Module 7: Micromechanics

Lecture 25: Strength of Materials Approach

Introduction

In the previous lecture we have introduced the concept of ***Representative Volume Element*** or ***Unit Cell.*** This is the basic building block in a micromechanical study. Further, we explained the need of micromechanical study. In the previous lecture we have obtained effective axial and transverse modulus and axial Poisson’s ratio using the strength of materials approach. In the present lecture we will derive the expressions for effective transverse and axial shear moduli. Further, we will derive the expressions for coefficients of thermal and hygral expansions as well. We will conclude this lecture with some numerical examples.

The Lecture Contains

- Effective Transverse Modulus
- Effective Axial Shear Modulus
- Coefficients of Thermal Expansion
- Coefficients of Hygral Expansion
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Effective Transverse Modulus E_2^*:

In the earlier lecture we have seen the first approach, where the deformation of individual constituent is independent of each other and the deformation in direction 1 is not considered. In this lecture we are going to derive an expression for effective transverse modulus using second approach as follows.

Second Approach:

In this approach, we consider the resulting deformation in direction 1. It should be noted that when the stress is applied in direction 2, the deformations of fibre and matrix in direction 1 are identical. The deformation in direction 1 is calculated from two dimensional state of stress in fibre and matrix. The deformations are shown in Figure 7.5.

The axial and transverse stresses in fibre and matrix can be given using planar constitutive relations as

$$\begin{aligned} \sigma_1^{(f)} &= Q_{11}^f \varepsilon_1^{(f)} + Q_{12}^f \varepsilon_2^{(f)}, \quad & \sigma_1^{(m)} &= Q_{11}^m \varepsilon_1^{(m)} + Q_{12}^m \varepsilon_2^{(m)}, \\ \sigma_2^{(f)} &= Q_{12}^f \varepsilon_1^{(f)} + Q_{22}^f \varepsilon_2^{(f)}, \quad & \sigma_2^{(m)} &= Q_{12}^m \varepsilon_1^{(m)} + Q_{11}^f \varepsilon_2^{(m)} \end{aligned} \tag{7.34}$$

where,

$$\begin{aligned} Q_{11}^f &= \frac{E_1^{(f)}}{1-\nu_{21}^{(f)}\nu_{12}^{(f)}}, \quad Q_{22}^f = \frac{E_2^{(f)}}{1-\nu_{21}^{(f)}\nu_{12}^{(f)}} \\ Q_{12}^f &= \frac{\nu_{21}^{(f)}E_1^{(f)}}{1-\nu_{21}^{(f)}\nu_{12}^{(f)}} = \frac{\nu_{12}^{(f)}E_2^{(f)}}{1-\nu_{21}^{(f)}\nu_{12}^{(f)}} \end{aligned} \quad \text{and} \quad Q_{11}^m = \frac{E^{(m)}}{1-\left(\nu^{(m)}\right)^2}, \quad Q_{12}^m = \frac{\nu^{(m)}E^{(m)}}{1-\left(\nu^{(m)}\right)^2} \tag{7.35}$$

To compute the effective transverse modulus E_2^* we need to find the total deformation ΔW as a function of the applied transverse stress $\bar{\sigma}_2$. It should be noted that the net force in the direction 1 is zero. Thus,

