Module 7: Micromechanics

Lecture 27: Hill's Concentration Factors Approach

Introduction

In the previous lecture we have introduced the concepts of statistical homogeneity, volumetric averaging and standard mechanics approach. In case of standard mechanics, the effective stiffness tensor for the composite is given in terms of local structure tensor and pointwise stiffness tensor. In this lecture we will introduce another approach of Hill's concentration factors. This approach is an extension of standard mechanics approach to two phase composites.

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Hill’s Concentration Factors Approach

The Hill’s concentration factors approach is based on the concept similar to the standard mechanics approach. In this approach, a composite with two elastic phases is considered. These phases are fibre and matrix.

The average stress in composite is given from Equation (7.74). In this equation, the stresses in individual phases are used to give following equation

$$\bar{\sigma}_{ij} = \frac{1}{V_{RVE}} \int_V \sigma_{ij}(\mathrm{x}) dV_{RVE} = \frac{1}{V_{RVE}} \int_{V_f} \sigma_{ij}^{(f)}(\mathrm{x}) dV + \frac{1}{V_{RVE}} \int_{V_m} \sigma_{ij}^{(m)}(\mathrm{x}) dV \quad (7.99)$$

where $\sigma_{ij}^{(f)}(\mathrm{x})$ and $\sigma_{ij}^{(m)}(\mathrm{x})$ are the local stresses in fibre and matrix, respectively. Now let us define the volume averaged stress in fibre as

$$\bar{\sigma}_{ij}^{(f)} = \frac{1}{v_f} \int_{v_f} \sigma_{ij}^{(f)}(\mathrm{x}) dV_f \quad (7.100)$$

and volume averaged stress in matrix as

$$\bar{\sigma}_{ij}^{(m)} = \frac{1}{v_m} \int_{v_m} \sigma_{ij}^{(m)}(\mathrm{x}) dV_m \quad (7.101)$$

Putting these two definitions in Equation (7.99) and adjusting the v_f and v_m terms properly, we get the average stress in composite as

$$\bar{\sigma}_{ij} = \frac{v_f}{V_{RVE}} \bar{\sigma}_{ij}^{(f)} + \frac{v_m}{V_{RVE}} \bar{\sigma}_{ij}^{(m)} = V_f\, \bar{\sigma}_{ij}^{(f)} + V_m\, \bar{\sigma}_{ij}^{(m)} \quad (7.102)$$

Similarly, we define the volume averaged strains in fibre and matrix as

$$\bar{\varepsilon}_{ij}^{(f)} = \frac{1}{v_f} \int_{v_f} \varepsilon_{ij}^{(f)}(\mathrm{x}) dV_f \text{ and } \bar{\varepsilon}_{ij}^{(m)} = \frac{1}{v_m} \int_{v_m} \varepsilon_{ij}^{(m)}(\mathrm{x}) dV_m \quad (7.103)$$

Note: The average stresses in Equation (7.100) and (7.101) and average strains in Equation (7.103) are also known as phase averaged stresses and phase averaged strains, respectively.

Putting these definitions for the definition of average strain in composite, we get

$$\bar{\varepsilon}_{ij} = V_f\, \bar{\varepsilon}_{ij}^{(f)} + V_m\, \bar{\varepsilon}_{ij}^{(m)} \quad (7.104)$$

Now let us derive the average stress in fibre and matrix using the pointwise constitutive equation for fibre and matrix in Equations (7.100) and (7.101) as

$$\bar{\sigma}_{ij}^{(f)} = \frac{1}{v_f}\int_{v_f} C_{ijkl}^{(f)}\,\varepsilon_{kl}^{(f)}(\mathrm{x})\,dV_f = C_{ijkl}^{(f)}\frac{1}{v_f}\int_{v_f}\varepsilon_{kl}^{(f)}(\mathrm{x})\,dV_f = C_{ijkl}^{(f)}\bar{\varepsilon}_{kl}^{(f)}$$
$$\bar{\sigma}_{ij}^{(m)} = \frac{1}{v_m}\int_{v_m} C_{ijkl}^{(m)}\,\varepsilon_{kl}^{(m)}(\mathrm{x})\,dV_m = C_{ijkl}^{(m)}\frac{1}{v_m}\int_{v_m}\varepsilon_{kl}^{(m)}(\mathrm{x})\,dV_m = C_{ijkl}^{(m)}\bar{\varepsilon}_{kl}^{(m)} \quad (7.105)$$

