

Introduction

In this lecture we will summarize some of the key points we have studied throughout this course which will be helpful for designing of laminated composites. Further, we will introduce some more key points which are very peculiar to laminated composites and very important from design and analysis point of view. Some of the considerations itself are a broad areas of study. However, in this lecture we will just introduce these issues for the sake of completeness. An additional reading on these topics is suggested to readers.

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Orthotropic and Monoclinic Behaviour:

One of the important differences of the laminated composite as compared to metals is that they are orthotropic in nature. Further, when these are transformed about an axis then the behaviour of the material changes from orthotropic to monoclinic material. The terms like C_{16} , C_{26} , C_{36} and C_{45} occur in constitutive (stiffness) relations when an orthotropic material is transformed about an axis. Thus, this makes the design and analysis little complicated and longer. Further, these terms result in unwanted coupling behaviour. The coupling effect is explained in the following.

Coupling Terms in Laminate Constitutive Relations:

The CLPT is widely used in laminate analysis. As we have studied earlier, we know that there are many coupling terms. These coupling terms are given in brief:

A_{16}, A_{26} - coupling between inplane normal and shear coupling for membrane action

B_{11}, B_{12}, B_{22} - coupling between membrane and bending actions

B_{16}, B_{26} - coupling between membrane and twisting actions

B_{16} - coupling between membrane shear and twisting actions

D_{16}, D_{26} - coupling between bending and twisting actions

The knowledge of these coupling is very much important as one coupling actions may lead to an unwanted action. At the same time it may helpful in design. For example, the extension-twisting coupling through B_{16}, B_{26} terms can be harnessed in the manufacturing of helicopter blades where a pre-twisting of blade is done.



Laminate Coefficients of Thermal and Hygral Expansions:

We have seen the variation of laminate coefficient of thermal and hygral expansion with the ply orientation. From these variations it is seen that these coefficients vary from positive to negative values. Further, it is observed that the coefficient of thermal expansion depends upon stacking sequence. This fact is very important from laminate designing point of view where it is used in an environment with large thermal gradient. One can choose a laminate sequence for which a coefficient of thermal expansion is zero. Similarly, one can choose the laminate sequence and material parameters to get a desired hygroscopic coefficient of expansion for laminates.

Thermal and Hygral Coefficients of Thermal Expansion in Off-axis Lamina:

The orthotropic materials, like isotropic materials have effects of thermal or hygral expansion only in principal material directions. They do not have shear effect in principal material directions. However, when the principal material directions are rotated about an axis then there exist a shear effect due to thermal or hygral actions. In such a rotated system one has α_{xy} or β_{xy} present in off axis laminae. Thus, a thermal or hygral change gives rise to shear effect in such off-axis laminae/laminate. This phenomenon should be studied carefully so that one can avoid it or harness is according to the design needs.



Positive and Negative Shear:

In the chapter Plane Stress Constitutive Relations we have seen the significance of positive and negative shear. In the following we revisit that section again. The figures are re-drawn for the sake of completeness of the explanation.

The direction of shear loads applied to a lamina, especially an off-axis lamina, is very important both from shear stiffness as well as strength point of view. This is explained with respect to an off-axis lamina. Here we have illustrated for $\theta = 45^\circ$.

The two cases of pure shear loading of a $\theta = 45^\circ$ lamina are shown in Figure 9.1. In these cases the direction of loading is reversed. The pure shear loading can be shown to be equivalent traction and compression loading along the 45° diagonals of a square element. This is depicted in Figure 9.1 for both cases. For the first case, the fibres are subjected to tensile normal stress and matrix is subjected to compressive normal stress, whereas for the second case, the fibres are subjected to compressive normal stress and matrix is subjected tensile normal stress. The first case of shear loading shown in Figure 9.1 is called is *Positive Shear* and the second case is called as **Negative Shear**.

In the case when fibres are oriented at $\pm 45^\circ$, either tensile or compressive normal stress is aligned along the fibres, thus resulting in higher shear stiffness at $\theta = \pm 45^\circ$. However, when the lamina is loaded in pure shear in principal material directions (as shown in Figure 9.1), the equivalent stress in fibre is neither pure normal tensile stress nor pure normal compressive stress. Thus, it results in lower shear stiffness, that is $\tau_{xy} = \tau_{12}$.