$$\int_{A_f+A_m} \sigma_1 \, dA = 0 \tag{7.36}$$

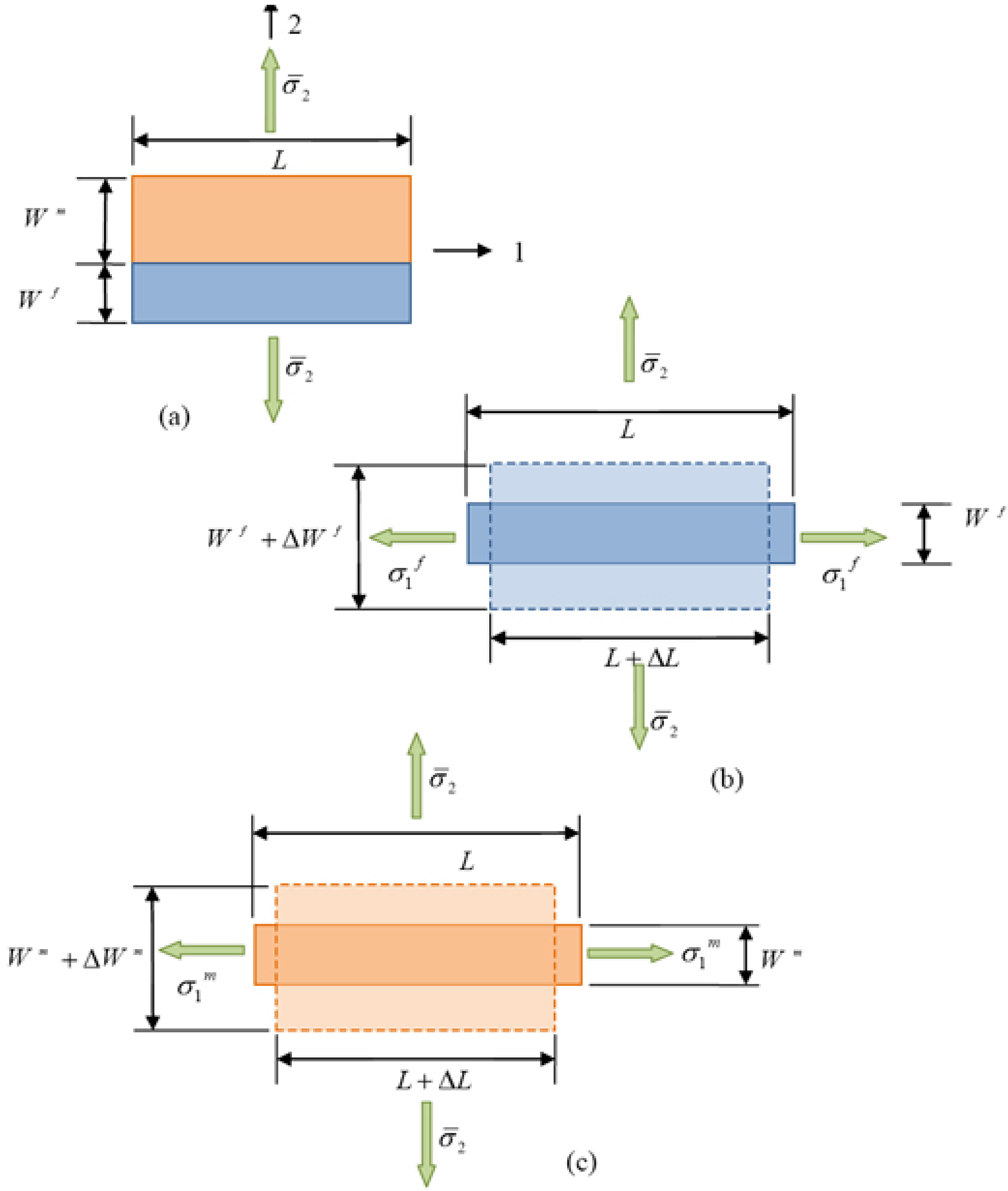

Figure 7.5: (a) Undeformed unit cell under uniform $\bar{\sigma}_2$ stress (b) and (c) deformed individual constituents of the unit cell

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Generally, the stresses are uniform in the fibre and matrix. Thus above equation reduces to

$$\sigma_1^{(f)} A_f + \sigma_1^{(m)} A_m = 0 \tag{7.37}$$

Further, for equilibrium in transverse direction, we have

$$\sigma_2^{(f)} = \sigma_2^{(m)} = \bar{\sigma}_2 \tag{7.38}$$

The axial and transverse strains in fibre and matrix are

$$\varepsilon_1^{(f)} = \varepsilon_1^{(m)} = \frac{\Delta L}{L} \quad \text{and} \quad \varepsilon_2^{(f)} = \frac{\Delta W^{(f)}}{W^{(f)}}, \quad \varepsilon_2^{(m)} = \frac{\Delta W^{(m)}}{W^{(m)}} \tag{7.39}$$

Using Equations (7.34) and (7.39) in (7.37), we get

$$\left(Q_{11}^f A_f + Q_{11}^m A_m\right)\frac{\Delta L}{L} + Q_{12}^f A_f \frac{\Delta W^{(f)}}{W^{(f)}} + Q_{12}^m A_m \frac{\Delta W^{(m)}}{W^{(m)}} = 0 \tag{7.40}$$

Further, using Equation (7.34) in Equation (7.38), we get

$$\begin{aligned} Q_{12}^f \frac{\Delta L}{L} + Q_{22}^f \frac{\Delta W^{(f)}}{W^{(f)}} &= \bar{\sigma}_2 \\ Q_{12}^m \frac{\Delta L}{L} + Q_{11}^m \frac{\Delta W^{(m)}}{W^{(m)}} &= \bar{\sigma}_2 \end{aligned} \tag{7.41}$$

