Module 7: Micromechanics

Lecture 29: Background of Concentric Cylinder Assemblage Model

Introduction

In this lecture we are going to introduce a new micromechanics model to determine the fibrous composite effective properties in terms of properties of its individual phases. In this model a composite is represented as an assemblage of concentric cylinders. The core of this cylinder is a fibre and surrounding annulus is a matrix material. This model is called ***concentric cylinder assemblage (CCA) model***.

In this lecture we give the introduction and back ground of this model.

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Concentric Cylinder Assemblage (CCA) Model

As we know, the unidirectional fibrous composite has fibres embedded in matrix material. The fibres are, in general, cylindrical in nature. Thus, Hashin and Rosen [1] introduced a micromechanical model in which the composite is represented as an assemblage of concentric cylinders. The inner cylinder represents the fiber and outer annulus is matrix. The fibres are considered to be infinitely long cylinders and matrix is considered to be continuous.

The model is shown in Figure 7.10(a). For each individual fibre of radius a, there is associated an annulus of matrix material of inner radius a and outer radius b . The individual cylinder, thus formed, is called as a composite cylinder. This is shown in Figure 7.10 (b). It should be noted that the all cylinders do not have the same radii. The radii of each cylinder vary in a way such that they completely fill the composite volume. However, the ratio of the fiber cylinder to the outer radius of matrix cylinder is same for all cylinders. This leads to the fact that all composite cylinders have the same volume fractions. Further, the resulting material is transversely isotropic in nature.

The advantage of this model is that analysis of one composite cylinder is sufficient to determine the four out of five effective elastic moduli of a transversely isotropic material.

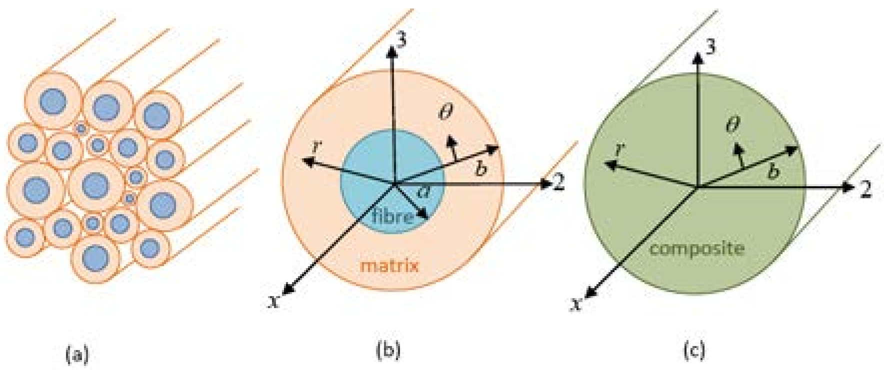

Figure 7.10: (a) Concentric cylinder assemblage model (b) composite cylinder and (c) homogeneous single cylinder

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Background of CCA Model

Here, we will relate five effective independent stiffness coefficients with the measurable effective engineering constants. In the following, for a transversely isotropic material we derive relations between the stiffness coefficients and engineering constants.

The unidirectional fibrous composite is transversely isotropic in nature in plane perpendicular to fibre direction (or in plane perpendicular to the plane in which fibres are placed together). The stress strain relations for the transversely isotropic material in *23* plane is written as

$$\begin{aligned}
\sigma_{11} &= C_{11}^*\varepsilon_{11} + C_{12}^*\varepsilon_{22} + C_{12}^*\varepsilon_{33}\\
\sigma_{22} &= C_{12}^*\varepsilon_{11} + C_{22}^*\varepsilon_{22} + C_{23}^*\varepsilon_{33}\\
\sigma_{33} &= C_{12}^*\varepsilon_{11} + C_{23}^*\varepsilon_{22} + C_{22}^*\varepsilon_{33}\\
\sigma_{33} &= (C_{22}^* - C_{23}^*)\varepsilon_{23}\\
\sigma_{13} &= 2C_{66}^*\varepsilon_{13}\\
\sigma_{12} &= 2C_{66}^*\varepsilon_{12}
\end{aligned} \tag{7.148}$$

Thus, there are five constants, $C_{11}^*, C_{12}^*, C_{22}^*, C_{23}^*$ and C_{66}^* are independent constants. These constants define the effective properties of the composite. Note that in above relations tensorial shear strains are used. The relation between these independent constants and effective engineering constants can be given as follows:

Consider an uniaxial stress state such that $\sigma_{11} \neq 0$ and $\sigma_{22} = \sigma_{33} = \sigma_{23} = \sigma_{13} = \sigma_{12} = 0$.