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In the above derivation it is assumed that the material behaviour is same everywhere for fibre and matrix. Further, Equation (7.103) has been used in above equation. The above equation can be written in terms of compliance of fibre and matrix material as

$$\begin{aligned}\bar{\varepsilon}_{ij}^{(f)} &= \frac{1}{v_f}\int_{v_f} S_{ijkl}^{(f)}\sigma_{kl}^{(f)}(\mathrm{x})dV_f = S_{ijkl}^{(f)}\frac{1}{v_f}\int_{v_f}\sigma_{kl}^{(f)}(\mathrm{x})dV_f = S_{ijkl}^{(f)}\bar{\sigma}_{kl}^{(f)}\\ \bar{\varepsilon}_{ij}^{(m)} &= \frac{1}{v_m}\int_{v_m} S_{ijkl}^{(m)}\sigma_{kl}^{(m)}(\mathrm{x})dV_m = S_{ijkl}^{(m)}\frac{1}{v_m}\int_{v_m}\sigma_{kl}^{(m)}(\mathrm{x})dV_m = S_{ijkl}^{(m)}\bar{\sigma}_{kl}^{(m)}\end{aligned} \tag{7.106}$$

Using Equation (7.105) in Equation (7.102) we get the average stress in composite in terms of volume fractions, stiffness tensor and phase averaged strains as

$$\bar{\sigma}_{ij} = V_f C_{ijkl}^{(f)}\bar{\varepsilon}_{kl}^{(f)} + V_m\, C_{ijkl}^{(m)}\,\bar{\varepsilon}_{kl}^{(m)} \tag{7.107}$$

Similarly, the average composite strain in terms of phase averaged stresses in the fiber and matrix, respective compliances and volume fractions using Equation (7.106) in Equation (7.104) is given as

$$\bar{\varepsilon}_{ij} = V_f S_{ijkl}^{(f)}\bar{\sigma}_{kl}^{(f)} + V_m\, S_{ijkl}^{(m)}\,\bar{\sigma}_{kl}^{(m)} \tag{7.108}$$

Note: It can be shown that if an RVE is subjected to homogeneous traction on its boundary, that is, $T_i = \sigma_{ij}^0 n_j$ with σ_{ij}^0 is a constant state of stress, then the average stress in composite is

$$\bar{\sigma}_{ij}^0 = \sigma_{ij}^0$$

Similarly, if an RVE is subjected to homogeneous displacement on its boundary, that is, $u_i = \varepsilon_{ij}^0\, x_j$ with ε_{ij}^0 is a constant strain, then the average strain in the composite is

$$\bar{\varepsilon}_{ij} = \varepsilon_{ij}^0$$

The local structure tensor used in Equation (7.94) in standard mechanics approach to define the local strains in terms of composite average strains. Hill [4] used this concept to relate the pointwise stresses and strains in fibre and matrix with average stresses and strains in composite through pointwise phase concentration factors. The pointwise strains in fibre and matrix are given as

$$\varepsilon_{ij}^{(f)}(\mathrm{x}) = A_{ijkl}^{(f)}(\mathrm{x})\bar{\varepsilon}_{kl} \text{ and } \varepsilon_{ij}^{(m)}(\mathrm{x}) = A_{ijkl}^{(m)}(\mathrm{x})\bar{\varepsilon}_{kl} \tag{7.109}$$

where $A_{ijkl}^{(f)}(\mathrm{x})$ and $A_{ijkl}^{(m)}(\mathrm{x})$ are the pointwise fibre and matrix strain concentration factors, respectively. Similarly, the pointwise stresses in fibre and matrix are given as

$$\sigma_{ij}^{(f)}(\mathrm{x}) = B_{ijkl}^{(f)}(\mathrm{x})\bar{\sigma}_{kl} \text{ and } \sigma_{ij}^{(m)}(\mathrm{x}) = B_{ijkl}^{(m)}(\mathrm{x})\bar{\sigma}_{kl} \tag{7.110}$$

where $B_{ijkl}^{(f)}(\mathrm{x})$ and $B_{ijkl}^{(m)}(\mathrm{x})$ are the pointwise fibre and matrix stress concentration factors, respectively.