We solve Equations (7.40) and (7.41) for $\frac{\Delta L}{L}$, $\frac{\Delta W^{(f)}}{W^{(f)}}$ and $\frac{\Delta W^{(m)}}{W^{(m)}}$. The transverse composite strain then is obtained as

$$\bar{\varepsilon}_2 = \frac{\bar{\sigma}_2}{E_2^*} = \frac{\Delta W}{W} = \frac{\Delta W^{(f)} + \Delta W^{(m)}}{W} \tag{7.42}$$

Finally, putting the values of $\frac{\Delta W^{(f)}}{W^{(f)}}$ and $\frac{\Delta W^{(m)}}{W^{(m)}}$ in the above equation, we get an expression for E_2^* as

$$\frac{1}{E_2^*} = \frac{\eta^{(f)} V_f}{E_2^{(f)}} + \frac{\eta^{(m)} V_m}{E^{(m)}} = \frac{\eta^{(f)} V_f}{E_2^{(f)}} + \frac{\eta^{(m)}\left(1 - V_f\right)}{E^{(m)}} \tag{7.43}$$

where,

$$\eta^{(f)} = \frac{E_1^{(f)} V_f + \left[\left(1 - \nu_{12}^{(f)} \nu_{21}^{(f)}\right) E^{(m)} + \nu^{(m)} \nu_{21}^{(f)} E_1^{(f)}\right]\left(1 - V_f\right)}{E_1^{(f)} V_f + E^{(m)}\left(1 - V_f\right)}$$

$$\eta^{(m)} = \frac{E^{(m)} V_m + \left[\left(1 - \left(\nu^{(m)}\right)^2\right) E_1^{(f)} - \left(1 - \nu^{(m)} \nu_{12}^{(f)}\right) E^{(m)}\right] V_f}{E_1^{(f)} V_f + E^{(m)}\left(1 - V_f\right)} \tag{7.44}$$

Equation (7.43) is an alternate equation for effective transverse modulus E_2^*. This is also a rule of mixtures equations. It should be noted that the factors $\eta^{(f)}$ and $\eta^{(m)}$ are the nondimensional factors.

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Effective Axial Shear Modulus G_{12}^{*}:

To derive the effective axial shear modulus of the composite the RVE is loaded in shear as shown in Figure 7.6(a). The fibre and matrix are assumed to deform independently. Figure 7.6(b) shows the overall deformation of the RVE. The overall axial deformation is denoted by ΔL. It is important to note that for equilibrium considerations the shear stresses acting on fibre and matrix are assumed to be identical.

Under the pure shear loading, that is, $\bar{\tau}_{12} \neq 0$ and other stress components are zero, the effective axial shear modulus is defined as

