The Lecture Contains:

Introduction

In the previous lecture we have derived the stiffness matrix in terms of engineering constants using the compliance relations. Further, we have seen the constraints over the engineering constants in orthotropic materials. Then we have visited the stress and strain transformation. In the present lecture we will see the stiffness and compliance transformation about an axis. Further, we will address the effect of thermal and hygroscopic actions on lamina constitutive equations.

Stiffness Transformation:

It is required to relate the stress components with strain components in global *xyz* directions. The stiffness matrix which relates the stress and strain components in global directions is called as transformed stiffness matrix. We will derive an expression for the transformed stiffness matrix as follows.

The constitutive equation in principal material coordinates, as given in Equation (3.11), is

$$\{\sigma\}_{123} = [C]\{\varepsilon\}_{123} \tag{3.70}$$

Now, we express $\{\sigma\}_{123}$ and $\{\varepsilon\}_{123}$ using Equation (3.63) to transform stresses and Equation (3.68) to transform strains. Substituting these equations, we get

$$[T_1]\{\sigma\}_{xyz} = [C][T_2]\{\varepsilon\}_{xyz} \tag{3.71}$$

Pre-multiplying both sides by $[T_1]^{-1}$, we get

$$\begin{aligned} \{\sigma\}_{xyz} &= [T_1]^{-1}[C][T_2]\{\varepsilon\}_{xyz} \\ \{\sigma\}_{xyz} &= [\bar{C}]\{\varepsilon\}_{xyz} \end{aligned} \tag{3.72}$$

where we define the transformed stiffness matrix $[\bar{C}]$ as

$$[\bar{C}] = [T_1]^{-1}[C][T_2] \tag{3.73}$$

The transformation matrices $[T_1]$ and $[T_2]$ can be inverted as follows

$$[T_i(\theta)]^{-1} = [T_i(-\theta)] \qquad i = 1, 2 \tag{3.74}$$

The final form of the transformed stiffness matrix is given in Equation (3.75).

(3.75)

$$\left[\overline{C}\right]=\begin{bmatrix}\overline{C}_{11} & \overline{C}_{12} & \overline{C}_{13} & 0 & 0 & \overline{C}_{16}\\ \overline{C}_{12} & \overline{C}_{22} & \overline{C}_{23} & 0 & 0 & \overline{C}_{26}\\ \overline{C}_{13} & \overline{C}_{23} & \overline{C}_{33} & 0 & 0 & \overline{C}_{36}\\ 0 & 0 & 0 & \overline{C}_{44} & \overline{C}_{45} & 0\\ 0 & 0 & 0 & \overline{C}_{45} & \overline{C}_{55} & 0\\ \overline{C}_{16} & \overline{C}_{26} & \overline{C}_{36} & 0 & 0 & \overline{C}_{66}\end{bmatrix}$$

The individual $\overline{C}_{ij}$ terms of this matrix are determined using $\left[T_1\right]^{-1}$, $\left[T_2\right]$ and relation for $\left[\overline{C}\right]$. The individual terms are given in Equation (3.76).

***Note*:** The transformed stiffness matrix is symmetric in nature.

Note: The transformed stiffness matrix given in Equation (3.75) has exactly the same form as a stiffness matrix for a monoclinic material. Thus, we can conclude that a transformation through an arbitrary angle θ about direction 3, leads to a monoclinic material behaviour.

The same can be seen from the plane of elastic symmetry considerations in *xyz* coordinate system. The given lamina is symmetric only about *xy* plane. Thus, the transformed stiffness matrix in Equation (3.75) is consistent with monoclinic material.