. Putting this in Equation (7.148) we can solve for the normal strains. The normal strains in terms of $C_{11}^*, C_{12}^*, C_{22}^*, C_{23}^*$ and σ_{11} are given as

$$\begin{aligned}
\varepsilon_{11} &= \frac{C_{22}^* + C_{23}^*}{C_{11}^*C_{22}^* - 2(C_{12}^*)^2 + C_{11}^*C_{23}^*}\sigma_{11}\\
\varepsilon_{22} &= \frac{C_{22}^* + C_{23}^*}{C_{11}^*C_{22}^* - 2(C_{12}^*)^2 + C_{11}^*C_{23}^*}\sigma_{11}\\
\varepsilon_{33} &= \frac{C_{22}^* + C_{23}^*}{C_{11}^*C_{22}^* - 2(C_{12}^*)^2 + C_{11}^*C_{23}^*}\sigma_{11}
\end{aligned} \tag{7.149}$$

From the first of the above equation, we can write

$$\sigma_{11} = \left(C_{11}^* - \frac{2(C_{12}^*)^2}{C_{22}^* + C_{23}^*}\right)\varepsilon_{11} \tag{7.150}$$

We define the effective axial modulus E_1^* through following equation as

$$\sigma_{11} = E_1^*\varepsilon_{11} \tag{7.151}$$

Comparing this equation with Equation (7.150) we can write for E_1^* as

$$E_1^* = C_{11}^* - \frac{2(C_{12}^*)^2}{C_{22}^* + C_{23}^*} \tag{7.152}$$

Similarly, we define the following the Poisson's ratios v_{12}^* and v_{13}^* as

$$v_{12}^* = -\frac{\varepsilon_{22}}{\varepsilon_{22}} \text{ and } v_{13}^* = -\frac{\varepsilon_{33}}{\varepsilon_{11}} \quad (7.153)$$

Thus, using Equation (7.149), we get

$$v_{12}^* = v_{13}^* - \frac{C_{12}^*}{C_{22}^* + C_{23}^*} \quad (7.154)$$

The other engineering constants that can be directly related to the effective stiffness coefficients are

$$G_{12}^* = G_{13}^* = G_{66}^* \text{ and } G_{23}^* = \frac{C_{22}^* + C_{23}^*}{2} \quad (7.155)$$

Equations (7.152), (7.154) and (7.155) are four equations with five effective stiffness constants. We need one more equation in effective stiffness constants that relates an effective engineering constant. Then we can solve for $C_{11}^*, C_{12}^*, C_{22}^*, C_{23}^*$ and C_{66}^* in terms of effective engineering constants.

We develop this equation as follows. Let us define the plane strain bulk modulus, K_{23}^* corresponding to the state of strain

$$\varepsilon_{11} = 0, \varepsilon_{22} = \varepsilon_{33} = \varepsilon \quad (7.156)$$

For this state of strain, from the constitutive equation in Equation (7.148) the normal stresses are given as

$$\begin{aligned} \sigma_{11} &= 2C_{12}^*\varepsilon \\ \sigma_{22} &= (C_{22}^* + C_{23}^*)\varepsilon = \sigma \\ \sigma_{33} &= (C_{22}^* + C_{23}^*)\varepsilon = \sigma \end{aligned} \quad (7.157)$$

Let us define $\sigma = 2K_{23}^*\varepsilon$ where

$$K_{23}^* = \frac{1}{2}(C_{22}^* + C_{23}^*) \quad (7.158)$$

is defined as effective plane strain bulk modulus.

Now, Equations (7.152), (7.154), (7.155) and (7.158) can be inverted to give

$$\begin{aligned} C_{11}^* &= E_1^* + 4(v_{12}^*)^2 K_{23}^* \\ C_{12}^* &= 2K_{23}^* v_{12}^* \\ C_{22}^* &= G_{23}^* + K_{23}^* \\ C_{23}^* &= -G_{23}^* + K_{23}^* \\ C_{66}^* &= G_{12}^* \end{aligned} \quad (7.159)$$

