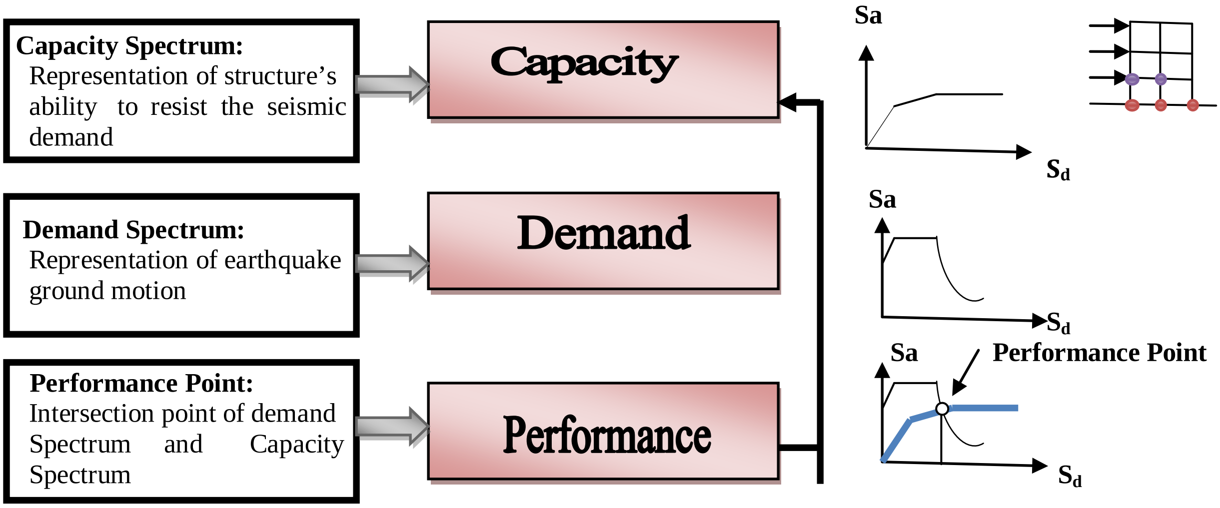



Figure 7.16 Determination of Performance Point.

There are three procedures described in ATC-40 to find the performance point.

***Procedure A***, which uses a set of equations described in ATC-40.

***Procedure B*** is also an iterative method to find the performance point, which uses the assumption that the yield point and the post yield slope of the bilinear representation, remains constant. This is adequate for most cases; however, in some cases this assumption may not be valid.

***Procedure C*** is graphical method that is convenient for hand as well as software analysis. SAP2000 uses this method for the determination of performance point. To find the performance point using Procedure C the following steps are used:

First of all, the single demand spectrum (variable damping) curve is constructed by doing the following for each point on the Pushover Curve:

1) Draw a radial line through a point (**P)** on the Pushover curve. This is a line of constant period.

2) Calculate the damping associated with the point (**P)** on the curve, based on the area under the curve up to that point.

3) Construct the demand spectrum, plotting it for the same damping level as associated with the point 'P' on the pushover curve.

4) The intersection point (P') for the radial line and associated demand spectrum represents a point on the Single Demand Spectrum (Variable Damping Curve).

A number of arbitrary points are taken on the Pushover curve. A curve is then drawn by joining through these points. The intersection of this curve with the original pushover curve gives the performance point of the structure as shown in Figure 7.16.