Note: Transformed stiffness coefficient terms are fourth order in the sine and cosine functions. It is very important to use appropriate precision level while calculating (in examinations and writing computer codes) these coefficients.

$$\begin{aligned}
\overline{C}_{11} &= m^4 C_{11} + 2m^2 n^2\left(C_{12} + 2\,C_{66}\right) + n^4 C_{22}\\
\overline{C}_{12} &= n^2 m^2\left(C_{11} + C_{22} - 4C_{66}\right) + \left(n^4 + m^4\right)C_{12}\\
\overline{C}_{13} &= m^2 C_{13} + n^2 C_{23}\\
\overline{C}_{16} &= nm\left[m^2\left(C_{11} - C_{12} - 2\,C_{66}\right) + n^2\left(C_{12} - C_{22} + 2\,C_{66}\right)\right]\\
\overline{C}_{22} &= n^4 C_{11} + 2m^2 n^2\left(C_{12} + 2\,C_{66}\right) + m^4 C_{22}\\
\overline{C}_{23} &= n^2 C_{13} + m^2 C_{23}\\
\overline{C}_{26} &= nm\left[n^2\left(C_{11} - C_{12} - 2\,C_{66}\right) + m^2\left(C_{12} - C_{22} + 2C_{66}\right)\right] \qquad (3.76)\\
\overline{C}_{33} &= C_{33}\\
\overline{C}_{36} &= mn\left(C_{13} - C_{23}\right)\\
\overline{C}_{44} &= m^2 C_{44} + n^2 C_{55}\\
\overline{C}_{45} &= mn\left(C_{55} - C_{44}\right)\\
\overline{C}_{55} &= n^2 C_{44} + m^2 C_{55}\\
\overline{C}_{66} &= n^2 m^2\left(C_{11} - 2\,C_{12} + C_{22}\right) + \left(n^2 - m^2\right)^2 C_{66}
\end{aligned}$$

The constitutive equation $\left\{\sigma\right\}_{xyz} = \left[\overline{C}\right]\left\{\varepsilon\right\}_{xyz}$ becomes

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{C}_{11} & \overline{C}_{12} & \overline{C}_{13} & 0 & \overline{C}_{16} \\ \overline{C}_{12} & \overline{C}_{22} & \overline{C}_{23} & 0 & \overline{C}_{26} \\ \overline{C}_{13} & \overline{C}_{23} & \overline{C}_{33} & 0 & \overline{C}_{36} \\ 0 & 0 & 0 & \overline{C}_{44} & \overline{C}_{45} \\ 0 & 0 & 0 & \overline{C}_{45} & \overline{C}_{55} \\ \overline{C}_{16} & \overline{C}_{26} & \overline{C}_{36} & 0 & \overline{C}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}$$

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Compliance Transformation:

We are going to follow a procedure to transform compliance matrix similar to one used for transformation of a stiffness matrix. We have the constitutive equation in principal material direction as in Equation (3.70). We can write this in inverted form as

$$\{\varepsilon\}_{123} = [S]\{\sigma\}_{123} \tag{3.77}$$

using Equation (3.63) and Equation (3.68) we get

$$[T_2]\{\varepsilon\}_{xyz} = [S][T_1]\{\sigma\}_{xyz} \tag{3.78}$$

Pre-multiplying both sides by $[T_2]^{-1}$, we get

$$\begin{aligned} \{\varepsilon\}_{xyz} &= [T_2]^{-1}[S][T_1]\{\sigma\}_{xyz} \\ \{\varepsilon\}_{xyz} &= [\bar{S}]\{\sigma\}_{xyz} \end{aligned} \tag{3.79}$$

where, we define the transformed compliance matrix $[\bar{S}]$ as

$$[\bar{S}] = [T_2]^{-1}\,[S]\,[T_1] \tag{3.80}$$

Alternately, we can find $[\bar{S}]$ by inverting the transformed stiffness matrix $[\bar{C}]$. Thus, inverting $[\bar{C}]$ from Equation (3.73), we get

$$\begin{aligned} [\bar{S}] &= [\bar{C}]^{-1} = \left([T_1]^{-1}[C][T_2]\right)^{-1} \\ &= [T_2]^{-1}[C]^{-1}\left([T_1]^{-1}\right)^{-1} \\ &= [T_2]^{-1}[S]\,[T_1] \end{aligned}$$

After carrying out the calculation for $[\bar{S}]$, it is easy to give its form as follows

$$[\bar{S}] = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{13} & 0 & 0 & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{23} & 0 & 0 & \bar{S}_{26} \\ \bar{S}_{13} & \bar{S}_{23} & \bar{S}_{33} & 0 & 0 & \bar{S}_{36} \\ 0 & 0 & 0 & \bar{S}_{44} & \bar{S}_{45} & 0 \\ 0 & 0 & 0 & \bar{S}_{45} & \bar{S}_{55} & 0 \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{36} & 0 & 0 & \bar{S}_{66} \end{bmatrix} \tag{3.81}$$

Note that $[\bar{S}]$ has the same symmetric form as the transformed stiffness matrix.

