Introduction

In the previous lecture we have seen maximum stress theory, maximum strain theory and Tsai-Hill theory. The former two theories are similar but do not have interaction with other stress or strain components. The Tsai-Hill theory is quadratic in stress and has interactive terms. In this lecture we will see some more failure theories for composite. Then we will see some numerical examples.

The Lecture Contains

- Hoffman Theory
- Tensor Polynomial Failure Criterion
- Examples
- Homework
- References

4. Hoffman Theory

The Hoffman criterion [1] is extension of Tsai-Hill theory. In Tsai-Hill theory the strength parameters are obtained without considering the difference in values in tension and compression. However, one should realistically consider the differing tension and compression strengths that characterize brittle behaviour. This can be done by adding odd functions of σ_1, σ_2 and σ_3 in the expression of Tsai-Hill criterion. Thus, Equation (6.13) becomes

$$\begin{aligned} &C_1(\sigma_2-\sigma_3)^2+C_2(\sigma_3-\sigma_1)^2+C_3(\sigma_1-\sigma_2)^2+C_4\,\sigma_1+C_5\,\sigma_2+C_6\,\sigma_3 \\ &+C_7\,\sigma_4^2+C_8\,\sigma_5^2+C_9\,\sigma_6^2=1 \end{aligned} \tag{6.24}$$

where, $C_1, C_2, \cdots\cdots C_9$ are the material parameters. These are uniquely determined from nine basic strength data, namely, three uniaxial tensile strengths, X_T, Y_T and Z_T; three uniaxial compressive strengths, X_C, Y_C and Z_C and three shear strength parameters, *Q*, *R* and *S*. To determine these material strength parameters, we need to do thought experiments as follows:

First, consider the state of stress such that $\sigma_6 \neq 0$ and all other stress components will be zero. For shear failure in this mode, we need $\sigma_6 = S$. Putting this in Equation (6.24), we get

$$C_9=\frac{1}{S^2} \tag{6.25}$$

Similarly, we get the constants

$$C_7=\frac{1}{Q^2} \tag{6.26}$$

$$C_8=\frac{1}{R^2} \tag{6.27}$$

Now, to find remaining constants we apply following state of stress. Let $\sigma_1 \neq 0$ and all other stress components be zero. For tensile failure, we need $\sigma_1 = X_T$ for this stress state. Putting this in Equation (6.24) we get

$$C_2\,X_T^2+C_3\,X_T^2+C_4X_T=1 \tag{6.28}$$

Similarly for this stress state, the compression failure requires $\sigma_1 = X_C$. This results the Equation (6.24) to give

$$C_2\,X_C^2+C_3\,X_C^2+C_4X_C=1 \tag{6.29}$$

Now, we have two stress states: First one as $\sigma_2 \neq 0$ and all other stress components are zero and the second one as $\sigma_3 \neq 0$ and all other stress components are zero. Again, as in previous case for failure in tension and compression, Equation (6.24) results into following conditions:

(6.30)

$$C_1\, Y_T^2 + C_3\, Y_T^2 + C_5 Y_T = 1$$
$$C_1\, Y_C^2 + C_3\, Y_C^2 + C_5 Y_C = 1$$
$$C_1\, Z_T^2 + C_2\, Z_T^2 + C_6 Z_T = 1$$
$$C_1\, Z_C^2 + C_2\, Z_C^2 + C_6 Z_C = 1$$