$$G_{12}^{*} = \frac{\bar{\tau}_{12}}{\bar{\gamma}_{12}} \qquad (7.45)$$

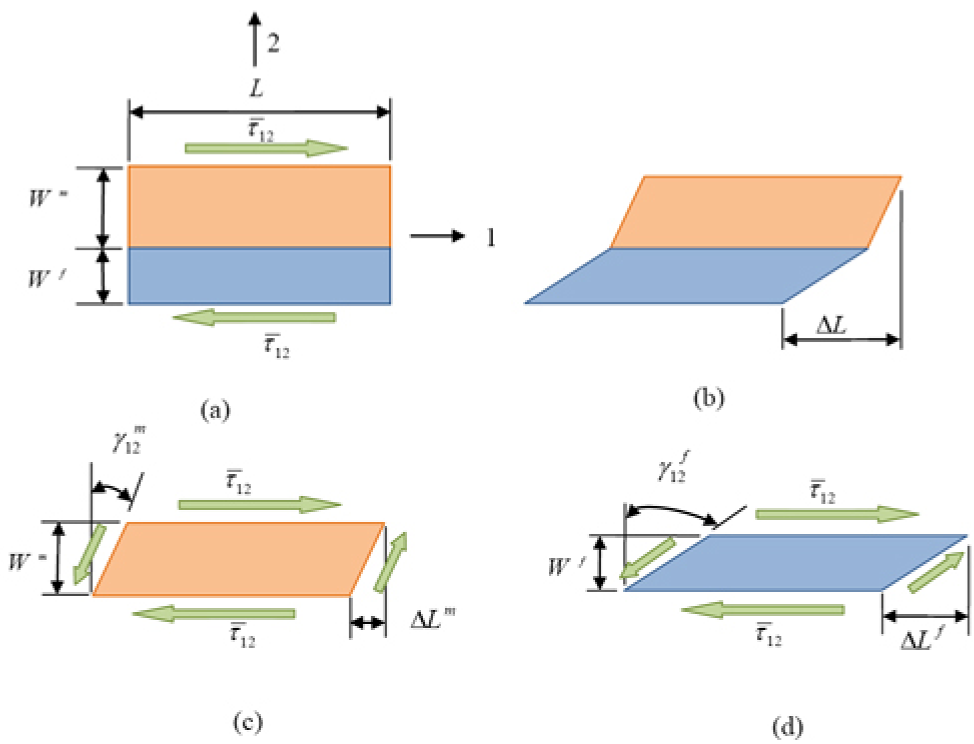

Figure 7.6: (a) Undeformed unit cell under uniform $\bar{\tau}_{12}$ stress (b) overall deformation of unit cell (c) and (d) deformed individual constituents of the unit cell

where, $\bar{\tau}_{12}$ and $\bar{\gamma}_{12}$ are the effective applied shear stress and the resulting effective shear strain in the composite, respectively. The effective shear strain in composite is obtained from the deformations of the fibre and matrix in RVE. The fibre and matrix undergo deformations $\Delta L^{(f)}$ and $\Delta L^{(m)}$, respectively. These deformations are shown in Figure 7.6(c) and (d), respectively.

The shear strains in the fiber and matrix are given as

$$\gamma_{12}^{(f)} = \frac{\bar{\tau}_{12}}{G_{12}^{(f)}} \quad \text{and} \quad \gamma^{(m)} = \frac{\bar{\tau}_{12}}{G^{(m)}} \qquad (7.46)$$

where, $G_{12}^{(f)}$ is the inplane shear modulus of the fibre and $G^{(m)}$ is the shear modulus of the matrix material. From Figure 7.6(c) and (d), we can write the individual deformations in fibre and matrix as

$$\Delta L^{(f)} = \gamma_{12}^{(f)} W^{(f)} \quad \text{and} \quad \Delta L^{(m)} = \gamma^{(m)} W^{(m)} \tag{7.47}$$

Using Equation (7.46) in above equation, the total axial deformation is given as

$$\Delta L = \Delta L^{(f)} + \Delta L^{(m)} = \frac{\bar{\tau}_{12}}{G_{12}^{(f)}} W^{(f)} + \frac{\bar{\tau}_{12}}{G^{(m)}} W^{(m)} = \left(\frac{W^{(f)}}{G_{12}^{(f)}} + \frac{W^{(m)}}{G^{(m)}} \right) \bar{\tau}_{12} \tag{7.48}$$

Now, we can give the overall shear strain of the RVE as

$$\bar{\gamma}_{12} = \frac{\Delta L}{W} = \left(\frac{W^{(f)}}{G_{12}^{(f)}} + \frac{W^{(m)}}{G^{(m)}} \right) \bar{\tau}_{12} = \left(\frac{V_f}{G_{12}^{(f)}} + \frac{V_m}{G^{(m)}} \right) \bar{\tau}_{12} \tag{7.49}$$

Finally, the effective axial shear modulus of the composite can be given from above equation as

$$\frac{1}{G_{12}^{*}} = \frac{\bar{\gamma}_{12}}{\bar{\tau}_{12}} = \frac{V_f}{G_{12}^{(f)}} + \frac{V_m}{G^{(m)}} = \frac{V_f}{G_{12}^{(f)}} + \frac{\left(1 - V_f\right)}{G^{(m)}} \tag{7.50}$$

This is the rule of mixtures equation for the effective axial shear modulus. This equation is analogous to Equation (7.33) for the effective transverse modulus of the composite.

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Coefficients of Thermal Expansion α_1^* **and** α_2^*:

The coefficients of thermal expansion are very important as the composite is fabricated as elevated temperature and cooled down to room temperature. In this process, due to difference in the coefficients of thermal expansion of fibre and matrix materials, the thermal residual stresses are developed. The determination of the coefficients of thermal expansion of the composite is dealt in this section.