The individual terms of the compliance matrix are obtained by carrying out multiplication of matrices as in Equation (3.82) and are given below.

$$
\begin{aligned}
\bar{S}_{11} &= m^4 S_{11} + m^2 n^2 \left(2\, S_{12} + S_{66}\right) + n^4 S_{22} \\
\bar{S}_{12} &= n^2 m^2 \left(S_{11} + S_{22} - S_{66}\right) + \left(n^4 + m^4\right) S_{12} \\
\bar{S}_{13} &= m^2 S_{13} + n^2 S_{23} \\
\bar{S}_{16} &= nm \left[m^2 \left(2\, S_{11} - 2\, S_{12} - S_{66}\right) + n^2 \left(2\, S_{12} - 2\, S_{22} + S_{66}\right)\right] \\
\bar{S}_{22} &= n^4 S_{11} + m^2 n^2 \left(2\, S_{12} + S_{66}\right) + m^4 S_{22} \\
\bar{S}_{23} &= n^2 S_{13} + m^2 S_{23} \\
\bar{S}_{26} &= nm \left[n^2 \left(2\, S_{11} - 2\, S_{12} - S_{66}\right) + m^2 \left(2\, S_{12} - 2 S_{22} + S_{66}\right)\right] \quad (3.82) \\
\bar{S}_{33} &= S_{33} \\
\bar{S}_{36} &= 2\, mn \left(S_{13} - S_{23}\right) \\
\bar{S}_{44} &= m^2 S_{44} + n^2 S_{55} \\
\bar{S}_{45} &= mn \left(S_{55} - S_{44}\right) \\
\bar{S}_{55} &= n^2 S_{44} + m^2 S_{55} \\
\bar{S}_{66} &= 4\, n^2 m^2 \left(S_{11} - 2 S_{12} + S_{22}\right) + \left(n^2 - m^2\right) S_{66}
\end{aligned}
$$

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Thermal Effects:

Thermal effects (effects due to change in temperature) are very important in composite materials for various reasons. The analysis of composites with thermal effects and effective thermal properties of the composite are two of the main reasons.

Important issues from analysis point of view:

1. The composite materials are used in environment where thermal gradients are unavoidable. For example, the helicopter containing composite fuselage operates at -50° C during winter at Leh and same helicopter can operate at +50° C during summer in the desert of Rajasthan. Thus, the effect of temperature gradient on the service performance of the composite is very important. In such service conditions, the layers of composite material tend to expand or contract but are restricted due to adjacent layers. Thus, it induces thermal stresses.

2. Most of the fabrication processes of polymer matrix composites have thermal cycles for matrix curing. A typical cycle involves raising the temperature to a certain level and holding it there for specified time and bringing it back to room temperature. It is well known that the fibre and matrix materials have different coefficients of thermal expansion (defined below). This mismatch produces residual thermal stresses because the fibres and matrix material are constrained in a composite.

Important issues from effective thermal properties point of view:

The second reason for the study of thermal effects is the effective properties of the composite materials.

1. Finding effective thermal properties of the composite theoretically to get an estimate requires sophisticated mathematical modeling when one considers:
    a. Difference in coefficients of thermal expansion of fibre and matrix materials

    b. The direction dependence of coefficients of thermal expansion in these materials

    c. Curing cycle temperature variations. This point is important because for some of the materials the coefficient of thermal expansion changes with temperature.

2. Finding the effective thermal properties for lamina in global direction with oriented fibres as shown in Figure 3.9 requires a special attention.

Further, finding these effective properties by laboratory test is also a challenge. Thus, for the various reasons mentioned above the study of thermal effect is very important. In the following, we develop a systematic way to handle effective thermal properties of a lamina along global directions.