In the above exercise, the measurable properties are and . However, one can

$E_1^*, v_{12}^*, K_{23}^*, G_{12}^*$ G_{23}^*

measure the other engineering properties and express the effective stiffness coefficients. Let us consider that a uniaxial tension normal to the fibre direction is applied such that $\sigma_{22} \neq 0$ and $\sigma_{11} = \sigma_{33} = \sigma_{23} = \sigma_{13} = \sigma_{12} = 0$. Putting this in Equation (7.148) we can solve for the normal strains. The normal strains in terms of $C_{11}^*, C_{12}^*, C_{22}^*, C_{23}^*$ and σ_{22} are given as

$$\begin{aligned} \varepsilon_{11} &= \frac{C_{12}^*}{-2(C_{12}^*)^2 + C_{11}^* C_{22}^* + C_{11}^* C_{23}^*}\sigma_{22} \\ \varepsilon_{22} &= \frac{-(C_{12}^*)^2 + C_{11}^* C_{22}^*}{(C_{22}^* - C_{23}^*) + [-2(C_{12}^*)^2 + C_{11}^* C_{22}^* + C_{11}^* C_{23}^*]}\sigma_{22} \\ \varepsilon_{33} &= \frac{-(C_{12}^*)^2 + C_{11}^* C_{22}^*}{(-C_{22}^* + C_{23}^*) + [-2(C_{12}^*)^2 + C_{11}^* C_{22}^* + C_{11}^* C_{23}^*]}\sigma_{22} \end{aligned} \qquad (7.160)$$

From the second of the above equation, we can write

$$E_2^* = \frac{\sigma_{22}}{\varepsilon_{22}} = C_{22}^* + \frac{(C_{22}^*)^2(-C_{22}^* + C_{23}^*) + C_{23}^*[-C_{11}^* C_{23}^* + (C_{22}^*)^2]}{C_{11}^* C_{22}^* - (C_{22}^*)^2} \qquad (7.161)$$

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Now we define the effective Poisson's ratios v_{21}^* and v_{23}^* as

$$v_{21}^* = -\frac{\varepsilon_{11}}{\varepsilon_{22}} \text{ and } v_{23}^* = -\frac{\varepsilon_{33}}{\varepsilon_{22}} \tag{7.162}$$

Thus, using Equation (7.149), we get

$$v_{21}^* = \frac{C_{12}^*(C_{22}^* - C_{23}^*)}{C_{11}^* C_{22}^* - (C_{12}^*)^2} \tag{7.163}$$

and

$$v_{23}^* = \frac{C_{11}^* C_{23}^* - (C_{12}^*)^2}{C_{11}^* C_{22}^* - (C_{12}^*)^2} \tag{7.164}$$

Further, from the symmetry consideration we have the following relations

$$v_{31}^* = v_{21}^* \text{ and } v_{32}^* = v_{23}^* \tag{7.165}$$

and from the reciprocal relations

$$\frac{v_{12}^*}{E_1^*} = \frac{v_{21}^*}{E_2^*} \tag{7.166}$$

Thus, we can write the following useful relations as

$$E_2^* = \frac{4G_{23}^* K_{23}^*}{K_{23}^* + G_{23}^* + \dfrac{4(v_{12}^*)^2 G_{23}^* K_{23}^*}{E_1^*}}, \qquad v_{23}^* = \frac{K_{23}^* - G_{23}^* - \dfrac{4(v_{12}^*)^2 G_{23}^* K_{23}^*}{E_1^*}}{K_{23}^* + G_{23}^* + 4(v_{12}^*)^2 G_{23}^* K_{23}^*}$$
$$v_{21}^* = \frac{4(v_{12}^*)^2 G_{23}^* K_{23}^*}{E_1^*(K_{23}^* + G_{23}^*) + 4(v_{12}^*)^2 G_{23}^* K_{23}^*} \qquad (v_{12}^*)^2 = \left(-v_{23}^* - \frac{1}{4}\frac{E_2^*}{K_{23}^*} + \frac{1}{4}\frac{E_2^*}{G_{23}^*}\right)\frac{E_1^*}{E_2^*} \tag{7.167}$$

Thus, if we know the effective engineering constants $E_1^*, v_{12}^*, K_{23}^*, G_{12}^*$ and G_{23}^*, then we can find the effective stiffness coefficients for transversely isotropic material. Further, the remaining engineering constants can also be determined.