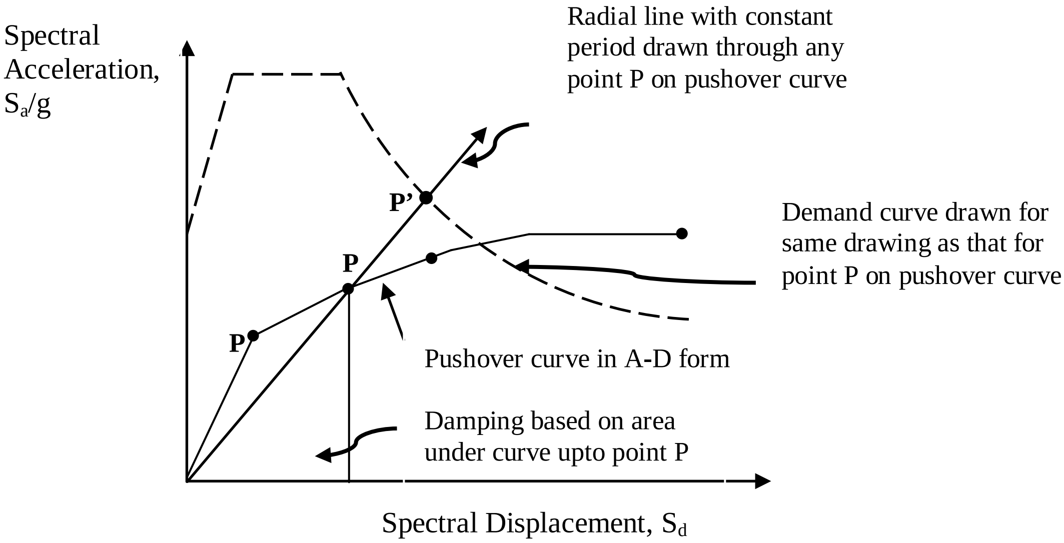



Figure 7.17 Capacity Spectrum Procedure 'C' to Determine Performance Point

It has been recognized that the inter-story drift performance of a multistory building is an important measure of structural and non-structural damage of the building under various levels of earthquake motion. In performance based design, inter-story drift performance has become a principal design consideration. The system performance levels of a multistory building are evaluated on the basis of the inter-story drift values along the height of the building under different levels of earthquake motion. Inter-storey drift is defined as the ratio of relative horizontal displacement of two adjacent floors (δ) and corresponding storey height (h).

$$\text{Inter-story Drift} = \frac{\delta}{h} = \frac{\delta_i - \delta_{i-1}}{h} \tag{7.39}$$

The sequence of plastic hinge formation and state of hinge at various levels of building performance can be obtained from SAP output. This gives the information about the weakest member and so the one which is to be strengthened in case of a building need to be retrofitted. Accordingly the detailing of the member can be done in order to achieve the desired pattern of failure of members in case of severe earthquakes. It is concluded that pushover analysis is a successful method in determination of the sequence of yielding of the components of a building, possible mode of failure, and final state of the building after a predetermined level of lateral load is sustained by the structure.

Following assumptions are made while analyzing a structure in the SAP: (i) The material is homogeneous, isotropic, (ii) All columns supports are considered as fixed at the foundation, (iii) Tensile strength of concrete is ignored in sections subjected to bending, (iv) The super structure is analyzed independently from foundation and soil medium, on the assumptions that foundations are fixed, (v) Pushover hinges are assigned to all the member ends. In case of Columns PMM hinges (i.e. Axial Force and Biaxial Moment Hinge) are provided while in case of beams M3 hinges (i.e. Bending Moment hinge) are provided, (vi) the maximum target displacement of the structure is calculated in accordance with the guidelines given by FEMA 356 for maximum roof level lateral drift. Performance of building has been classified into 5 levels, viz. (i) Operational (OP), (ii) Immediate Occupancy (IO), (iii) Damage Control (DC), (iv) Life Safety (LS) and (v) Collapse Prevention (CP).

Structural Performance Level S-1, Immediate Occupancy, means the post earthquake damage state in which only very limited structural damage has occurred. The basic vertical- and lateral-force-resisting systems of the building retain nearly all of their pre-earthquake strength and stiffness. In the primary concrete frames, there will be hairline cracking. There may be a few locations where the rebar will yield, but the crushing of concrete is not expected. The transient drift will be about 1%, with negligible permanent drift. In the brick infill walls, there will be minor cracking and minor spalling of plaster. The risk of life-threatening injury as a result of structural damage is very low, and although some minor structural repairs may be appropriate, these would generally not be required prior to re-occupancy.

Damage Control Performance Range (S-2) means the continuous range of damage states that entail less damage than that defined for the Life Safety level, but more than that defined for the Immediate Occupancy level. Design for Damage Control performance may be desirable to minimize repair time and operation interruption; as a partial means of protecting valuable equipment and contents; or to preserve important historic features when the cost of design for Immediate Occupancy is excessive. Acceptance criteria for this range may be obtained by interpolating between the values provided for the Immediate Occupancy (S-1) and Life Safety (S-3) levels.