It is well known that when a material is subjected to thermal gradient, it undergoes a deformation. The strain due to thermal changes is called thermal strain (denoted by superscript (*T*)). In general, the thermal strain is proportional to the temperature change ΔT. The constant of proportionality is called coefficient of thermal expansion. Thus, we can write the thermal strains in principal material directions for an orthotropic material as

$$\{\varepsilon^{(T)}\}_{123} = \{\alpha\}_{123}\,\Delta T \tag{3.83}$$

where $\{\alpha\}_{123} = \{\alpha_1, \alpha_2, \alpha_3, 0, 0, 0\}^T$ denote the coefficients of thermal expansion in principal material directions. It should be noted that for an orthotropic material in principal directions there are no shear strains due to thermal effects like in an isotropic material. For an isotropic material the coefficient of thermal expansion is same in any direction. However, for an orthotropic material $\alpha_1 >> \alpha_2, \alpha_3$. The thermal expansion of an elemental cube in principal directions for an isotropic and orthotropic material is shown in Figure 3.10.

These thermal strains will not produce stresses unless these are constrained. The thermal strains which do not produce stresses are known as free thermal strains. However, in case of composites the fibres and matrix are constrained in a lamina and layers are constrained in a laminate. Thus, in composite the thermal strains produce the thermal stresses.

The thermal strains are given in principal material directions as given in Equation (3.83). Let us consider that we need to find these strains in a global coordinate system (refer Figure 3.9). We need to transform them from 123 coordinate system to *xyz* coordinate system by a rotation θ about 3-axis. Thus, similar to Equation (3.68), we can write

$$\begin{aligned}\{\varepsilon^{(T)}\}_{123} &= [T_2]\{\varepsilon^{(T)}\}_{xyz}\\ \{\varepsilon^{(T)}\}_{xyz} &= [T_2]^{-1}\{\varepsilon^{(T)}\}_{123}\end{aligned} \tag{3.84}$$

Substituting Equation (3.83) in the above equation,

$$\begin{aligned}\{\varepsilon^{(T)}\}_{xyz} &= [T_2]^{-1}\{\alpha\}_{123}\,\Delta T\\ &= \{\alpha\}_{xyz}\,\Delta T\end{aligned} \tag{3.85}$$

where

$$\{\alpha\}_{xyz} = [T_2]^{-1}\{\alpha\}_{123} \quad \{\alpha\}_{xyz} = \{\alpha_{xx}\ \alpha_{yy}\ \alpha_{zz}\ 0\ 0\ \alpha_{xy}\}^T \text{ and } \{\alpha\}_{123} = \{\alpha_1\ \alpha_2\ \alpha_3\ 0\ 0\ 0\}^T \tag{3.86}$$

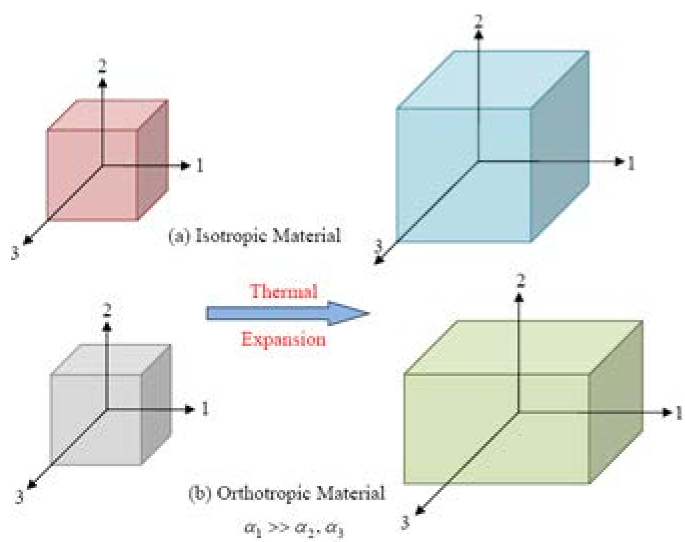

Figure 3.10: Thermal expansion in an isotropic and

orthotropic material

On substitution of $[T_2]^{-1}$ in the above equation, we get the following individual terms of coefficients thermal expansion in *xyz* directions.