The effective stiffness can be determined from the composite cylinder either using the concept of equivalence of strain energy or the basic definitions for each of the engineering property. For the equivalence of strain energy approach the strain energy of the composite cylinder must be equal to the strain energy of the single homogeneous cylinder. The strain energy in a given volume of the cylinder is given as

$$U = \int_V (\sigma_{xx}\varepsilon_{xx} + \sigma_{\theta\theta}\varepsilon_{\theta\theta} + \sigma_{rr}\varepsilon_{rr} + \tau_{\theta r}\varepsilon_{\theta r} + \tau_{xr}\varepsilon_{xr} + \tau_{x\theta}\varepsilon_{x\theta})dV \tag{7.168}$$

In the case of a composite cylinder, the integral over volume extends over the volume of the core cylinder plus the volume of the annulus and in case of homogeneous cylinder it is over the one cylinder. It should be noted that both cylinder systems are subjected to same deformations. This approach can be complicated in the case of concentric cylinders as it involves both the geometry and elastic properties of two cylinders.

In the following sections we derive the effective engineering constants using the CCA model and basic definitions of engineering properties as follows. In this, the concentric cylinders are axially loaded. The composite axial modulus can be defined as axial force divided by axial strain produced by this axial force. Further, in this loading the assumption is that no other stresses are applied and the cylinders are free to deform.

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Analysis of Concentric Cylinders

When the concentric cylinders are subjected to either an axial load or a uniform radial stress then there are no shear stresses produced. Further, in an infinite cylinder away from the ends, the stresses are independent of x. For this state of stress, there will be one equilibrium equation given as

$$\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \tag{7.169}$$

where, σ_{rr} and $\sigma_{\theta\theta}$ are radial and circumferential stresses, respectively. For a cylindrical coordinate system with no shear stresses, we can write normal stresses for transversely isotropic material as

$$\begin{aligned} \sigma_{xx} &= C_{11}^{*}\varepsilon_{xx} + C_{12}^{*}\varepsilon_{\theta\theta} + C_{12}^{*}\varepsilon_{rr} \\ \sigma_{\theta\theta} &= C_{12}^{*}\varepsilon_{xx} + C_{22}^{*}\varepsilon_{\theta\theta} + C_{23}^{*}\varepsilon_{rr} \\ \sigma_{rr} &= C_{12}^{*}\varepsilon_{xx} + C_{23}^{*}\varepsilon_{\theta\theta} + C_{22}^{*}\varepsilon_{rr} \end{aligned} \tag{7.170}$$

Similarly, for transversely isotropic fibre and isotropic matrix the normal stresses are given as

$$\begin{aligned} \sigma_{xx}^{(f)} &= C_{11}^{f}\varepsilon_{xx} + C_{12}^{f}\varepsilon_{\theta\theta} + C_{12}^{f}\varepsilon_{rr}, & \sigma_{xx}^{(m)} &= C_{11}^{m}\varepsilon_{xx} + C_{12}^{m}\varepsilon_{\theta\theta} + C_{12}^{m}\varepsilon_{rr} \\ \sigma_{\theta\theta}^{(f)} &= C_{12}^{f}\varepsilon_{xx} + C_{22}^{f}\varepsilon_{\theta\theta} + C_{23}^{f}\varepsilon_{rr}, & \sigma_{\theta\theta}^{(m)} &= C_{12}^{m}\varepsilon_{xx} + C_{11}^{m}\varepsilon_{\theta\theta} + C_{12}^{m}\varepsilon_{rr} \\ \sigma_{rr}^{(f)} &= C_{12}^{f}\varepsilon_{xx} + C_{23}^{f}\varepsilon_{\theta\theta} + C_{22}^{f}\varepsilon_{rr}, & \sigma_{rr}^{(m)} &= C_{12}^{m}\varepsilon_{xx} + C_{12}^{m}\varepsilon_{\theta\theta} + C_{11}^{m}\varepsilon_{rr} \end{aligned} \tag{7.171}$$

Now, we write the strain displacement relations in cylindrical coordinates as

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} & \varepsilon_{\theta\theta} &= \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{w}{r}, & \varepsilon_{rr} &= \frac{\partial w}{\partial r} \\ \gamma_{\theta r} &= \frac{1}{r}\left(\frac{\partial w}{\partial \theta} - v + r\frac{\partial v}{\partial r}\right), & \gamma_{xr} &= \frac{\partial u}{\partial r} + \frac{\partial w}{\partial x}, & \gamma_{x\theta} &= \frac{\partial w}{\partial x} + \frac{1}{r}\frac{\partial u}{\partial \theta} \end{aligned} \tag{7.172}$$

where, u, v and w are axial, circumferential or tangential and radial displacements, respectively. Further, with no shear effects and axisymmetry the tangential displacement is zero. It should be noted that the stresses are not function of x. Hence, the strains are independent of x. Hence, the displacement w can be a function of r. However, u can be function of r and, at most, a linear function of x. Hence, in the expression for strain ε_{xx} there is partial derivative and not in the expression for ε_{rr}. For the expression of ε_{rr} we have the full derivative with respect to r .