Life Safety Performance Level (S-3) means the post-earthquake damage state in which significant damage to the structure has occurred, but some margin against either partial or total structural collapse remains. Some structural elements and components are severely damaged, but this has not resulted in large falling debris hazards, either within or outside the building. In the primary concrete frames, there will be extensive damage in the beams. There will be spalling of concrete cover and shear cracking in the ductile columns. The transient drift will be around 2%, with 1% being permanent. In the brick infill walls, there will be extensive cracking and some crushing. But the walls are expected to remain in place. The transient drift will be about 0.5%, with 0.3% being permanent. Injuries may occur during the earthquake; however, it is expected that the overall risk of life threatening injury as a result of structural damage is low. It should be possible to repair the structure; however, for economic reasons this may not be practical. While the damaged structure is not an imminent collapse risk, it would be prudent to implement structural repairs or install temporary bracing prior to re-occupancy.

Collapse Prevention Performance Level (S-5) means the building is on the verge of experiencing partial or total collapse. Substantial damage to the structure has occurred, potentially including significant degradation in the stiffness and strength of the lateral-force-resisting system, large permanent lateral deformation of the structure and to more limited extent degradation in vertical-load-carrying capacity. However, all significant components of the gravity load-resisting system must continue to carry their gravity load demands. In the primary concrete frames, there will be extensive cracking and formation of hinges in the ductile elements. There will be about 4% inelastic drift, transient or permanent. There will be extensive cracking and crushing in the brick infill walls. Walls may dislodge due to out-of-plane bending. There will be 0.6% inelastic drift, transient or permanent. Significant risk of injury due to falling hazards from structural debris may exist. The structure may not be technically practical to repair and is not safe for re-occupancy, as aftershock activity could induce collapse. Figure 7.18 depicts various performance levels and damage functions.

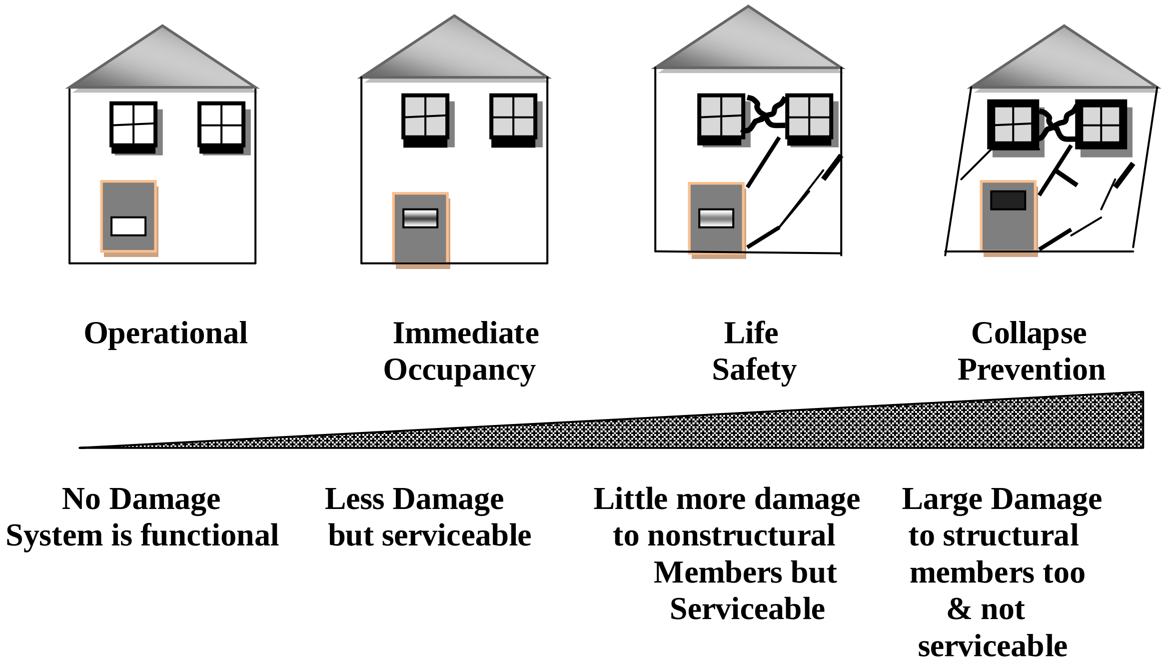



Figure 7.18 Performance levels and damage Functions.