Thus, Equations (6.28), (6.29) and (6.30) give a set of six simultaneous equations in $C_1, C_2, C_3, C_4, C_5, C_6$. Solving these, we get

$$\begin{aligned} C_1 &= \frac{1}{2}\left[\frac{1}{X_C\, X_T} - \frac{1}{Y_C\, Y_T} - \frac{1}{Z_C\, Z_T}\right] \\ C_2 &= \frac{1}{2}\left[-\frac{1}{X_C\, X_T} + \frac{1}{Y_C\, Y_T} - \frac{1}{Z_C\, Z_T}\right] \\ C_3 &= \frac{1}{2}\left[-\frac{1}{X_C\, X_T} - \frac{1}{Y_C\, Y_T} + \frac{1}{Z_C\, Z_T}\right] \end{aligned} \qquad (6.31)$$

and

$$C_4 = \frac{1}{X_T} + \frac{1}{X_C}, \quad C_5 = \frac{1}{Y_T} + \frac{1}{Y_C}, \quad C_6 = \frac{1}{Z_T} + \frac{1}{Z_C} \qquad (6.32)$$

Thus, the Hoffman criterion as given in Equation (6.24) becomes

$$\begin{aligned} &\frac{1}{2}\left[\frac{1}{X_C\, X_T} - \frac{1}{Y_C\, Y_T} - \frac{1}{Z_C\, Z_T}\right](\sigma_2 - \sigma_3)^2 + \frac{1}{2}\left[-\frac{1}{X_C\, X_T} + \frac{1}{Y_C\, Y_T} - \frac{1}{Z_C\, Z_T}\right]\,(\sigma_3 - \sigma_1)^2 \\ &+ \frac{1}{2}\left[-\frac{1}{X_C\, X_T} - \frac{1}{Y_C\, Y_T} + \frac{1}{Z_C\, Z_T}\right]\,(\sigma_1 - \sigma_2)^2 + \left(\frac{1}{X_T} + \frac{1}{X_C}\right)\sigma_1 + \left(\frac{1}{Y_T} + \frac{1}{Y_C}\right)\sigma_2 \\ &+ \left(\frac{1}{Z_T} + \frac{1}{Z_C}\right)\sigma_3 + \frac{\sigma_4^2}{Q^2} + \frac{\sigma_5^2}{R^2} + \frac{\sigma_6^2}{S^2} = 1 \end{aligned} \qquad (6.33)$$

Now consider transverse isotropy of the material in 2-3 plane. Thus, $Y_T = Z_T$ and $Y_C = Z_C$. For shear strength, we have $R = S$. Then for plane stress condition $\sigma_3 = \sigma_4 = \sigma_5 = 0$, the criterion in Equation. (6.33) becomes

$$\frac{\sigma_1\sigma_2 - \sigma_1^2}{X_T\, X_C} - \frac{\sigma_2^2}{Y_T\, Y_C} + \sigma_1\frac{X_T + X_C}{X_T\, X_C} + \sigma_2\frac{Y_T + Y_C}{Y_T\, Y_C} + \frac{\sigma_6^2}{S^2} = 1 \qquad (6.34)$$

Equation (6.34) represents Hoffman criterion for planar state of stress in transversely isotropic materials. It should be noted that in this criterion there is no need to check the sign of the stress components to decide whether a tensile or compressive strength is to be used.

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***5. Tensor Polynomial Failure Criterion*:**

Tensor polynomial criterion attempts to mathematically overcome one of the shortcomings of the quadratic criteria that they do not account for differences between tensile and compressive strengths.

In the present lecture we will see the second order tensor polynomial criterion proposed by Tsai and Wu [2]. This is a complete quadratic tensor polynomial which includes the linear terms.

The failure surface in the stress space has the following scalar form:

$$f(\sigma_k) = F_i\, \sigma_i \,+\, F_{ij}\, \sigma_i\, \sigma_j = 1 \qquad (6.35)$$

where, F_i and F_{ij} are strength tensors of the second and fourth order, respectively. The expanded form of the above equation is