First, we will derive an expression for the coefficients of thermal expansion in transverse direction, α_2^*. We will use the deformation same as shown in Figure 7.5. The only difference is that the effective stress in transverse direction must be zero. This is because that the thermal expansion should occur without any applied stress. Thus, for the deformations as shown in Figure 7.5, we take $\bar{\sigma}_2 = 0$. Now the deformation in fibre and matrix in transverse direction can be given as

$$\Delta W^{(f)} = \alpha_2^{(f)} \Delta T \Delta W^{(f)} \quad \text{and} \quad \Delta W^{(m)} = \alpha^{(m)} \Delta T \Delta W^{(m)} \qquad (7.51)$$

where, $\alpha_2^{(f)}$ is the coefficient of thermal expansion of fibre in transverse direction and $\alpha^{(m)}$ is the coefficient of thermal expansion of matrix. It should be noted that the matrix is assumed to be isotropic in nature. ΔT is increase in temperature. Let us define the coefficient of expansion for composite in transverse direction, α_2^* as

$$\alpha_2^* = \frac{\Delta W / W}{\Delta T} \qquad (7.52)$$

Using Equation (7.51) in above equation, we get

$$\alpha_2^* = \frac{\left(\Delta W^{(f)} + \Delta W^{(f)}\right)/W}{\Delta T} = \alpha_2^{(f)} \frac{\Delta W^{(f)}}{W} + \alpha^{(m)} \frac{\Delta W^{(m)}}{W} \qquad (7.53)$$

Thus, using the volume fraction definitions, we get

$$\alpha_2^* = \alpha_2^{(f)} V_f + \alpha^{(m)} V_m = \alpha_2^{(f)} V_f + \alpha^{(m)} \left(1 - V_f\right) \qquad (7.54)$$

This is the rule of mixtures for the transverse coefficient of thermal expansion. It should be noted that in this derivation the interaction between fibre and matrix under the temperature effect is not constant. Thus, this derivation neglects the thermally induced stresses in fibre and matrix. However, this is not true as thermal stresses will be induced in fibre and matrix. We take into account this fact and derive alternate expression for this coefficient of thermal expansion as follows.

The stresses in fibre can be given for the temperature change of ΔT as

(7.55)

$$\sigma_1^{(f)} = Q_{11}^f\left(\varepsilon_1^f - \alpha_1^{(f)}\Delta T\right) + Q_{12}^f\left(\varepsilon_2^f - \alpha_2^{(f)}\Delta T\right)$$
$$\sigma_2^{(f)} = Q_{12}^f\left(\varepsilon_1^f - \alpha_1^{(f)}\Delta T\right) + Q_{22}^f\left(\varepsilon_2^f - \alpha_2^{(f)}\Delta T\right)$$

and in matrix can be given as

$$\begin{aligned}\sigma_1^{(m)} &= Q_{11}^m\left(\varepsilon_1^m - \alpha^{(m)}\Delta T\right) + Q_{12}^m\left(\varepsilon_2^m - \alpha^{(m)}\Delta T\right)\\ \sigma_2^{(m)} &= Q_{12}^m\left(\varepsilon_1^m - \alpha^{(m)}\Delta T\right) + Q_{11}^m\left(\varepsilon_2^m - \alpha^{(m)}\Delta T\right)\end{aligned} \qquad (7.56)$$

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Now it should be noted that due to temperature change the force in direction 1 should be zero. Further, the stresses in transverse direction in fibre and matrix should be identically zero. The first condition leads to

$$\int_{A_f + A_m} \sigma_1 \, dA = 0 \tag{7.57}$$

Further, assuming that the stresses in fibre and matrix are uniform, the above equation becomes

$$\sigma_1^{(f)} A_f + \sigma_1^{(m)} A_m = 0 \tag{7.58}$$

The second condition that the stresses in transverse direction are zero leads to

$$\sigma_2^{(f)} = \sigma_2^{(m)} = \bar{\sigma}_2 = 0 \tag{7.59}$$

Putting the first of Equation (7.55) and Equation (7.56) in Equation (7.58), we get

$$\begin{aligned} &\left(Q_{11}^f A_f + Q_{11}^m A_m\right)\frac{\Delta L}{L} + Q_{12}^f A_f \frac{\Delta W^{(f)}}{W^{(f)}} + Q_{12}^m A_m \frac{\Delta W^{(m)}}{W^{(m)}} \\ &= \left[\left(Q_{11}^f \alpha_1^{(f)} + Q_{12}^f \alpha_2^{(f)}\right) A_f + \left(Q_{11}^m \alpha^{(m)} + Q_{12}^m \alpha^{(m)}\right) A_m\right]\Delta T \end{aligned} \tag{7.60}$$

Using the second of Equation (7.55) and Equation (7.56) in Equation (7.59), we get

$$\begin{aligned} Q_{12}^f \frac{\Delta L}{L} + Q_{22}^f \frac{\Delta W^{(f)}}{W^{(f)}} &= \left(Q_{12}^f \alpha_1^{(f)} + Q_{22}^f \alpha_2^{(f)}\right)\Delta T \\ Q_{12}^m \frac{\Delta L}{L} + Q_{11}^m \frac{\Delta W^{(m)}}{W^{(m)}} &= \left(Q_{11}^m \alpha^{(m)} + Q_{12}^m \alpha^{(m)}\right)\Delta T \end{aligned} \tag{7.61}$$