### 7.6.1 Procedure of Pushover Analysis using SAP 2000

The procedure of Pushover Analysis using SAP 2000 software is summarized below:

1. Select the type of model and scale of system
2. Fill in the blanks and click the '+' to select the sections and their properties

   to create required structure / structural frame/ model
3. Select the bottom Joints and assign the support conditions as per requirement

   Assign – Joint – Restraint
4. For Loading (Response Spectrum or Time History)

   Define – Functions
5. For Dead Load, Live Loads, Wind Load etc

   Define- Load Patterns
6. To assign loads to beam / slab

   Define – Load Case
7. Add New Load Case to add Response Spectrum/ Time History and details
8. Define – Load Combinations – Add New Combination
9. Analyze – Set Analysis Options – Analysis Options

10. Analyze – Run Analysis
11. Design – Concrete Frame Design – View/Revise Preference..... to Select IS code
12. Design – Concrete Frame Design – Start Design / Check of Design to design
13. Click Unlock Model
14. Select Beams and Columns – Assign – Frame – Hinges – Assign Hinges
15. Define – Load Cases – Select Dead Load – Assign Non Linear – OK
16. Define – Load Cases - Add new Load Case

    Type - Push X in Load Case Name

    Click – Non Linear in Analysis Type

    Select - Accel in Load Applied – Load Type and write -1 in Scale Column

    In Other Parameters Column - Select Modify / Show of Load Application

    And then click Displacement control and then OK

    In Other Parameters Column - Select Modify / Show of Results saved

    And then click Multiple States and then OK

    Now Click Ok twice to come out of Load Case window.
17. Click Run Analysis and then select Response Spectrum / Time History and click Run / Do no Run Case and then Click Run Now Button to run analysis for Pushover Analysis
18. Now Click Display- Show Static Pushover Curve to see Pushover curve by FEMA 356, Capacity Curve using ATC 40
19. To get performance point – Click – Modify / show Parameters and then change Ca and Cv to get performance point.
20. Click – File – Display Tables – and check the $T_{eff}$ (i.e. T*)at which performance point is achieved

    and see the step number and then click OK twice to come out of the Pushover Curve Window.
21. Now Click – Display – Show Deformed Shape and select Push X in Case / Combo Window and then select same step to get hinge pattern at that particular step and conclude with respect to the hinge pattern.

### *Example 7.4*

Perform the pushover analysis and draw pushover curve, capacity curve and demand curve using SAP 2000 for a two storied RCC frame (refer Figure 7.19) having the properties.

i) RCC frame with single bay and two storied
ii) Floor to floor height is 3.5m and bay width is 4m
iii) Reinforcement – Fe 415 and Concrete – M20
iv) Column Size – 400mm x 230mm
v) Beam Size – 300mm x 230mm
vi) Response Spectra- IS:1893 (Part 1)-2002
vii) Soil strata- Hard Rock
viii) Zone – V
ix) Importance Factor- 1
x) Lumped Mass – 1500kg at each floor
xi) Modal Combination – Square root of sum of squares (SRSS)
xii) Directional Combination - Square root of sum of squares (SRSS)
xiii) Load Combination- 1.5 (DL+EL) as per IS: 1893-2002

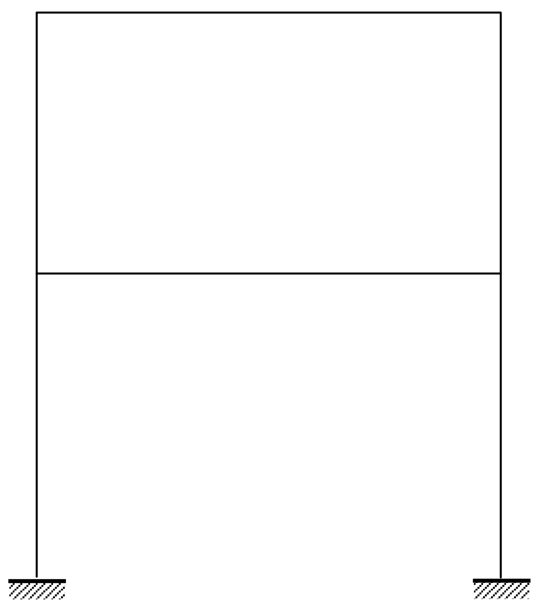


Figure 7.19 Model of the frame.