$$\begin{aligned}
&F_1\, \sigma_1 \,+ F_2\, \sigma_2 \,+ F_3\, \sigma_3 \,+ F_4\, \sigma_4 \,+ F_5\, \sigma_5 \,+ F_6\, \sigma_6 \\
&+F_{11}\, \sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + 2F_{13}\sigma_1\sigma_3 + 2F_{14}\sigma_1\sigma_4 + 2F_{15}\sigma_1\sigma_5 + 2F_{16}\sigma_1\sigma_6 \\
&+F_{22}\, \sigma_2^2 + 2F_{23}\sigma_2\sigma_3 + 2F_{24}\sigma_2\sigma_4 + 2F_{25}\sigma_2\sigma_5 + 2F_{26}\sigma_2\sigma_6 \\
&+F_{33}\, \sigma_3^2 + 2F_{34}\sigma_3\sigma_4 + 2F_{35}\sigma_3\sigma_5 + 2F_{36}\sigma_3\sigma_6 \\
&+F_{44}\, \sigma_4^2 + 2F_{45}\sigma_4\sigma_5 + 2F_{46}\sigma_4\sigma_6 \\
&+F_{55}\, \sigma_5^2 + 2F_{56}\sigma_5\sigma_6 \\
&+F_{66}\, \sigma_6^2 = 1
\end{aligned} \qquad (6.36)$$

It should be noted that the linear term σ_i take into account the difference between tensile stress and compressive stress induced failures. The quadratic terms $\sigma_i\sigma_j$ define an ellipsoid in the stress space.

The features of this theory are given below:

1. The resulting criterion is a scalar function and thus an invariant. Further, the interactions between various components are independent unlike in Tsai-Hill theory where interactions are fixed and in case of maximum stress or maximum strain theory these interactions are not possible.

2. The strength components are expressed as tensors; one can use the transformation rules as discussed earlier for their transformations. Further, the invariants of these strength tenors are also well defined.

3. The property of symmetry of strength tenors and number of independent and non-zero components can be derived in similar way that we carried out for anisotropic materials earlier.

4. One can either transform the strength parameters from F_i to F_i' and F_{ij} to F_{ij}' or transform σ_i to σ_i'. Most of the existing criteria are limited to specially orthotropic materials. Such criteria can be applied only by transforming the external stresses to material axes. However, the transformations of strength criteria cannot be done as this transformation is not known.

5. Since Equation (6.36) is an invariant, hence, an invariant for any other coordinate system. This also holds for curvilinear coordinate system as well.

6. The theory has also introduced constraints over the magnitude of the strength interaction terms in the following manner :

$$F_{ii}\, F_{jj} - F_{ij}^2 \geq 1 \quad \text{no sum over } i \text{ and } j \qquad (6.37)$$

The terms F_{ii} represent diagonal terms and are positive terms, whereas the off-diagonal terms can be positive or negative depending upon their interaction with other terms. However, their magnitudes are constrained by Equation (6.37). Further, the inequality in Equation (6.37) is very important as it assures the failure envelope in Equation (6.35) intersects all stress axes. The surface formed by Equation (6.35) is an ellipsoid and Equation (6.37) ensures that it is a closed surface unlike a hyperboloid. It should be noted that the positive definiteness is also imposed for F_i terms.

7. Gol'denbalt and Kopnov [3] proposed a tensorial criterion in a general form of

$$(F_i\, \sigma_i)^\alpha + \left(F_{ij}\, \sigma_i\, \sigma_j\right)^\beta + \left(F_{ijk}\, \sigma_i\, \sigma_j\, \sigma_k\right)^\gamma + \cdots\cdots = 1 \quad (6.38)$$

This form of the equation is more complicated than in Equation (6.35). Further, the size of the strength terms is enormously very high to handle and the additional terms do not yield more generality than the linear and quadratic terms.

Note: Statement 4 above equivalently says that the Tsai-Wu strength criterion can be given in transformed coordinate system. The transformed criterion may be given as

$$f(\sigma_k') = F_i'\, \sigma_i' + F_{ij}'\, \sigma_i'\, \sigma_j' = 1 \qquad (6.39)$$

And the strength terms can be transformed using the following relations

$$\begin{aligned} F_i' &= a_{ji}\, F_j \\ F_{ij}' &= a_{ki}\, a_{lj}\, F_{kl} \end{aligned} \qquad (6.40)$$

However, the transformation of any other strength criteria may not hold true.