The local strains and stresses in fibre and matrix as given in Equation (7.109) and Equation (7.110) can be integrated over their respective volumes to give the phase averaged strains and stresses in terms of phase averaged concentration factors $\bar{A}_{ijkl}^{(f)}, \bar{A}_{ijkl}^{(m)}$ and $\bar{B}_{ijkl}^{(f)}, \bar{B}_{ijkl}^{(m)}$. The phase averaged concentration factors as defined are given below.

$$\bar{A}_{ijkl}^{(f)} = \frac{1}{v_f} \int_{v_f} A_{ijkl}^{(f)}(\mathrm{x})\, dv_f \text{ and } \bar{A}_{ijkl}^{(m)} = \frac{1}{v_m} \int_{v_m} A_{ijkl}^{(m)}(\mathrm{x})\, dv_m$$
$$\bar{B}_{ijkl}^{(f)} = \frac{1}{v_f} \int_{v_f} B_{ijkl}^{(f)}(\mathrm{x})\, dv_f \text{ and } \bar{B}_{ijkl}^{(m)} = \frac{1}{v_m} \int_{v_m} B_{ijkl}^{(m)}(\mathrm{x})\, dv_m \qquad (7.111)$$

Thus, using this definition the phase averaged strains in fibre and matrix can be written using Equation. (7.109) and Equation (7.110) as

$$\begin{aligned} &\bar{\varepsilon}^{(f)}_{ijkl}\ \bar{A}^{(f)}_{ijkl}\bar{\varepsilon}_{kl}, \qquad \bar{\varepsilon}^{(m)}_{ij} = \bar{A}^{(m)}_{ijkl}\bar{\varepsilon}_{kl} \\ &\bar{\sigma}^{(f)}_{ijkl}\ \bar{B}^{(f)}_{ijkl}\bar{\sigma}_{kl}, \qquad \bar{\sigma}^{(m)}_{ij} = \bar{B}^{(m)}_{ijkl}\bar{\sigma}_{kl} \end{aligned} \tag{7.112}$$

Now, using the first of Equation (7.112) in Equation (7.107) composite average stress is given as

$$\begin{aligned} \bar{\sigma}_{ij} &= V_f C^{(f)}_{ijpm}\ \bar{A}^{(f)}_{pmkl}\bar{\varepsilon}_{kl} + V_m\ C^{(m)}_{ijpm}\ \bar{A}^{(m)}_{pmkl}\ \bar{\varepsilon}_{kl} \\ &= \left[V_f\ C^{(f)}_{ijpm}\ \bar{A}^{(f)}_{pmkl} + V_m\ C^{(m)}_{ijpm}\ \bar{A}^{(m)}_{pmkl}\right]\bar{\varepsilon}_{kl} \\ &= C^{*}_{ijkl}\ \bar{\varepsilon}_{kl} \end{aligned} \tag{7.113}$$

Similarly, using the second of Equation (7.112) in Equation (7.108) composite average strain is given as

$$\begin{aligned} \bar{\varepsilon}_{ij} &= V_f S^{(f)}_{ijpm}\ \bar{B}^{(f)}_{pmkl}\bar{\sigma}_{kl} + V_m\ C^{(m)}_{ijpm}\ \bar{B}^{(m)}_{pmkl}\ \bar{\sigma}_{kl} \\ &= \left[V_f\ S^{(f)}_{ijpm}\ \bar{B}^{(f)}_{pmkl} + V_m\ S^{(m)}_{ijpm}\ \bar{B}^{(m)}_{pmkl}\right]\bar{\sigma}_{kl} \\ &= S^{*}_{ijkl}\ \bar{\sigma}_{kl} \end{aligned} \tag{7.114}$$