Table 7.1 Data of Push Over Curve.

| Step | Displacement (m) | Base Force (kN) | OP | IO | LS | CP | Beyond CP | Total |
|---|---|---|---|---|---|---|---|---|
| 0 | 4.06E-06 | 0 | 12 | 0 | 0 | 0 | 0 | 12 |
| 1 | 0.006007 | 21.524 | 11 | 1 | 0 | 0 | 0 | 12 |
| 2 | 0.015512 | 39.235 | 8 | 4 | 0 | 0 | 0 | 12 |
| 3 | 0.016973 | 40.611 | 7 | 5 | 0 | 0 | 0 | 12 |
| 4 | 0.019389 | 41.097 | 6 | 6 | 0 | 0 | 0 | 12 |
| 5 | 0.047389 | 41.838 | 6 | 6 | 0 | 0 | 0 | 12 |
| 6 | 0.075389 | 42.569 | 6 | 2 | 4 | 0 | 0 | 12 |
| 7 | 0.103389 | 43.292 | 6 | 0 | 6 | 0 | 0 | 12 |
| 8 | 0.131389 | 44.007 | 6 | 0 | 4 | 2 | 0 | 12 |
| 9 | 0.15703 | 44.654 | 6 | 0 | 1 | 3 | 2 | 12 |
| 10 | 0.173724 | 44.663 | 6 | 0 | 0 | 3 | 3 | 12 |

*SAP 2000 Advanced 14.2.0 do not provide data for Damage Control Level (DC)

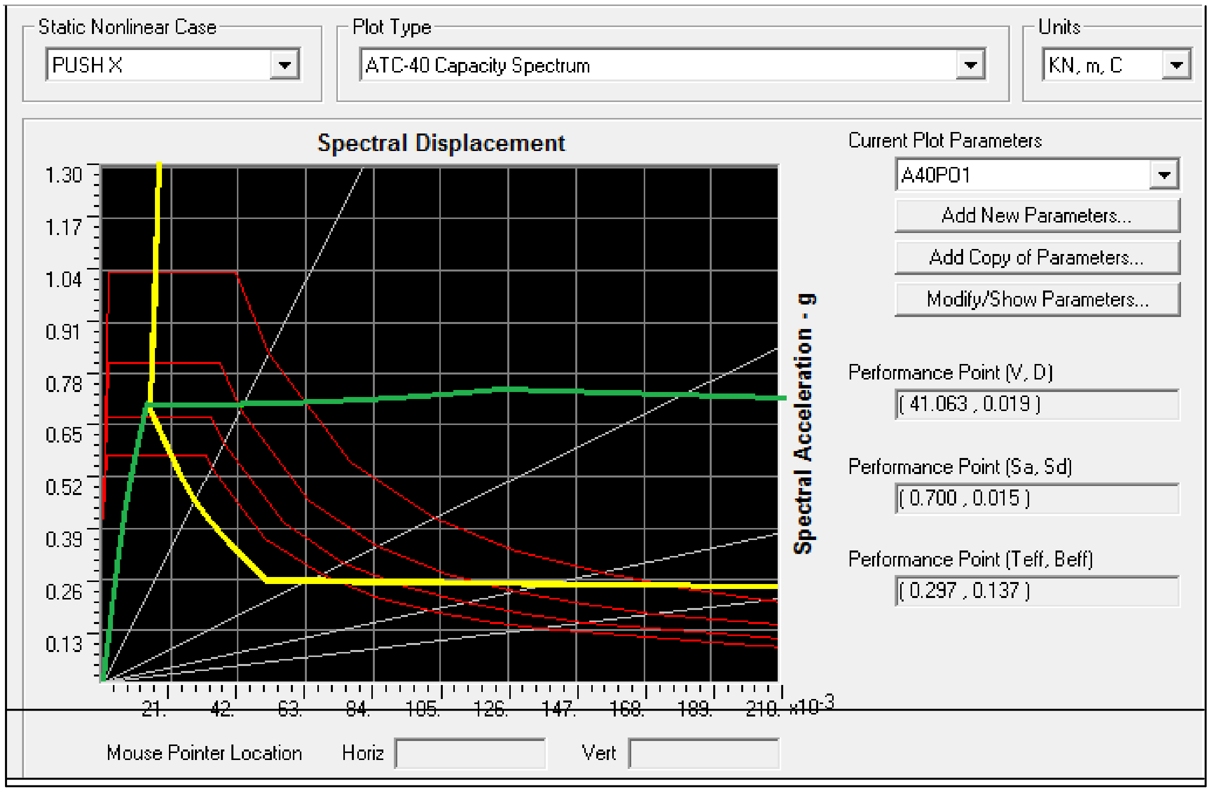



Figure 7.20 Capacity spectrum curve.