It is well known that fibres are good in traction and weak in compressive loading. Thus, it is desirable from designing point of view that the shear loading should results in an equivalent loading in which the fibres are subjected to tensile normal stress. This kind of shear loading of an off-axis lamina will ensure the higher shear strength of the lamina. In case of $\pm 45^\circ$ off-axis lamina the fibres are in pure tensile for their positive shear loading. Thus, it results into the highest shear strength.

The positive and negative shear loading has no effect in case of unidirectional lamina as shown in Figure 9.2.

The loading of an off-axis lamina in pure shear should be, in general, positive shear. This is one of the important design considerations.

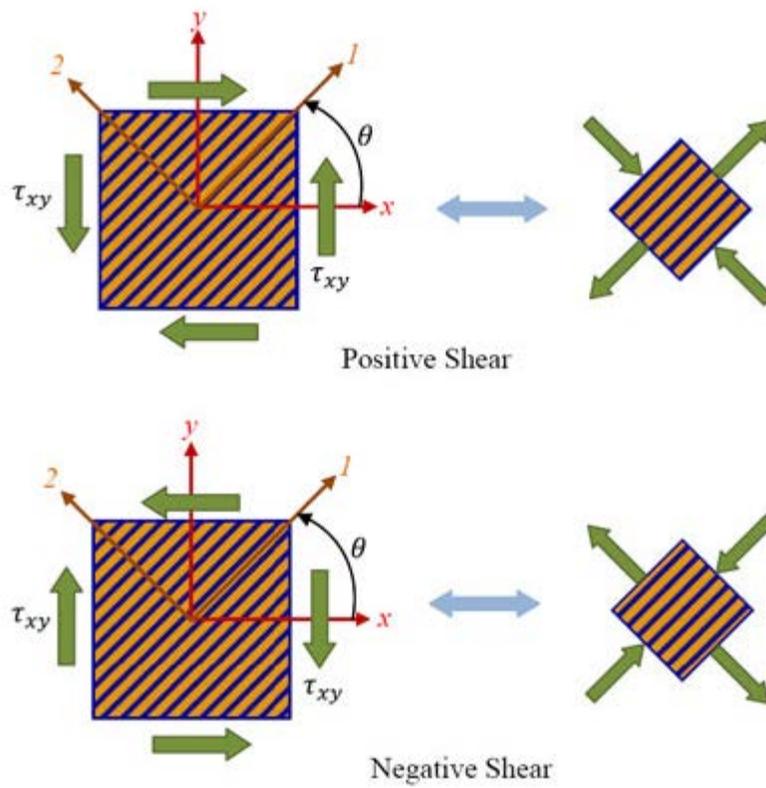


Figure 9.1: Off-axis lamina loaded in pure shear

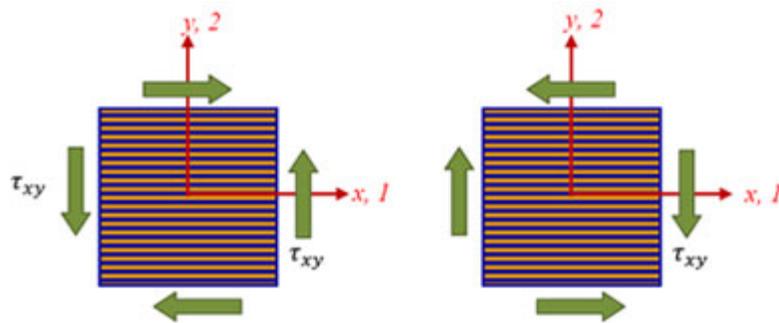


Figure 9.2: Unidirectional lamina loaded in pure shear

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Module 9: Design Considerations in Laminated Composites

Lecture 40: Design Considerations in Composites

Interlaminar Stresses:

The inplane loads applied to angle ply laminate develop the interlaminar stresses. This is because the bending-stretching coupling can give rise to transverse stresses in the interface. A schematic illustration of how axial tensile loading of angle ply laminates cause rotation of the plies is shown in Figure 6.5(b). This rotation of the plies generates the interlaminar shear stresses, which is one of the crucial factors in delamination. The interlaminar stresses are very pronounced in the region of free edges. The development of these stresses under axial extension is discussed in the following.