Equation (7.102) can be written using the second of Equation (7.112) as

$$\begin{aligned} \bar{\sigma}_{ij} &= V_f\ \bar{\sigma}^{(f)}_{ij} + V_m\ \bar{\sigma}^{(m)}_{ij} \\ &= V_f\ \bar{B}^{(f)}_{ijkl}\ \bar{\sigma}_{kl} + V_m\ \bar{B}^{(m)}_{ijkl}\ \bar{\sigma}_{kl} \\ &= \left[V_f\ \bar{B}^{(f)}_{ijkl} + V_m\ \bar{B}^{(m)}_{ijkl}\right]\bar{\sigma}_{kl} \end{aligned} \tag{7.115}$$

It should be noted that in above equation the stresses on left and right hand side are the composite average stresses. Hence, they are same. Thus, the bracketed term in above equation is an identity tensor of fourth order, that is,

$$V_f \bar{B}^{(f)}_{ijkl} + V_m \bar{B}^{(m)}_{ijkl} = I_{ijkl} \tag{7.116}$$

Likewise, from Equation (7.104) and the first of Equation (7.112) we can write

$$V_f \bar{A}^{(f)}_{ijkl} + V_m \bar{A}^{(m)}_{ijkl} = I_{ijkl} \tag{7.117}$$

Now using the Eq. (7.117) in Eq. (7.113), we can write

$$C^{*}_{ijkl} = C^{(m)}_{ijkl} + V_f\left(C^{(f)}_{ijrs} - C^{(m)}_{ijrs}\right)\bar{A}^{(f)}_{rskl} \tag{7.118}$$

Similarly, using Equation (7.116) in Equation (7.114), we can write

(7.119)

$$S^{*}_{ijkl} = S^{(m)}_{ijkl} + V_f \left(S^{(f)}_{ijrs} - S^{(m)}_{ijrs} \right) \bar{B}^{(f)}_{rskl}$$

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Voigt Approximation

Voigt [5] assumed that the strains are constants throughout the composite. Thus, we can say that

$$\bar{\varepsilon}_{ij}^{(f)} = \bar{\varepsilon}_{ij}^{(m)} = \bar{\varepsilon}_{ij} \tag{7.120}$$

From the first of Equation (7.112), this leads to

$$\bar{A}_{ijkl}^{(f)} = \bar{A}_{ijkl}^{(m)} \tag{7.121}$$

Now, Equation (7.117) is written as

$$V_f \bar{A}_{ijkl}^{(f)} + V_m \bar{A}_{ijkl}^{(m)} = V_f \bar{A}_{ijkl}^{(f)} + (1 - V_f)\bar{A}_{ijkl}^{(f)} = \bar{A}_{ijkl}^{(f)} = I_{ijkl} \tag{7.122}$$

Thus, we can write

$$\bar{A}_{ijkl}^{(f)} = \bar{A}_{ijkl}^{(m)} = I_{ijkl} \tag{7.123}$$

Using this relation in Equation (7.118) we write

$$C_{ijkl}^{*} = C_{ijkl}^{(m)} + V_f\left(C_{ijkl}^{(f)} - C_{ijkl}^{(m)}\right) = V_f C_{ijkl}^{(f)} + V_m C_{ijkl}^{(m)} \tag{7.124}$$

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Reuss Approximation

Reuss [6] assumed that the stresses are constant throughout the composite. This assumption leads to the relation

$$\bar{B}_{ijkl}^{(f)} = \bar{B}_{ijkl}^{(m)} \quad (7.125)$$

which upon substitution in Equation (7.116) leads to the relation

$$\bar{B}_{ijkl}^{(f)} = \bar{B}_{ijkl}^{(m)} = I_{ijkl} \quad (7.126)$$

Putting the above relation in Equation (7.119) gives

$$S_{ijkl}^{*} = S_{ijkl}^{(m)} + V_f\left(S_{ijkl}^{(f)} - S_{ijkl}^{(m)}\right) = V_f S_{ijkl}^{(f)} + V_m S_{ijkl}^{(m)} \quad (7.127)$$

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Examples

Example 7.2: For AS4 fibre and 3501-6 Epoxy material with 0.6 fibre volume fraction calculate all effective engineering constants of the composite using **a) Voigt and b) Reuss approximations**. The properties are given in Table 7.1 and Table 7.2.