Thus, Equation (7.60) and Equation (7.61) are three simultaneous equations in $\frac{\Delta L}{L}$, $\frac{\Delta W^{(f)}}{W^{(f)}}$ and $\frac{\Delta W^{(m)}}{W^{(m)}}$. We solve these three equations and define the coefficients of thermal expansions for composites as

$$\alpha_1^* = \frac{\Delta L / L}{\Delta T} = \frac{\alpha_1^{(f)} E_1^{(f)} V_f + \alpha^{(m)} E^{(m)} V_m}{E_1^{(f)} V_f + E^{(m)} V_m} = \frac{\left(\alpha_1^{(f)} E_1^{(f)} - \alpha^{(m)} E^{(m)}\right) V_f + \alpha^{(m)} E^{(m)}}{\left(E_1^{(f)} - E^{(m)}\right) V_f + E^{(m)}} \tag{7.62}$$

and

(7.63)

$$\alpha_2^* = \frac{\Delta W / W}{\Delta T} = \frac{\left(\Delta W^{(f)} + \Delta W^{(m)}\right) W}{\Delta T}$$
$$= \alpha_2^{(f)} V_f + \alpha^{(m)} V_m + \left(\frac{E_1^{(f)} \nu^{(m)} - E^{(m)} \nu_{12}^{(f)}}{E_1^*}\right)\left(\alpha^{(m)} - \alpha_1^{(f)}\right)\left(1 - V_f\right) V_f$$

The above expressions are the rule of mixture for coefficients of thermal expansions for composite in terms of individual coefficients of thermal expansion, volume fractions and other properties. In the above equation E_1^* is the effective axial modulus, as given earlier. Comparing Equation (7.54) and Equation (7.63), it is clear that the last term in Equation (7.63) is due to the imposition of deformation constraint under thermal loading.

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Coefficients of Hygral Expansion β_1^* and β_2^*:

When the composite is subjected to hygroscopic environment, it absorbs the moisture and deforms. It should be noted that the deformation of the fibre and matrix depends upon the amount of moisture absorbed. Further, the amount of moisture absorbed by fibre and matrix in same environment are, in general, not same. It will be shown that, unlike the coefficients of thermal expansion, the moisture content will enter into the expressions of coefficients of hygral expansion of composite.

The derivation also follows similar procedure as used in the derivation of coefficients of thermal expansion. The deformation constraints, similar to the derivation of coefficients of thermal expansion, are also imposed in this derivation.

The force in axial direction in composite should be zero. This condition leads to the equation

$$\sigma_1^{(f)} A_f + \sigma_1^{(m)} A_m = 0 \qquad (7.64)$$

The axial stresses in fibre and matrix due to moisture absorption alone are given as

$$\begin{aligned} \sigma_1^{(f)} &= E_1^{(f)}\left(\varepsilon_1^f - \beta_1^{(f)}\Delta M^{(f)}\right) \\ \sigma_1^{(m)} &= E^{(m)}\left(\varepsilon_1^m - \beta^{(m)}\Delta M^{(m)}\right) \end{aligned} \qquad (7.65)$$

where, $\beta_1^{(f)}$ and $\beta^{(m)}$ are the coefficients of hygral expansion of fibre and matrix, respectively. $\Delta M^{(f)}$ and $\Delta M^{(m)}$ are the per weight % moisture absorption for fibre and matrix, respectively. Putting this in Equation (7.64) and knowing that the axial strain in fibre and matrix are equal, that is, $\varepsilon_1^f = \varepsilon_1^m = \bar{\varepsilon}_1$

$$\bar{\varepsilon} = \frac{\beta_1^{(f)}\Delta M^{(f)} V_f E_1^{(f)} + \beta^{(m)}\Delta M^{(m)} V_m E^{(m)}}{E_1^{(f)} V_f + E^{(m)} V_m} \qquad (7.66)$$

The effective coefficient of hygral expansion in axial direction is defined as

$$\beta_1^* = \frac{\bar{\varepsilon}_1}{\Delta M} \qquad (7.67)$$

where, ΔM is the per weight % moisture absorption for composite. Thus, the above equation becomes

$$\beta_1^* = \frac{\beta_1^{(f)}\Delta M^{(f)} V_f E_1^{(f)} + \beta^{(m)}\Delta M^{(m)} V_m E^{(m)}}{\left(E_1^{(f)} V_f + E^{(m)} V_m\right)\Delta M} \qquad (7.68)$$