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Now we will seek simplifications to Equation (6.36) as follows. The strength tensors can be written in the form

$$F_i = \{F_1 \quad F_2 \quad F_3 \quad F_4 \quad F_5 \quad F_6\}^T \tag{6.41}$$

and

$$F_{ij} = \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} & F_{15} & F_{16} \\ & F_{22} & F_{23} & F_{24} & F_{25} & F_{26} \\ & & F_{33} & F_{34} & F_{35} & F_{36} \\ & & & F_{44} & F_{45} & F_{46} \\ & & & & F_{55} & F_{56} \\ & & & & & F_{66} \end{bmatrix} \tag{6.42}$$

where, both strength tensors are assumed to be symmetric with 6 and 21 independent constants for F_i and F_{ij}, respectively.

The number of independent strength parameters can be further reduced if we have some form of material symmetry. We consider a special case of specially orthotropic material. Thus, for specially orthotropic material the terms F_4, F_5 and F_6 will vanish. Further, the off-diagonal terms which give normal shear coupling like F_{14}, F_{15}, F_{16}, etc. will also become zero if we assume that the sign of shear stress does not change the failure stress. Further, with same reasoning we assume that the shear strengths are also uncoupled leading to $F_{45} = F_{46} = F_{56} = 0$. Thus, with this material symmetry, we have

$$F_4 = F_5 = F_6 = 0$$

$$F_{14} = F_{15} = F_{16} = F_{24} = F_{25} = F_{26} = F_{34} = F_{35} = F_{36} = 0 \text{ and}$$

$$F_{45} = F_{46} = F_{56} = 0$$

The number of independent strength parameters are now 3 and 9 for F_i and F_{ij}, respectively. Thus, for orthotropic material the criterion becomes

$$\begin{aligned} f(\sigma_k) = & F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 \\ & + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 \\ & + 2F_{12}\sigma_1\sigma_2 + 2F_{13}\sigma_1\sigma_3 + 2F_{23}\sigma_2\sigma_3 = 1 \end{aligned} \tag{6.43}$$

Now we will determine the strength parameters by thought experiments. First, we apply $\sigma_1 \neq 0$ and other stress components being zero. For this state of stress, the failure in tension requires $\sigma_1 = X_T$. Thus, Equation (6.43) becomes

$$F_1 X_T + F_{11} X_T^2 = 1 \tag{6.44}$$

Similarly, for this state of stress, the failure in compression requires $\sigma_1 = X_C$. This results in Equation (6.43) to become

$$F_1 \, X_C + F_{11} \, X_C^2 = 1 \qquad (6.45)$$

Equation (6.44) and Equation (6.45) are two simultaneous equations with F_1 and F_{11} as two unknowns.

Solution of these two equations gives

$$F_1 = \frac{1}{X_T} + \frac{1}{X_C}, \quad F_{11} = -\frac{1}{X_T \, X_C} \qquad (6.46)$$

Likewise, if we apply stress states as $\sigma_2 \neq 0$ and $\sigma_3 \neq 0$ with other stresses being zero, it will give us the following constants:

$$F_2 = \frac{1}{Y_T} + \frac{1}{Y_C}, \quad F_{22} = -\frac{1}{Y_T\, Y_C} \tag{6.47}$$

$$F_3 = \frac{1}{Z_T} + \frac{1}{Z_C}, \quad F_{33} = -\frac{1}{Z_T\, Z_C} \tag{6.48}$$

Similarly, if we apply $\sigma_4 \neq 0$, $\sigma_5 \neq 0$, $\sigma_6 \neq 0$ and with other stress components as zero, as three separate states of stress, then we get following constants:

$$F_{44} = \frac{1}{Q^2}, \quad F_{55} = \frac{1}{R^2}, \quad F_{66} = \frac{1}{S^2} \tag{6.49}$$

So far we have developed expressions for 3 strength terms for F_i and diagonal terms of F_{ij}. Now the expressions for off-diagonal terms of F_{ij} require combined state of stress to be applied. The pure axial or shear state of stress will not be sufficient. In other criteria the interaction terms like F_{12} are assumed to be dependent or terms like F_{16} are zero.