Solution:

a) Voigt Approximation:

According to this approximation the effective stiffness tensor for composite is given as

$$C^{*}_{ijkl} = V_f C^{(f)}_{ijkl} + V_m C^{(m)}_{ijkl}$$

The stiffness matrices for fibre and matrix are calculated using the respective engineering constants and are given below.

For this purpose it is better to calculate first the compliance matrices for fibre and matrix materials and invert them to get the stiffness matrices. We know that getting stiffness from compliance can be easier than remembering individual stiffness entries in terms of engineering constants. The compliance matrices for fibre and matrix material are calculated as below.

$$S^{(f)}_{ij} = \begin{bmatrix} 44.444 & -8.888 & -8.888 & 0 & 0 & 0 \\ -8.888 & 666.666 & -47.619 & 0 & 0 & 0 \\ -8.888 & -47.619 & 666.666 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1428.572 & 0 & 0 \\ 0 & 0 & 0 & 0 & 666.666 & 0 \\ 0 & 0 & 0 & 0 & 0 & 666.666 \end{bmatrix} \times 10^{-4} \frac{1}{GPa}$$

$$S^{(m)}_{ij} = \begin{bmatrix} 23.809 & -8.095 & -8.095 & 0 & 0 & 0 \\ -8.095 & 23.809 & -8.095 & 0 & 0 & 0 \\ -8.095 & -8.095 & 23.809 & 0 & 0 & 0 \\ 0 & 0 & 0 & 63.809 & 0 & 0 \\ 0 & 0 & 0 & 0 & 63.809 & 0 \\ 0 & 0 & 0 & 0 & 0 & 63.809 \end{bmatrix} \times 10^{-2} \frac{1}{GPa}$$

Now the stiffness matrices of fibre and matrix are:

$$C^{(f)}_{ij} = \begin{bmatrix} 226.299 & 3.249 & 3.249 & 0 & 0 & 0 \\ 3.249 & 15.123 & 1.123 & 0 & 0 & 0 \\ 3.249 & 1.123 & 15.123 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 15.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15.0 \end{bmatrix} GPa$$

$$C_{ij}^{(m)} = \begin{bmatrix} 6.464 & 3.331 & 3.331 & 0 & 0 & 0 \\ 3.331 & 6.464 & 3.331 & 0 & 0 & 0 \\ 3.331 & 3.331 & 6.464 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.567 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.567 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.567 \end{bmatrix} GPa$$

Thus, the effective stiffness matrix according to Voigt approximation for fibre volume fraction of 0.6 is

$$C_{ij}^{*} = \begin{bmatrix} 138.365 & 3.282 & 3.282 & 0 & 0 & 0 \\ 3.282 & 11.659 & 2.006 & 0 & 0 & 0 \\ 3.282 & 2.006 & 11.659 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.826 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9.626 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9.626 \end{bmatrix} GPa$$

The inverse of this effective stiffness matrix is

$$S_{ij}^{*} = \begin{bmatrix} 7.311 & -1.755 & -1.755 & 0 & 0 & 0 \\ -1.755 & 88.802 & -14.785 & 0 & 0 & 0 \\ -1.755 & -14.785 & 88.802 & 0 & 0 & 0 \\ 0 & 0 & 0 & 207.173 & 0 & 0 \\ 0 & 0 & 0 & 0 & 103.875 & 0 \\ 0 & 0 & 0 & 0 & 0 & 103.875 \end{bmatrix} \times 10^{-3} \frac{1}{GPa}$$