The above equation can be further simplified by expressing the percentage weight moisture absorbed by composite to the moisture absorbed by fibre and matrix. Thus,

$$\Delta M\, w_c = \Delta M^{(f)}\, w_f + \Delta M^{(m)}\, w_m \tag{7.69}$$

where, w_c, w_f and w_m are the masses (as defined earlier) of composite, fibre and matrix, respectively. The above equation can be written as

$$\Delta M = \Delta M^{(f)}\, W_f + \Delta M^{(m)}\, W_m \tag{7.70}$$

Thus, Equation (7.68) becomes

$$\beta_1^{*} = \frac{\beta_1^{(f)}\Delta M^{(f)} V_f E_1^{(f)} + \beta^{(m)}\Delta M^{(m)} V_m E^{(m)}}{\left(E_1^{(f)} V_f + E^{(m)} V_m\right)\left(\Delta M^{(f)}\, W_f + \Delta M^{(m)}\, W_m\right)} \tag{7.71}$$

The mass fractions are written in terms of volume fractions with the use of densities of composite, fibre and matrix. Then the above equation becomes

$$\beta_1^{*} = \frac{\beta_1^{(f)}\Delta M^{(f)} V_f E_1^{(f)} + \beta^{(m)}\Delta M^{(m)} V_m E^{(m)}}{\left(E_1^{(f)} V_f + E^{(m)} V_m\right)\left(\Delta M^{(f)}\, \rho_f\, V_f + \Delta M^{(m)}\, \rho_m\, V_m\right)} \tag{7.72}$$

The coefficient of thermal expansion in transverse direction, β_2^{*} is given as

$$\beta_1^{*} = \frac{\beta_1^{(f)}\Delta M^{(f)} V_f E_1^{(f)} + \beta^{(m)}\Delta M^{(m)} V_m E^{(m)}}{\left(E_1^{(f)} V_f + E^{(m)} V_m\right)\left(\Delta M^{(f)}\, \rho_f\, V_f + \Delta M^{(m)}\, \rho_m\, V_m\right)} \tag{7.73}$$

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Examples:

Table 7.1: Mechanical and thermal properties of fibres [5]

Property/Fibre	AS4	T300	E-Glass 21xK43 Gevetex	Silenka E-Glass 1200tex
E_1 GPa	225	230	80	74
E_2 GPa	15	15	80	74
G_{12} GPa	15	15	33.33	30.8
G_{23} GPa	7	7	33.33	30.8
ν_{12}	0.2	0.2	0.2	0.2
$\alpha_1 \times 10^{-6}$ / °C	-0.5	-0.7	4.9	4.9
$\alpha_2 \times 10^{-6}$ / °C	15	12	4.9	4.9

Table 7.2: Mechanical and thermal properties of matrix [5]

Property/Matrix	3501-6 epoxy	BSL914C epoxy	LY556/HT907/DY063 epoxy	MY750/HY917/DY063 epoxy
E GPa	4.2	4.0	3.35	3.35
G GPa	1.567	1.481	1.24	1.24
ν	0.34	0.35	0.35	0.35
$\alpha \times 10^{-6}$ / °C	45	55	58	58

Example 7.1: For AS4/3501-6 Epoxy with 0.6 fibre volume fraction calculate all mechanical and thermal properties using strength of materials approach of composite.

Solution:

1. Effective axial modulus:

$$\begin{aligned} E_1^* &= E_1^{(f)} V_f + E^{(m)} V_m = E_1^{(f)} V_f + E^{(m)} \left(1 - V_f\right) \\ &= 225 \times 0.6 + 4.2 \times 0.4 \\ &= 136.68 \ \text{GPa} \end{aligned}$$

2. Effective transverse modulus without deformation constraint satisfied

$$\frac{1}{E_2^*} = \frac{V_f}{E_2^{(f)}} + \frac{V_m}{E_2^{(m)}} = \frac{V_f}{E_2^{(f)}} + \frac{\left(1-V_f\right)}{E^{(m)}}$$
$$= \frac{0.6}{15} + \frac{0.4}{4.2}$$
$$E_2^* = 7.395 \text{ GPa}$$