Figure 7.20 shows that performance point is at $T_{eff}$ = 0.297 sec which is close value of $T_{eff}$ at Step No. 4. Hence, it is required to see the hinge formations at Step No. 4. From Fig. 7.20, it also becomes clear that hinges formed in beams and columns are below immediate occupation level. Hence, structure is very safe to use.

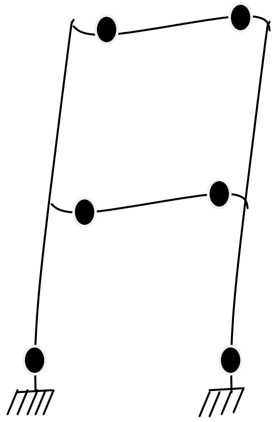


Figure 7.20 Hinge formations at Step No. 4.

Table 7.2 Modal Periods and Frequencies.

| **Output Case** | **Step Type** | **Step Num** | **Period** | **Frequency** | **Circ Freq** | **Eigen value** |
|---|---|---|---|---|---|---|
| **Text** | **Text** | **Unitless** | **Sec** | **Cyc/sec** | **rad/sec** | **rad2/sec2** |
| MODAL | Mode | 1 | 0.263335 | 3.7974 | 23.86 | 569.3 |
| MODAL | Mode | 2 | 0.075421 | 13.259 | 83.308 | 6940.2 |
| MODAL | Mode | 3 | 0.017003 | 58.814 | 369.54 | 136560 |
| MODAL | Mode | 4 | 0.016885 | 59.224 | 372.12 | 138470 |
| MODAL | Mode | 5 | 0.009896 | 101.05 | 634.94 | 403150 |
| MODAL | Mode | 6 | 0.008829 | 113.26 | 711.64 | 506430 |
| MODAL | Mode | 7 | 0.006795 | 147.17 | 924.72 | 855100 |
| MODAL | Mode | 8 | 0.006789 | 147.31 | 925.55 | 856640 |

The mode shapes for all eight steps are as shown in Figure 7.21 and deformed shapes and hinge formation has been shown in Figure 7.22.