Effective engineering constants:

$$E_1 = -\frac{1}{S_{11}^*} = 136.78 \text{ GPa}, \qquad G_{23} = \frac{1}{S_{44}^*} = 4.826 \text{ GPa}$$
$$E_2 = -\frac{1}{S_{22}^*} = 11.26 \text{ GPa}, \qquad G_{13} = \frac{1}{S_{55}^*} = 9.626 \text{ GPa}$$
$$E_3 = -\frac{1}{S_{33}^*} = 11.26 \text{ GPa}, \qquad G_{12} = \frac{1}{S_{66}^*} = 9.626 \text{ GPa}$$

$$v_{12} = -\frac{S_{21}^*}{S_{11}^*} = 0.24, \qquad v_{21} = \frac{S_{12}^*}{S_{22}^*} = 0.0197$$
$$v_{13} = -\frac{S_{31}^*}{S_{11}^*} = 0.24, \qquad v_{31} = \frac{S_{13}^*}{S_{33}^*} = 0.0197$$
$$v_{23} = -\frac{S_{32}^*}{S_{22}^*} = 0.166, \qquad v_{32} = \frac{S_{23}^*}{S_{33}^*} = 0.166$$

b) Reuss Approximation:

According to this approximation the effective compliance tensor for composite is given as

$$S^*_{ijkl} = V_f S^{(f)}_{ijkl} + V_m S^{(m)}_{ijkl}$$

Using the compliance matrices for fibre and matrix we get the effective compliance for composite as

$$S^*_{ij} = \begin{bmatrix} 9.791 & -3.292 & -3.292 & 0 & 0 & 0 \\ -3.292 & 13.524 & -3.524 & 0 & 0 & 0 \\ -3.292 & -3.524 & 13.524 & 0 & 0 & 0 \\ 0 & 0 & 0 & 34.095 & 0 & 0 \\ 0 & 0 & 0 & 0 & 29.523 & 0 \\ 0 & 0 & 0 & 0 & 0 & 29.523 \end{bmatrix} \times 10^{-2} \frac{1}{GPa}$$

Effective engineering constants:

$$E_1 = -\frac{1}{S^*_{11}} = 10.214 \text{ GPa}, \qquad G_{23} = \frac{1}{S^*_{44}} = 2.933 \text{ GPa}$$
$$E_2 = -\frac{1}{S^*_{22}} = 7.394 \text{ GPa}, \qquad G_{13} = \frac{1}{S^*_{55}} = 3.387 \text{ GPa}$$
$$E_3 = -\frac{1}{S^*_{33}} = 7.394, \text{GPa}, \qquad G_{12} = \frac{1}{S^*_{66}} = 3.387 \text{ GPa}$$

$$v_{12} = -\frac{S^*_{21}}{S^*_{11}} = 0.336, v_{21} = -\frac{S^*_{12}}{S^*_{22}} = 0.243$$
$$v_{13} = -\frac{S^*_{31}}{S^*_{11}} = 0.336, v_{31} = -\frac{S^*_{13}}{S^*_{33}} = 0.243$$
$$v_{23} = -\frac{S^*_{32}}{S^*_{22}} = 0.261, v_{32} = -\frac{S^*_{23}}{S^*_{33}} = 0.261$$

Example 7.3: Plot the variation of following effective stiffness terms against the fibre volume fractions for both Voigt and Reuss approximation. $C_{11}^*, C_{12}^*, C_{13}^*, C_{22}^*, C_{23}^*, C_{44}^* \text{and} C_{66}^*$.

Solution: The plots of $C_{11}^*, C_{22}^*, C_{44}^*$ and C_{66}^* are shown in Figure 7.10 and plots of C_{12}^*, C_{13}^*, and C_{23}^* are shown in Figure 7.11. The Voigt approximation gives upper bound for the terms $C_{11}^*, C_{22}^*, C_{44}^*$ and C_{66}^* whereas Reuss approximation gives the lower bound for these terms. However, for the terms C_{12}^*, C_{13}^*, and C_{23}^* Voigt approximation gives lower bound and Reuss approximation gives upper bound.

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***Example 7.4*:** Plot the variation of following effective compliance terms against the fibre volume fractions for both Voigt and Reuss approximation. $S^*_{11}, S^*_{12}, S^*_{13}, S^*_{22}, S^*_{23}, S^*_{44}$ and S^*_{66}.