Thus, the normal strains for the displacements in this case simplify to

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \varepsilon_{\theta\theta} = \frac{w}{r}, \varepsilon_{rr} = \frac{dw}{dr}$$

When these strain-displacement relations are used in stress-strain relations then the equilibrium equation in Equation (7.169) gives an ordinary differential equation in w as

$$\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}-\frac{w}{r^2}=0 \tag{7.173}$$

The solution to this equation is

$$w(r) = Ar + \frac{B}{r} \tag{7.174}$$

Thus, the solution for fibre is

$$w^{(f)}(r) = A^f r + \frac{B^f}{r} 0 \le r \le a \tag{7.175}$$

and the solution for the matrix is

$$w^{(m)}(r) = A^m r + \frac{B^m}{r} a \le r \le b \tag{7.176}$$

The axial displacement in fibre and matrix can be determined by integrating the first of Equation (7.172) with respect to x. In this equation the left hand side is independent of x.
Thus,

$$\begin{aligned} u^{(f)}(x,r) &= \varepsilon_{xx}^{f} x + g^f(r) \quad 0 \le r \le a \\ u^{(m)}(x,r) &= \varepsilon_{xx}^{m} x + g^m(r) \quad a \le r \le b \end{aligned} \tag{7.177}$$

It should be noted that ε_{xx}^{f} and ε_{xx}^{m} are constants and $g^f(r)$ and $g^m(r)$ are the arbitrary constants of integration. The constants $A^f, B^f, A^m, B^m, \varepsilon_{xx}^{f}$ and ε_{xx}^{m} and functions $g^f(r)$ and $g^m(r)$ are unknown. These can be determined from specific boundary conditions.

The w displacement in fibre as given in Equation (7.175) should be bounded when $r = 0$. This requires the condition that

$$B^f = 0 \tag{7.178}$$

Further, the displacements are continuous at the interface of fibre and matrix. This results in

$$\begin{aligned} w^{(f)}(a) &= w^{(m)}(a) \\ u^{(f)}(a) &= u^{(m)}(a) \end{aligned} \tag{7.179}$$

Using Equation (7.178) and Equation (7.179), the continuity conditions become

$$A^f a = A^m a + \frac{B^m}{a}$$
$$\varepsilon_{xx}^{(f)} x + g^f(r) = \varepsilon_{xx}^{(m)} x + g^m(r) \quad (7.180)$$

On equating the terms in x and r in the second of Equation (7.180), we get

$$\varepsilon_{xx}^{(f)} = \varepsilon_{xx}^{(m)} \varepsilon_{xx}$$
$$g^f(r) = g^m(r) = g(r) \quad (7.181)$$

This means that the strains in x-direction in fibre and matrix are given by same function of r. Further, the unknown constant ε_{xx} in fibre and matrix is same. This constant, in fact, is the axial strain. Therefore, it is denoted as ε_{xx}.

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The third continuity condition required is the continuity of the stress component normal to the interface between the fibre and matrix, that is,

$$\sigma_{rr}^{(f)}(a) = \sigma_{rr}^{(m)}(a) \quad (7.182)$$

Using, Equation (7.171) and the unknown constants in above condition, we get

$$C_{12}^{f}\varepsilon_{xx} + \left(C_{22}^{f} + C_{23}^{f}\right)A^{f} = C_{12}^{m}\varepsilon_{xx} + \left(C_{11}^{m} + C_{12}^{m}\right)A^{m} + \left(C_{12}^{m} - C_{11}^{m}\right)\frac{B^{m}}{a^{2}} \quad (7.183)$$

The unknown constants $A^{f}, A^{m}, B^{m}, \varepsilon_{xx}$ and $g^{f}(r)$

$$C_{12}^{f}\varepsilon_{xx} + \left(C_{22}^{f} + C_{23}^{f}\right)A^{f} = C_{12}^{m}\varepsilon_{xx} + \left(C_{11}^{m} + C_{12}^{m}\right)A^{m} + \left(C_{12}^{m} - C_{11}^{m}\right)\frac{B^{m}}{a^{2}} \quad (7.183)$$

can be determined from the first of Equation (7.180) and Equation (7.183) along with additional equations that will result due to the application of specific load or deformation, which in turn will depend upon which of the engineering property is to be determined.