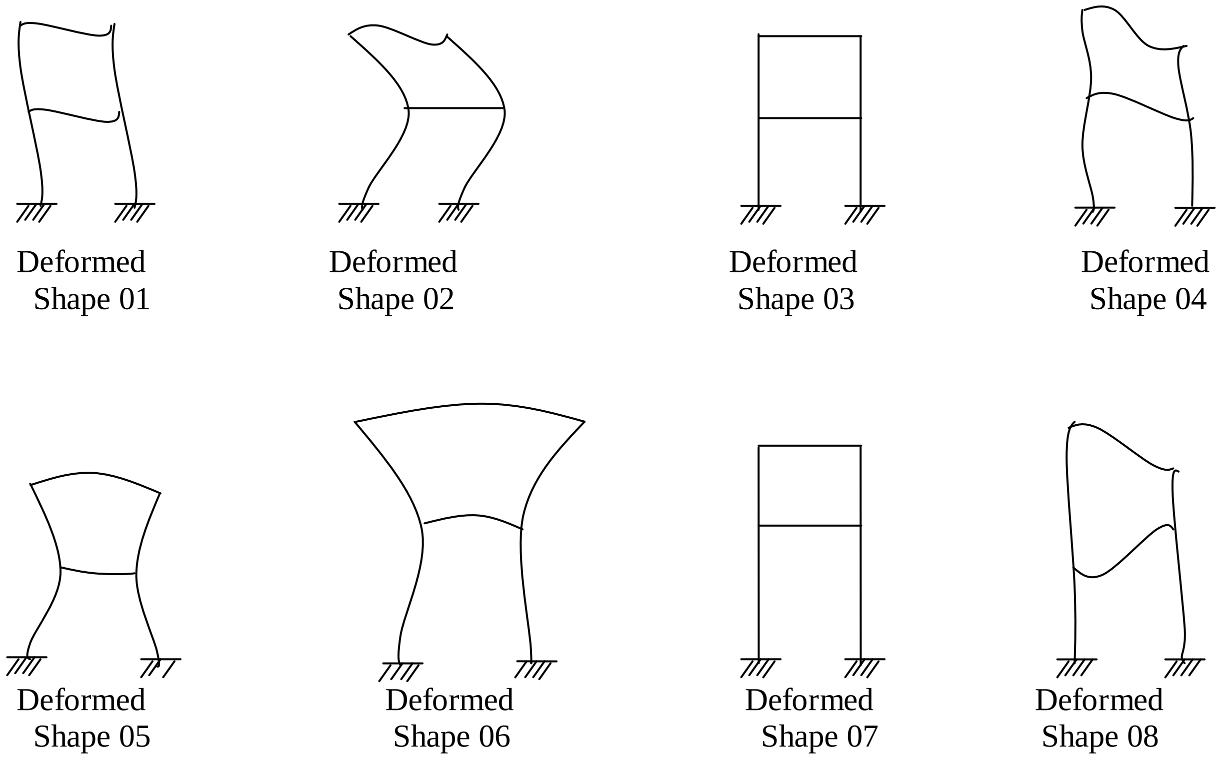



Figure 7.21 Deformed Shapes as per Modal Analysis (Mode Shapes)

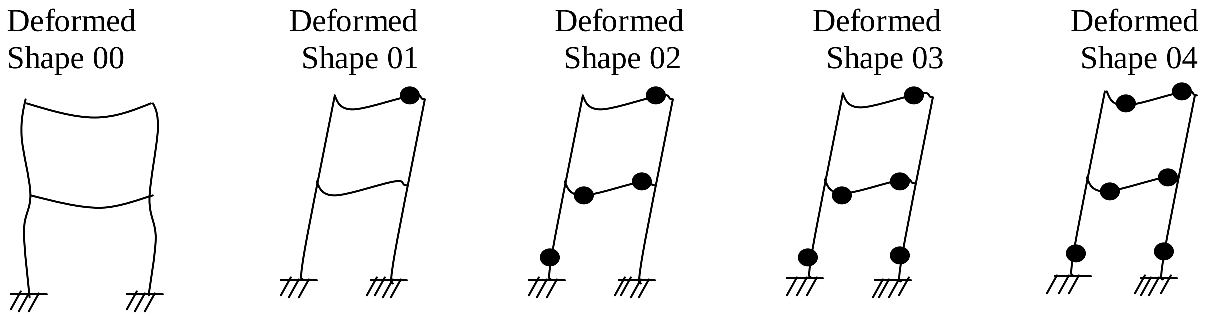



Figure 7.22 Deformed Shapes and Hinge Formation due to Push X .

## 7.7 Tutorial Problems

**Q1.** Consider an elasto-plastic SDOF system having mass = 1kg, elastic stiffness = 157.914 N/m and damping constant = 1.1257 N.sec/m. Determine the maximum response of the system under the El-Centro, 1940 motion for (i) yield displacement = 0.025m and (ii) yield displacement = 0.0125m.

**Q2.** A three-story building is modeled as 3-DOF system and rigid floors as shown in Figure 7.23. Determine the maximum floor displacements under El-Centro, 1940 earthquake ground motion. Take the inter-story lateral stiffness of floors are modeled as elasto-plastic with elastic stiffness i.e. $k_1 = k_2 = k_3 = 16357.5$ kN/m and the yield displacement of 0.005m. The floor masses are $m_1 = m_2 = 10000$ kg and $m_3 = 5000$ kg.

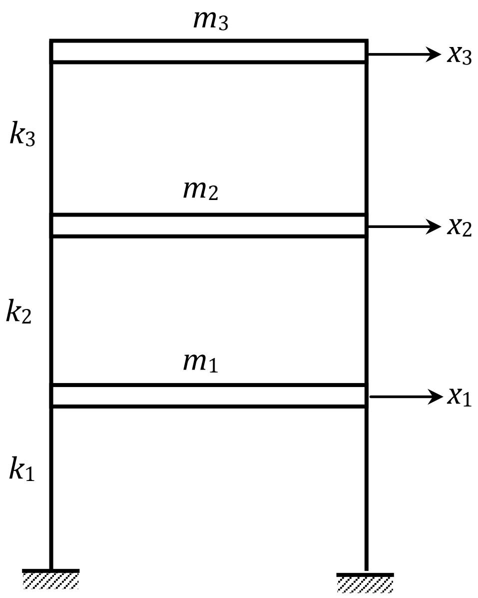



Figure 7.23

$m_1 = m_2 = 10000\ kg$
$m_3 = 5000\ kg$
$k_1 = k_2 = k_3 = 16357.5 \quad kN/m$

**Q3.** For a three storied frame (refer Figure 7.24), perform pushover analysis and draw pushover curve, capacity curve and demand curve using SAP 2000.