***Solution*:** The plots of $S^*_{11}, S^*_{22}, S^*_{44}$ and S^*_{66} are shown in Figure 7.12 and plots of S^*_{12}, S^*_{13} and S^*_{23} are shown in Figure 7.13. The Voigt approximation gives lower bound and Reuss approximation gives upper bound for $S^*_{11}, S^*_{22}, S^*_{44}$ and S^*_{66} terms. Further, for terms S^*_{12}, S^*_{13} and S^*_{23} Voigt approximation gives upper bound and Reuss approximation gives lower bound.

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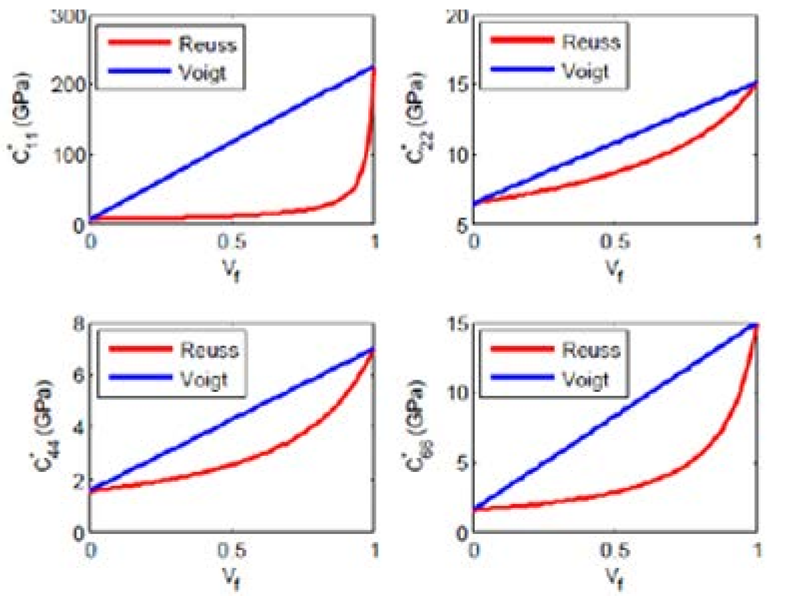

Figure 7.10: Variation of $C^*_{11}, C^*_{22}, C^*_{44}$ and C^*_{66} terms with fibre volume fractions for AS4/3501-6 Epoxy composite

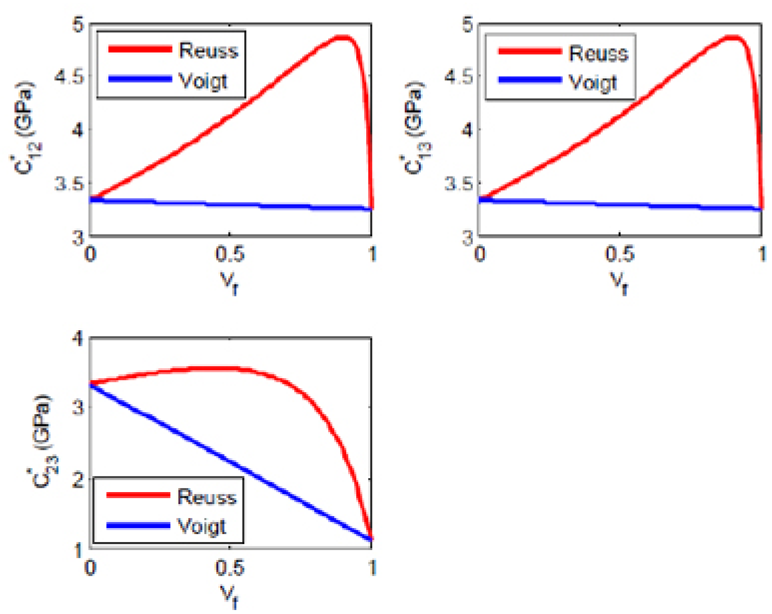

Figure 7.11: Variation of $C^*_{12}, C^*_{13},$ and C^*_{23} terms with fibre volume fraction for AS4/3501-6 Epoxy composite