There are an infinite combinations of the stresses from which these terms can be obtained. However, one should choose those combinations which can yield the desired result in a reliable and easy manner. In the following, we will see typical combinations of stresses to find F_{12}.

Consider the equivi-biaxial stress state $\sigma_1 = \sigma_2 = \sigma$ and other stress components are zero. Putting this in Equation (6.43), we get

$$\sigma^2(F_{11} + F_{22} + 2F_{12}) + \sigma(F_1 + F_2) = 1 \tag{6.50}$$

Solving for F_{12}, we get

$$F_{12} = \frac{1}{2\sigma^2}\left[1 - \sigma\left(\frac{1}{X_T} + \frac{1}{X_C} + \frac{1}{Y_T} + \frac{1}{Y_C}\right) + \sigma^2\left(\frac{1}{X_T X_C} + \frac{1}{Y_T Y_C}\right)\right] \tag{6.51}$$

Similarly, if we apply equivi-biaxial stress states in 1-3 and 2-3 planes, then we get the following constants:

$$F_{13} = \frac{1}{2\sigma^2}\left[1 - \sigma\left(\frac{1}{X_T} + \frac{1}{X_C} + \frac{1}{Z_T} + \frac{1}{Z_C}\right) + \sigma^2\left(\frac{1}{X_T X_C} + \frac{1}{Z_T Z_C}\right)\right] \tag{6.52}$$

$$F_{23} = \frac{1}{2\sigma^2}\left[1 - \sigma\left(\frac{1}{Y_T} + \frac{1}{Y_C} + \frac{1}{Z_T} + \frac{1}{Z_C}\right) + \sigma^2\left(\frac{1}{Y_T Y_C} + \frac{1}{Z_T Z_C}\right)\right] \tag{6.53}$$

We can find the constants F_{12}, F_{13} and F_{23} by imposing the equivi-biaxial state of stress. However, practically it is very difficult to impose such a state of stress. Hence, many researchers have proposed tests on $45°$ angle specimens to determine these strength parameters.

Consider the stress state

$$\sigma_1 = \sigma_2 = \sigma_6 = \frac{U}{2} \tag{6.54}$$

and other stresses are zero. Here, U is the axial tensile strength of $45°$ specimen. The stress state in Equation (6.54) is obtained by applying U in axial direction for $45°$ specimen. One should be able to get the state of stress in Equation (6.54) from our earlier stress transformation equations. Putting Equation (6.54) in Equation (6.43), we get

$$\frac{U^2}{4}(F_{11} + F_{22} + 2F_{12} + F_{66}) + \frac{U}{2}(F_1 + F_2) = 1 \tag{6.55}$$

which upon solving for F_{12} gives

$$F_{12} = \frac{2}{U^2}\left[1 - \frac{U}{2}\left(\frac{1}{X_T} + \frac{1}{X_C} + \frac{1}{Y_T} + \frac{1}{Y_C}\right) + \frac{U^2}{4}\left(\frac{1}{X_T\, X_C} + \frac{1}{Y_T\, Y_C} - \frac{1}{S^2}\right)\right] \tag{6.56}$$

A similar expression can be derived with compressive strength of 45^0 specimen. One can further find this constant using the in-plane shear strength of 45^0 specimen, V which produces the stress state as

$$\sigma_1 = -\sigma_2 = V, \sigma_3 = \sigma_4 = \sigma_5 = \sigma_6 = 0 \tag{6.57}$$