$$\begin{aligned}
\alpha_{xx} &= m^2\alpha_1 + n^2\alpha_2 \\
\alpha_{yy} &= n^2\alpha_1 + m^2\alpha_2 \\
\alpha_{zz} &= \alpha_3 \\
\alpha_{xy} &= 2mn\left(\alpha_1 - \alpha_2\right)
\end{aligned} \tag{3.87}$$

Using Equation (3.87) in Equation (3.85), the engineering thermal strains in global coordinates are given as

$$\begin{Bmatrix} \varepsilon_{xx}^{(T)} \\ \varepsilon_{yy}^{(T)} \\ \varepsilon_{zz}^{(T)} \\ \gamma_{yz}^{(T)} \\ \gamma_{xz}^{(T)} \\ \gamma_{xy}^{(T)} \end{Bmatrix} = \begin{Bmatrix} m^2\alpha_1 + n^2\alpha_2 \\ n^2\alpha_1 + m^2\alpha_2 \\ \alpha_3 \\ 0 \\ 0 \\ 2mn\left(\alpha_1 - \alpha_2\right) \end{Bmatrix} \Delta T \tag{3.88}$$

Thus, from this equation it should be noted that the transformation of thermal strains in global coordinates gives normal strain components and a shear strain component in *xy* plane for an orthotropic material with $\alpha_1 \neq \alpha_2$ and fiber orientations other than $0°$ and $90°$.

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Thermo-Elastic Constitutive Equation:

Let us assume that the total strain, $\{\varepsilon\}$ in a composite is a superposition of the free thermal strain $\{\varepsilon^{(T)}\}$ and the strain due to mechanical loads (also known as mechanical strains) $\{\varepsilon^{(\sigma)}\}$. Thus,

$$\{\varepsilon\}_{123} = \{\varepsilon^{(\sigma)}\}_{123} + \{\varepsilon^{(T)}\}_{123}$$

Now, for mechanical strains we use the constitutive equation as

$$\{\varepsilon^{(\sigma)}\}_{123} = [S]\{\sigma\}_{123}$$

Thus, we can write the total thermo-elastic strain as

$$\{\varepsilon\}_{123} = [S]\{\sigma\}_{123} + \{\varepsilon^{(T)}\}_{123} \qquad (3.89)$$

Equation (3.89) can be written as

$$[S]\{\sigma\}_{123} = \{\varepsilon\}_{123} - \{\varepsilon^{(T)}\}_{123}$$

Premultiplying the above expression by $[S]^{-1}$, we get the stresses as

$$\begin{aligned}\{\sigma\}_{123} &= [S]^{-1}\left(\{\varepsilon\}_{123} - \{\varepsilon^{(T)}\}_{123}\right) \\ \{\sigma\}_{123} &= [C]\left(\{\varepsilon\}_{123} - \{\varepsilon^{(T)}\}_{123}\right)\end{aligned} \qquad (3.90)$$

Equation (3.90) is the basic constitutive equation for thermo-elastic stress analysis.

Using Equation (3.90) and similar to Equation (3.72) we can find the stresses due to thermo-elastic effects in global directions as,

$$\{\sigma\}_{xyz} = [\bar{C}]\left(\{\varepsilon\}_{xyz} - \{\varepsilon^{(T)}\}_{xyz}\right) \qquad (3.91)$$

where

$$\{\varepsilon\}_{xyz} - \{\varepsilon^{(T)}\}_{xyz} = \left\{\varepsilon_{xx} - \varepsilon_{xx}^{(T)} \quad \varepsilon_{yy} - \varepsilon_{yy}^{(T)} \quad \varepsilon_{zz} - \varepsilon_{zz}^{(T)} \quad \gamma_{yz} \quad \gamma_{xz} \quad \gamma_{xy} - \gamma_{xy}^{(T)}\right\}^T$$

Equation (3.91) is inverted to give the total strains in terms of the mechanical and free thermal strains as

$$\{\varepsilon\}_{xyz} = [\bar{S}]\{\sigma\}_{xyz} + \{\varepsilon^{(T)}\}_{xyz} \qquad (3.92)$$

Effect of Moisture:

The polymer matrix composite materials, during their service can absorb moisture from the environment. The effect of absorption of moisture is to degrade the various material properties of the composite. Further, this results in an expansion. It is called hygroscopic expansion. However, this expansion is again constrained as in thermal expansion. Hence, when dealing with the hygroscopic expansions, a treatment similar to thermal expansion is used.