i) RCC frame with single bay and three storied

ii) Floor to floor height is 4m and bay width is 4m

iii) Reinforcement – Fe 415 and Concrete – M20

iv) Column Size – 400mm x 230mm

v) Beam Size – 300mm x 230mm

vi) El-Centro Time History

vii) Lumped Mass – 10000kg at each floor

viii) Load Combination- 1.5 (DL+EL) as per IS: 1893-2002

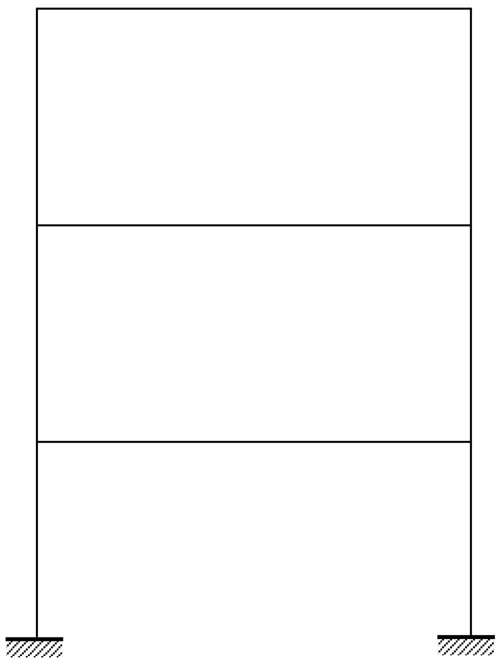


Figure 7.24

## 7.8 Answers to Tutorial Problems

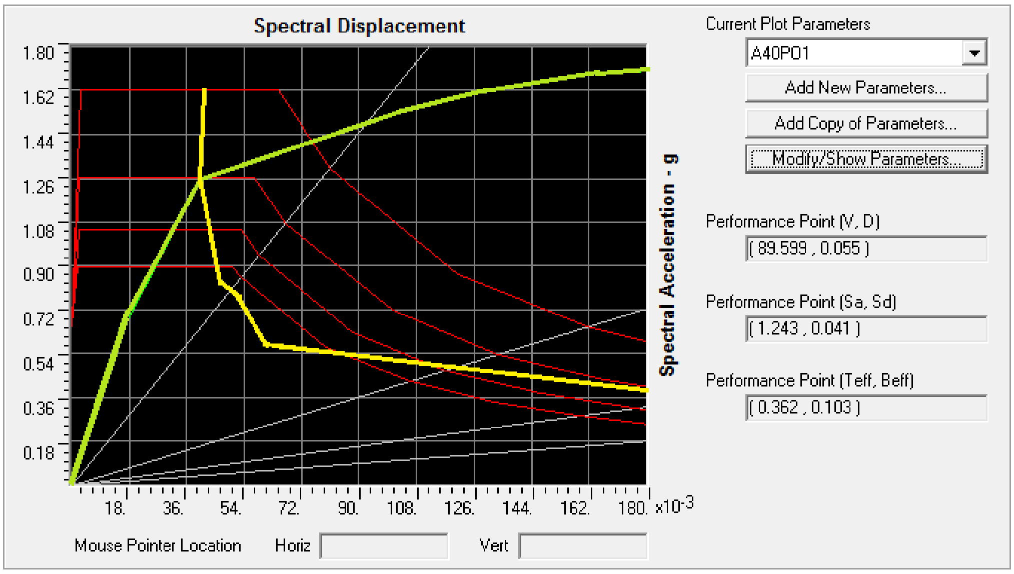
**Q1.** (i) Maximum displacement =0.04277 m
(ii) Maximum displacement = 0.038 m

**Q2.** Maximum displacement, $x_3$ = -0.0176m

Maximum displacement, $x_2$ = -0.0153m

Maximum displacement, $x_2$ = -0.0115m

**Q3.**




Performance point is very close to $T_{eff}$ equal to 0.362 sec which is at step number 3 and as per the hinge formation at Step 3, structure is safe from the pushover analysis.