Effective transverse modulus with deformation constraint satisfied

$$\frac{1}{E_2^*} = \frac{\eta^{(f)} V_f}{E_2^{(f)}} + \frac{\eta^{(m)} V_m}{E^{(m)}} = \frac{\eta^{(f)} V_f}{E_2^{(f)}} + \frac{\eta^{(m)}\left(1-V_f\right)}{E^{(m)}}$$

where,

$$\eta^{(f)} = \frac{E_1^{(f)} V_f + \left[\left(1-\nu_{12}^{(f)}\nu_{21}^{(f)}\right) E^{(m)} + \nu^{(m)}\nu_{21}^{(f)} E_1^{(f)}\right]\left(1-V_f\right)}{E_1^{(f)} V_f + E^{(m)}\left(1-V_f\right)}$$

$$\eta^{(m)} = \frac{E^{(m)} V_m + \left[\left(1-\left(\nu^{(m)}\right)^2\right) E_1^{(f)} - \left(1-\nu^{(m)}\nu_{12}^{(f)}\right) E^{(m)}\right] V_f}{E_1^{(f)} V_f + E^{(m)}\left(1-V_f\right)}$$

$$\eta^{(f)} = \frac{225\times 0.6 + \left[\left(1-0.2\times 0.0133\right)\times 4.2 + 0.34\times 0.0133\times 225\right]\left(1-0.6\right)}{225\times 0.6 + 4.2\times\left(1-0.6\right)} = 1.0029$$

$$\eta^{(m)} = \frac{4.2\times 0.34 + \left[\left(1-\left(0.34\right)^2\right)\times 225 - \left(1-0.34\times 0.2\right)\times 4.2\right]\times 0.6}{225\times 0.6 + 4.2\times\left(1-0.6\right)} = 0.8686$$

Putting these values, we get

$$\frac{1}{E_2^*} = \frac{\eta^{(f)} 0.6}{15} + \frac{\eta^{(m)} 0.4}{4.2} = 0.1288$$
$$E_2^* = 8.14 \text{ GPa}$$

3. Effective axial shear modulus

$$\frac{1}{G_{12}^*} = \frac{\bar{\gamma}_{12}}{\bar{\tau}_{12}} = \frac{V_f}{G_{12}^{(f)}} + \frac{V_m}{G^{(m)}} = \frac{V_f}{G_{12}^{(f)}} + \frac{\left(1-V_f\right)}{G^{(m)}}$$
$$= \frac{0.6}{15} + \frac{0.4}{1.567}$$
$$G_{12}^* = 7.395 \text{ GPa}$$

4. Effective axial coefficient of thermal expansion with deformation constraint satisfied

$$\alpha_1^* = \frac{\alpha_1^{(f)} E_1^{(f)} V_f + \alpha^{(m)} E^{(m)} V_m}{E_1^*}$$
$$= \frac{\left(15\times10^{-6}\right)\times 225\times0.6 + \left(45\times10^{-6}\right)\times4.2\times0.4}{136.68}$$
$$= 5.9263\times10^{-8}\ /°\mathrm{C}$$

5. Effective transverse coefficient of thermal expansion without deformation constraint satisfied

$$\alpha_2^* = \alpha_2^{(f)} V_f + \alpha^{(m)} V_m$$
$$= \left(-0.5\times10^{-6}\right)\times0.6 + \left(45\times10^{-6}\right)\times0.4$$
$$= 2.7\times10^{-5}\ /°\mathrm{C}$$

Effective transverse coefficient of thermal expansion with deformation constraint satisfied

$$\alpha_2^* = \alpha_2^{(f)} V_f + \alpha^{(m)} V_m + \left(\frac{E_1^{(f)} \nu^{(m)} - E^{(m)} \nu_{12}^{(f)}}{E_1^*}\right)\left(\alpha^{(m)} - \alpha_1^{(f)}\right)\left(1 - V_f\right) V_f$$
$$= 15\times10^{-6}\times0.6 + 45\times10^{-6}\times0.4 + \left(\frac{225\times0.34 - 4.2\times0.2}{136.68}\right)\times\left(45-15\right)\times10^{-6}\times0.4\times0.6$$
$$= 3.3045\times10^{-5}\ /°\mathrm{C}$$

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Home Work:

1. Using strength of materials approach, derive the expression for effective transverse modulus with deformation constrains satisfied.
2. Derive an expression for effective axial shear modulus of the composite using strength of materials approach.
3. Using strength of materials approach, derive the expressions for effective coefficients of thermal and hygral expansions in axial and transverse directions.
4. For fibre volume fraction of 0.6, determine all the effective mechanical and thermal properties of the fibre and matrix materials given in Table 7.1 and Table 7.2 and compare them with the experimental effective properties as reported in Soden et al [5]. Calculate percentage difference for all properties with respect to experimental effective properties.

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