The hygroscopic strains $\left\{\varepsilon^{(H)}\right\}$ are assumed to be proportional to the percentage moisture absorbed, ΔM. This percentage is measured in terms of weight of the moisture. The constant of proportionality, $\{\beta\}$ is the coefficient of hygroscopic expansion.

Thus, in principal coordinates the hygroscopic strains are

$$\left\{\varepsilon^{(H)}\right\}_{123} = \{\beta\}_{123} \Delta M \tag{3.93}$$

where

$$\{\beta\}_{123} = \{\beta_1\ \beta_2\ \beta_3\ 0\ 0\ 0\}^T \tag{3.94}$$

denotes the coefficients of hygroscopic expansion in principal material directions. Following a similar procedure for thermal strains, we can write strains due to hygroscopic expansion in *xyz* coordinates as

$$\left\{\varepsilon^{(H)}\right\}_{xyz} = \{\beta\}_{xyz} \Delta M \tag{3.95}$$

We can write $\{\beta\}_{xyz} = \left\{\beta_{xx}\ \beta_{yy}\ \beta_{zz}\ 0\ 0\ \beta_{xy}\right\}^T$ using values of in 123 directions as $\{\beta\}_{xyz} = [T_2]^{-1}\{\beta\}_{123}$.

Thus, comparing Equation (3.85) and Equation (3.95), it is easy to conclude that the coefficients of hygroscopic expansion will vary similar to the coefficients of thermal expansion as a function of orientation of fibres.

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Hygro-Thermo-Elastic Constitutive Equation:

This is the most general formulation for the mechanical, thermal and hygral effects on stress analysis in composites. Here, we superimpose the strains due to these three effects to give us the total strain as

$$\{\varepsilon\}_{123} = \left\{\varepsilon^{(\sigma)}\right\}_{123} + \left\{\varepsilon^{(T)}\right\}_{123} + \left\{\varepsilon^{(H)}\right\}_{123} \tag{3.96}$$

Using constitutive equation for mechanical strains, we get

$$\{\varepsilon\}_{123} = [S]\{\sigma\}_{123} + \left\{\varepsilon^{(T)}\right\}_{123} + \left\{\varepsilon^{(H)}\right\}_{123} \tag{3.97}$$

The stresses in the composite can be given as

$$\{\sigma\}_{123} = [C]\left(\{\varepsilon\}_{123} - \left\{\varepsilon^{(T)}\right\}_{123} - \left\{\varepsilon^{(H)}\right\}_{123}\right) \tag{3.98}$$

These stresses in global coordinates *xyz* can be written as

$$\{\sigma\}_{xyz} = \left[\bar{C}\right]\left(\{\varepsilon\}_{xyz} - \left\{\varepsilon^{(T)}\right\}_{xyz} - \left\{\varepsilon^{(H)}\right\}_{xyz}\right) \tag{3.99}$$

where

$$\{\varepsilon\}_{xyz} - \left\{\varepsilon^{(T)}\right\}_{xyz} - \left\{\varepsilon^{(H)}\right\}_{xyz} = \begin{Bmatrix} \varepsilon_{xx} - \varepsilon_{xx}^{(T)} - \varepsilon_{xx}^{(H)} \\ \varepsilon_{yy} - \varepsilon_{yy}^{(T)} - \varepsilon_{yy}^{(H)} \\ \varepsilon_{zz} - \varepsilon_{zz}^{(T)} - \varepsilon_{zz}^{(H)} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} - \gamma_{xy}^{(T)} - \gamma_{xy}^{(H)} \end{Bmatrix} \tag{3.100}$$

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Examples:

Example 2: Transform the stiffness and compliance matrix of Example 1 about axis 3 by an angle of
θ = 30°.