Let us write the strain displacement relations for fibre and matrix using Equation (7.172) and Equation (7.175) through Equation (7.177).

$$\begin{aligned}
\varepsilon_{xx}^{(f)} &= \frac{\partial u^{(f)}}{\partial x} = \varepsilon_{xx}, & \varepsilon_{xx}^{(m)} &= \frac{\partial u^{(m)}}{\partial x} = \varepsilon_{xx} \\
\varepsilon_{\theta\theta}^{(f)} &= \frac{w^{(f)}}{r} = A^{f}, & \varepsilon_{\theta\theta}^{(m)} &= \frac{w^{(m)}}{r} = A^{m} + \frac{B^{m}}{r^{2}} \\
\varepsilon_{rr}^{(f)} &= \frac{dw^{(f)}}{dr} = A^{f}, & \varepsilon_{rr}^{(m)} &= \frac{dw^{(m)}}{dr} = A^{m} - \frac{B^{m}}{r^{2}}
\end{aligned} \quad (7.184)$$

Note that the strains in the fibre are spatially uniform.

Using above relation in Equation (7.171) we get the stresses in fibre and matrix as

$$\begin{aligned}
\sigma_{xx}^{(f)} &= C_{11}^{f}\varepsilon_{xx} + 2C_{12}^{f}A^{f}, & \sigma_{xx}^{(m)} &= C_{11}^{m}\varepsilon_{xx} + 2C_{12}^{m}A^{m} \\
\sigma_{\theta\theta}^{(f)} &= C_{12}^{f}\varepsilon_{xx} + \left(C_{22}^{f}C_{23}^{f}\right)A^{f}, & \sigma_{\theta\theta}^{(m)} &= C_{12}^{m}\varepsilon_{xx} + \left(C_{11}^{m} + C_{12}^{m}\right)A^{m} + \left(C_{11}^{m} - C_{12}^{m}\right)\frac{B^{m}}{r^{2}} \\
\sigma_{rr}^{(f)} &= C_{12}^{f}\varepsilon_{xx} + \left(C_{22}^{f}C_{23}^{f}\right)A^{f}, & \sigma_{rr}^{(m)} &= C_{12}^{m}\varepsilon_{xx} + \left(C_{11}^{m} + C_{12}^{m}\right)A^{m} + \left(C_{12}^{m} - C_{11}^{m}\right)\frac{B^{m}}{r^{2}}
\end{aligned} \quad (7.185)$$

From the above equation, it is easy to see that like strains, the stresses are also spatially uniform in the fibre. Further, the radial and hoop/transverse stresses are identical for this case.

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Lecture 29: Background of Concentric Cylinder Assemblage Model

Home Work:

1. What is a CCA model?
2. Give a brief description of the background for CCA model.

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References:

- *Z Hashin, BW Rosen. The elastic moduli of fibre-reinforced materials. J. Appl. Mech. 1964, Vol. 31, pp. 223-232.*
- *Z Hashin. Analysis of properties of fiber composites with anisotropic constituents. J. Appl. Mech. 1979, Vol. 46, pp. 543-550.*
- *RM Christensen. Mechanics of Composite Materials. Krieger Publishing Company, Florida, 1991.*
- *Z. Hashin: On elastic behaviour of fibre reinforced materials of arbitrary transverse phase geometry. J. Mech. Phys. Solids, 1965, Vol. 13, pp. 119-134.*
- *Z. Hashin: Analysis of composite materials-a survey. J. Mech. Phys. Solids, 1983, Vol. 50, pp. 481-505.*
- *YC Fung. Foundations of Solid Mechanics. Prentice Hall International, Inc., 1965.*
- *CT Herakovich. Mechanics of Fibrous Composites, John Wiley & Sons, Inc. New York, 1998.*
- *MW Hyer, AM Waas. Micromechanics of linear elastic continuous fibre composites, in Comprehensive Composite Materials. Vol. 1: Fiber Reinforcements and General Theory of Composites, Eds. A Kelly, C Zweben. Elsevier, 2000.*
- *JC Halpin Affdl, JL Kardos. The Halpin-Tsai equations: A review. 1976, Vol. 16(5), pp. 344-352.*
- *Z Hashin, S Shtrikman. A variational approach to the theory of elastic behaviour of multiphase materials. J. Mech. Phys. Solids, 1963, Vol. 11, pp. 127-140.*

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