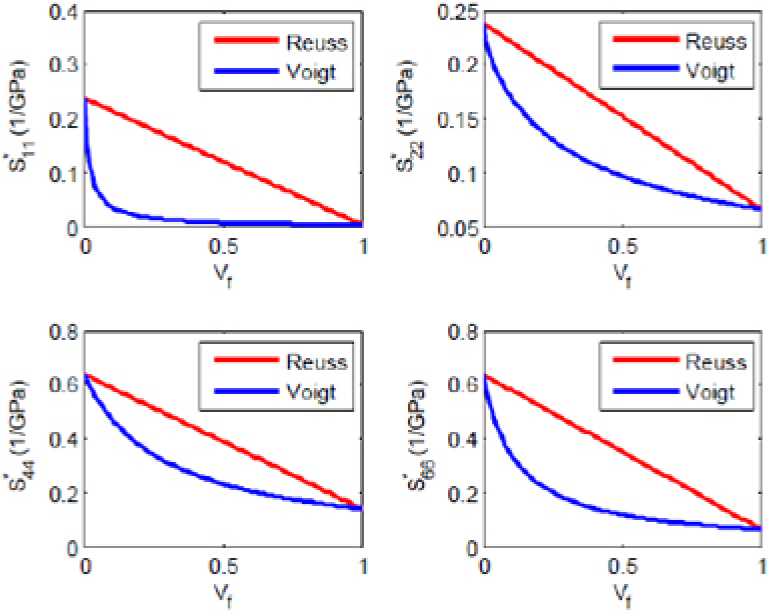

Figure 7.12: Variation of $S^*_{11}, S^*_{22}, S^*_{44}$ and S^*_{66} terms with fibre volume fractions for AS4/3501-6 Epoxy composite

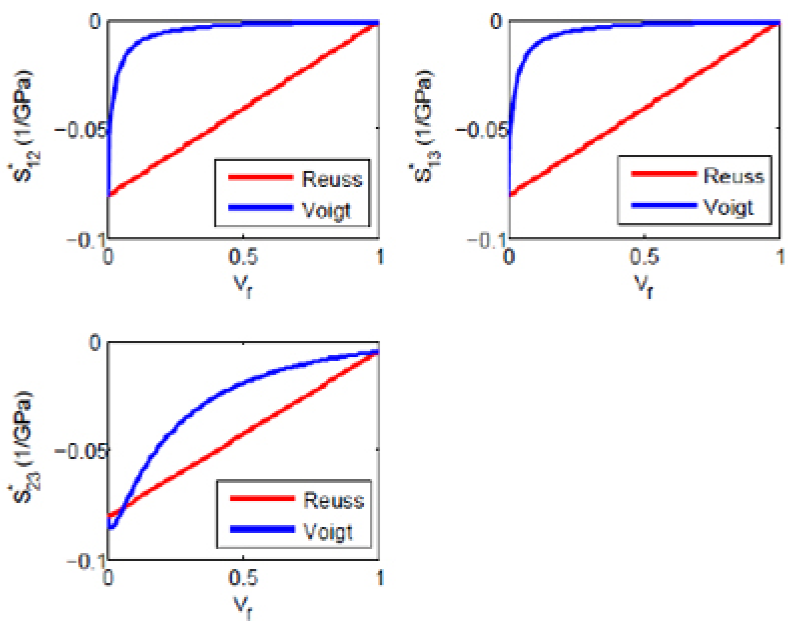

Figure 7.13: Variation of S^*_{12}, S^*_{13} and S^*_{23} terms with fibre volume fraction for AS4/3501-6 Epoxy composite

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Lecture 27: Hill's Concentration Factors Approach

Home Work:

1. Explain in detail the Hill’s concentration factors approach.
2. What are Reuss and Voigt approximations in connection with Hill’s concentration factors approach?
3. For fibre volume fraction of 0.6, determine all the effective mechanical properties for the fibre and matrix materials given in Table 7.1 and Table 7.2 and compare them with the experimental effective properties as reported in Soden et al [7]. Calculate percentage difference for all properties. Use Voigt and Reuss approximation for this exercise.

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