This reduces Equation (6.43) to

$$V^2(F_{11} + F_{22} - 2F_{12}) + V(F_1 - F_2) = 1 \tag{6.58}$$

The solution F_{12} from this equation is

$$F_{12} = -\frac{1}{2V^2}\left[1 - V\left(\frac{1}{X_T} + \frac{1}{X_C} - \frac{1}{Y_T} - \frac{1}{Y_C}\right) + V^2\left(\frac{1}{X_T X_C} + \frac{1}{Y_T Y_C}\right)\right] \tag{6.59}$$

***Note*:** In case of anisotropic materials the constant F_{16} is no longer zero. This can be obtained by a tension-torque test such that it results in following stress state

$$\sigma_1 = \sigma_6 = T, \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = 0 \tag{6.60}$$

Using this in Equation (6.43), we get

$$T^2(F_{11} + F_{66} + 2F_{16}) + T(F_1 + F_6) = 1 \tag{6.61}$$

which gives

$$F_{16} = \frac{1}{2T^2}\left[1 - T\left(\frac{1}{X_T} + \frac{1}{X_C}\right) - T^2\left(\frac{1}{X_T X_C} + \frac{1}{S^2}\right)\right] \tag{6.62}$$

The Tsai-Wu criterion for planar state of stress can be given as

$$f(\sigma_k) = F_1\sigma_1 + F_2\sigma_2 + F_{11}\ \sigma_1^2 + F_{22}\ \sigma_2^2 + F_{66}\ \sigma_6^2 + 2F_{12}\sigma_1\sigma_2 = 1 \tag{6.63}$$

The strength parameters are as given above. If the strength term $F_{12} = 0$ then, the criterion becomes

$$f(\sigma_k) = F_1\sigma_1 + F_2\sigma_2 + F_{11}\ \sigma_1^2 + F_{22}\ \sigma_2^2 + F_{66}\ \sigma_6^2 = 1 \tag{6.64}$$

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Examples

***Example 6.3*:** For the data in Example 6.1, find whether lamina will fail if a) Hoffman and b) Tsai-Wu theory is used.

Solution: The strain transformation and $[Q]$ are as given in Example 6.1.

Hoffman Theory:

For planar stress the Hoffman theory is

$$\frac{\sigma_1\sigma_2 - \sigma_1^2}{X_T\,X_C} - \frac{\sigma_2^2}{Y_T\,Y_C} + \sigma_1\frac{X_T + X_C}{X_T\,X_C} + \sigma_2\frac{Y_T + Y_C}{Y_T\,Y_C} + \frac{\sigma_6^2}{S^2} = 1$$

Using the strength parameters and stresses we get

$$\begin{aligned}&\frac{653.96 \times 115.72 - (-653.96)^2}{(1950)(-1480)} - \frac{(115.72)^2}{(48)(-200)} + 653.96\frac{1950 - 1480}{(1950)(-1480)}\\&+115.72\frac{48 - 200}{(48)(-200)} + \frac{(-60.46)^2}{(79)^2}\\&= 2.836\end{aligned}$$

Thus, the failure index is greater than unity. Hence, according to this theory, for the given state of strain/stress the lamina fails.

Tsai-Wu Theory:

Using the stresses and strength parameters, for planar stress state the Tsai-Wu theory with term $F_{12} = 0$ is given as

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\,\sigma_1^2 + F_{22}\,\sigma_2^2 + F_{66}\,\sigma_6^2 = 1$$

The strength parameters are calculated as

$$F_1 = \frac{1}{X_T} + \frac{1}{X_C} = \frac{1}{1950} + \frac{1}{(-1480)} = -1.6286 \times 10^{-4}\,\frac{1}{MPa}$$

$$F_2 = \frac{1}{Y_T} + \frac{1}{Y_C} = \frac{1}{48} + \frac{1}{(-200)} = 0.0158\,\frac{1}{MPa}$$

$$F_{11} = -\frac{1}{X_T\,X_C} = -\frac{1}{(1950)\,(-1480)} = 3.465 \times 10^{-7}\,\frac{1}{MPa^2}$$

$$F_{22} = -\frac{1}{Y_T\,Y_C} = -\frac{1}{(48)\,(-200)} = 1.0417 \times 10^{-4}\,\frac{1}{MPa^2}$$

$$F_{66} = \frac{1}{S^2} = \frac{1}{(79)^2} = 1.6023 \times 10^{-4} \frac{1}{MPa^2}$$