Solution:

Approach 1: One can find the transformation matrices $[T_1]$ and $[T_2]$ and do the matrix multiplication as given in Equation (3.73) for transformed stiffness matrix and then inverse this matrix or do the matrix multiplication as given in Equation (3.80) to get the transformed compliance matrix. The use of Equation (3.73) and Equation (3.80) is suggested because remembering $[T_1]$ and $[T_2]$ is not so difficult. Further, their inverse can be easily found with the help of Equation (3.74).
For

$$\theta = 30^\circ, m = \cos 30^\circ = 0.86603, n = \sin 30^\circ = 0.5$$

Thus

$$[T_1] = \begin{bmatrix} 0.75 & 0.25 & 0 & 0 & 0 & 0.86603 \\ 0.25 & 0.75 & 0 & 0 & 0 & -0.86603 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.86603 & -0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.86603 & 0 \\ -0.43301 & 0.43301 & 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$$[T_2] = \begin{bmatrix} 0.75 & 0.25 & 0 & 0 & 0 & 0.43301 \\ 0.25 & 0.75 & 0 & 0 & 0 & -0.43301 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.86603 & -0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.86603 & 0 \\ -086603 & 0.86603 & 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$$[T_1]^{-1} = \begin{bmatrix} 0.75 & 0.25 & 0 & 0 & 0 & -0.86603 \\ 0.25 & 0.75 & 0 & 0 & 0 & 0.86603 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.86603 & 0.5 & 0 \\ 0 & 0 & 0 & -0.5 & 0.86603 & 0 \\ 0.43301 & -0.43301 & 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$$[T_2]^{-1} = \begin{bmatrix} 0.75 & 0.25 & 0 & 0 & 0 & -0.43301 \\ 0.25 & 0.75 & 0 & 0 & 0 & 0.43301 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.86603 & 0.5 & 0 \\ 0 & 0 & 0 & -0.5 & 0.86603 & 0 \\ 086603 & -0.86603 & 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$$[\overline{C}] = [T_1]^{-1}[C][T_2] = \begin{bmatrix} 80.282 & 25.005 & 5.303 & 0 & 0 & 36.439 \\ & 22.465 & 5.402 & 0 & 0 & 13.632 \\ & & 13.309 & 0 & 0 & -0.086 \\ & & & 4.539 & 1.157 & 0 \\ & Symmetric & & & 5.932 & 0 \\ & & & & & 26.352 \end{bmatrix}$$

Unit of all transformed stiffness coefficients is *GPa*.

$$[\overline{S}] = [T_2]^{-1}[S][T_1] = \begin{bmatrix} 0.03772 & -0.01126 & -0.01075 & 0 & 0 & -0.04636 \\ & 0.07921 & -0.02782 & 0 & 0 & -0.02548 \\ & & 0.09091 & 0 & 0 & 0.02956 \\ & & & 0.22878 & -0.04462 & 0 \\ & symmetric & & & 0.17727 & 0 \\ & & & & & 0.11534 \end{bmatrix}$$

Unit of all transformed compliance coefficients is *1/GPa*.

***Approach 2*:** You can write the expanded form for transformed stiffness and compliance coefficients in Equation (3.76) and Equation (3.82). However, the readers are suggested to use this approach only when they are confident of remembering these terms.

***Example 3*:** The coefficients of moisture absorption for T300/5208 composite material are

$\beta_1 = 0.0 / wt\%$, $\beta_2 = \beta_3 = 6.67 \times 10^{-3} / wt\%$. Plot the variation these coefficients between $-90^\circ < \theta < 90^\circ$.

Solution:

We have the expression for variation of the coefficients of moisture absorption as

$$\beta_{xx} = m^2 \beta_1 + n^2 \beta_2$$
$$\beta_{yy} = n^2 \beta_1 + m^2 \beta_2$$
$$\beta_{zz} = \beta_3$$
$$\beta_{xy} = 2mn(\beta_1 - \beta_2)$$

where, $m = \cos\theta$ and $n = \sin\theta$. We plot the above variation using a computer code. The final plot is shown in Figure 3.11.