Thus, putting the stress values, we get

$$\begin{aligned}&(-1.6286 \times 10^{-4})(653.96) + (0.0158)(115.72) \\ &+(3.465 \times 10^{-7})(653.96)^2 + (1.0417 \times 10^{-4})(115.72)^2 + (1.6023 \times 10^{-4})(-60.46)^2 \\ &= 2.809\end{aligned}$$

Since, the value of failure index by Tsai-Wu theory is greater than unity, the lamina will fail.

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Example 6.4: For the data in Example 6.2, find the maximum value of $P > 0$ if a) Hoffman and b) Tsai-Wu theory is used.

Solution:

Hoffman Theory:

For planar stress the Hoffman theory is rearranged as

$$\frac{\sigma_1\sigma_2 - \sigma_1^2}{X_T\,X_C} - \frac{\sigma_2^2}{Y_T\,Y_C} + \frac{\sigma_6^2}{S^2} + \sigma_1\frac{X_T + X_C}{X_T\,X_C} + \sigma_2\frac{Y_T + Y_C}{Y_T\,Y_C} = 1$$

Using the stresses and strength parameters,

$$LHS = \left[\frac{(1.7141)(-2.7141) - (1.7141)^2}{(1950)(-1480)} - \frac{(-2.7141)^2}{(48)(-200)} + \frac{(-4.1651)^2}{(79)^2}\right]P^2 + \left[(1.7141)\frac{1950 - 1480}{(1950)(-1480)} + (-2.7141)\frac{48 - 200}{(48)(-200)}\right]P = 1$$

Thus, we get

$$0.0035\,P^2 - 0.0433\,P = 1$$

Solving this quadratic equation for P we get $P = 24.185$.

Tsai-Wu Theory:

For planar stress state the Tsai-Wu theory with term $F_{12} = 0$ is given as

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\,\sigma_1^2 + F_{22}\,\sigma_2^2 + F_{66}\,\sigma_6^2 = 1$$

The strength parameters are as given in Example 6.3. Putting the stresses, we get

$$\begin{aligned}&-1.6286\times10^{-4}(1.7141)P + 0.0158(-2.7141)P\\&+3.465\times10^{-7}(1.7141)^2P^2 + 1.0417\times10^{-4}(-2.7141)^2P^2 + 1.6023\times10^{-4}(-4.1651)^2P^2\\&= -0.0433\,P + 0.0036\,P^2 = 1\end{aligned}$$

Solving this for P, we get $P = 23.73$.

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Home Work:

1. Explain in detail the following failure theories.
   a. Hoffman Theory
   b. Tsai-Wu Theory
2. What is the difference between Tsai-Hill and Hoffman theory?
3. What are the key features of the Tsai-Wu theory?
4. What are the different methods to find the strength parameter F_{12} in Tsai-Wu theory?
5. Explain the methods to find the strength parameter F_{16} in Tsai-Wu theory.
6. A ply of AS4/3506-1 material with $45°$ fibre orientation is in the planar state of stress. The strains are $\{\epsilon\}_{xy} = 10^{-2}\{1.2 \quad 0.2 \quad 0.1\}^T$. Check that whether lamina will fail if a) Hoffman theory b) Tsai-Wu theory is used.
7. Find the maximum value of $P > 0$ if a state of stress of $\sigma_{xx} = 2P, \sigma_{yy} = -3P,$ and $\tau_{xy} = 4P,$ is applied to the $45°$ lamina of AS4/3506-1 material using a) Hoffman theory b) Tsai-Wu theory.

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