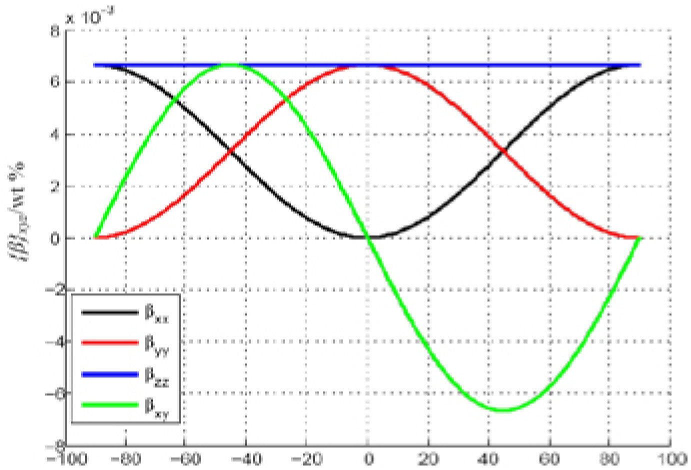

Figure 3.11: Variation of coefficients of moisture expansion with orientation of fibres

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Homework:

1. Verify the result given in Equation (3.74).

2. Using the invariance property of strain energy density function, show that

$$[T_1]^{-1} = [T_2]^T \text{ and } [T_2]^{-1} = [T_1]^T$$

3. Obtain the individual terms of transformed stiffness and compliance matrices using Equation (3.73) and Equation (3.80), respectively and verify it with Equation (3.76) and Equation (3.82), respectively.

4. Obtain the strain transformation matrix $[T_2]$ using tensorial shear strains. Further, using this transformation matrix obtain the transformed stiffness and compliance matrix in the form similar to Equation (3.75) and Equation (3.81). Compare the new matrices and comment on the observations with justifications.

5. Calculate the stiffness and compliance coefficients for following transversely isotropic materials given in Table 3.1.

Table 3.1: Properties for unidirectional transversely isotropic lamina

Property\Material	T300/BSL914C Epoxy	E-glass/LY556/ HT907/DY063 Epoxy	S-glass/MY750/ HY917/DY063 Epoxy
E_1 (GPa)	138	53.48	45.6
$E_2=E_3$(GPa)	11	17.7	16.2
$G_{12} = G_{13}$(GPa)	5.5	5.83	5.83
$\nu_{12}= \nu_{13}$	0.28	0.278	0.278
ν_{23}	0.4	0.4	0.4
α_1 (×10^{-6}/°C)	-1	8.6	8.6
$\alpha_2 = \alpha_3$ (×10^{-6}/°C)	26	26.4	26.4

6. The stiffness matrix for an orthotropic material is given as

$$[C]=\begin{bmatrix} 141.3602 & 3.6453 & 3.6453 & 0 & 0 & 0 \\ 3.6453 & 10.2763 & 4.1029 & 0 & 0 & 0 \\ 3.6453 & 4.1029 & 10.2763 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4.082 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.082 \end{bmatrix} GPa$$

Find:

a. The compliance matrix
b. The engineering constants $E_1, E_2, E_3, G_{12}, G_{31}, G_{23}, \nu_{12}, \nu_{31}\ \nu_{23}$.

7. Why are the thermal effects important in composite materials? Explain in detail.

8. Plot variation of $\alpha_x, \alpha_y, \alpha_z$ and α_{xy} between $-90^\circ < \theta < 90^\circ$ for T300/BSL914C Epoxy and E-glass/LY556/ HT907/DY063 Epoxy. See Table 3.1 for the required thermal properties.

9. Why are the hygral effects important in composite materials? Explain.

10. Search literature to get the coefficients of moisture absorption for at least two composite materials and plot its variation between $-90^\circ < \theta < 90^\circ$

11. Write a computer code to read the properties of a transversely isotropic material and calculate all the terms of stiffness and compliance matrix. Verify your results with the results given in Example 1. Then use this code to get the stiffness and compliance matrices of T300/BSL914C Epoxy and S-glass/MY750/ HY917/DY063 Epoxy.

12. Add another module to the code written for above problem to calculate the transformed stiffness and compliance matrices. Plot all the coefficients between $-90^\circ < \theta < 90^\circ$. Compare the corresponding terms of these materials and comment.

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