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Free Edge Effects:

Most of the composite laminate analysis uses the CLPT. CLPT assumes a planar state of stress along with the kinematic assumptions resulting from Kirchhoff assumptions. The determination of transverse stresses is not possible. Further, the CLPT is inadequate in determining the interlaminar stresses. The transverse stresses become significant in case of thick laminates under transverse loading. Further, these stresses are significant in case of geometric discontinuities like re-entrant corners, cut-outs, notches, ply-drops and material discontinuity as in the interfaces near the free edges. Thus, the state of stress in such regions is highly three dimensional and decays very fast inside the laminate. The CLPT fails to capture these stresses accurately in these regions. However, it is a good approximation away from these regions. It has been shown in the literature that the free-edge stress fields decays rapidly away from the laminate edges.

The free edge effects are due to discontinuous change of elastic material properties in the adjacent layers of the laminate. The development of interlaminar stresses near the free edges has been shown with the help of cross-ply symmetric laminate in the following paragraphs.

Consider a $[0/90]_S$ laminate under axial extension as shown in Figure 9.3. The x axis is along the length of the laminate, y axis is along the width and z axis is along the thickness of the laminate. The y coordinate is measured inward from the laminate edge. The CLPT predicts only intralaminar, that is, planar stresses whereas it neglects the interlaminar transverse normal (σ_{zz}) and shear (τ_{xz}, τ_{yz}) stresses. However, with the use of equilibrium equations one can determine these interlaminar stresses. It should be noted that the quality of these stresses depends upon the quality with which the planar stresses are determined.

The laminate shown has thickness of t and all the laminae are of equal thickness. Further, the laminate is sufficiently long. Therefore, the displacement components v and w are independent of x coordinate. First consider that the laminae are not bonded to each other and are subjected to axial extension. Under these constraints, each layer is free to deform individually. From our earlier studies we know that, the transverse contraction of outer 0° layers contract more than the inner 90° layers. This is because the contraction in y direction is perpendicular to the principal material directions of the 0° layers' material. Therefore, the displacement components v will be discontinuous across the thickness. However, this is not true in practical situation as the layers are perfectly bonded leading to continuous displacement in the thickness direction. Hence, to maintain a continuous displacement one should apply tensile stresses σ_{yy} on 0° layers whereas compressive stresses σ_{yy} on 90° layers. This fact is depicted in figure 9.3(b). These stresses are exactly predicted by the CLPT and together maintain the compatibility of the v displacement. The absolute values of these stresses in 0° and 90° layers are identical and thus the resultant value through the thickness vanishes, that is,

$$\int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_{yy} dz = 0 \quad (9.1)$$

Now it should be noted that the free edge is traction free. Therefore, the stresses σ_{yy} should

actually vanish. Thus, for equilibrium of forces in y direction there must be interlaminar shear stresses τ_{yz} in the interface of the 0° and 90° layers (at $z = \frac{t}{4}$). This is depicted in the exploded view of Figure 9.3(a)

Thus, we can write

$$\int_0^y \tau_{yz} d\bar{y} = \int_{\frac{t}{4}}^{\frac{t}{2}} \sigma_{yy} dz \quad (9.2)$$

This holds true for high values of y coordinate, that is, near the free edge. The variation of τ_{yz} is shown in Figure 9.4(a).

It can be seen that these stress resultants do not share a common line of action, thus it leads to the bending moment about x axis. Hence, for the equilibrium of this moment interlaminar stress σ_{zz} as depicted in Figure 9.3(c) arises at the interface between 0° and 90° layers (at $z = \frac{t}{4}$). Thus, we can write

$$\int_0^y \sigma_{zz} \bar{y} d\bar{y} = - \int_{\frac{t}{4}}^{\frac{t}{2}} \sigma_{yy} \left(z - \frac{t}{4} \right) dz \quad (9.3)$$

The interlaminar stress σ_{zz} acts only in z - direction at the interface $z = \frac{t}{4}$, the upper 0° layer as shown in the free body diagram of Figure 9.3(d) must be sufficiently long and the resultant must vanish. Thus,

$$\int_0^y \sigma_{zz} d\bar{y} = 0 \quad (9.4)$$

From this it is clear that the interlaminar stress σ_{zz} must change its sign along y - direction. The variation of σ_{zz} is shown in Figure 9.4(b). The interlaminar stress σ_{zz} shows a higher tensile value at the free edge.

Similarly one can show the development of other interlaminar shear stress τ_{xz} in angle ply laminates. For details, see [2,3].

A thorough understanding of the free edge effect is essential for a designer so that there are minimum interlaminar stresses in the structure.

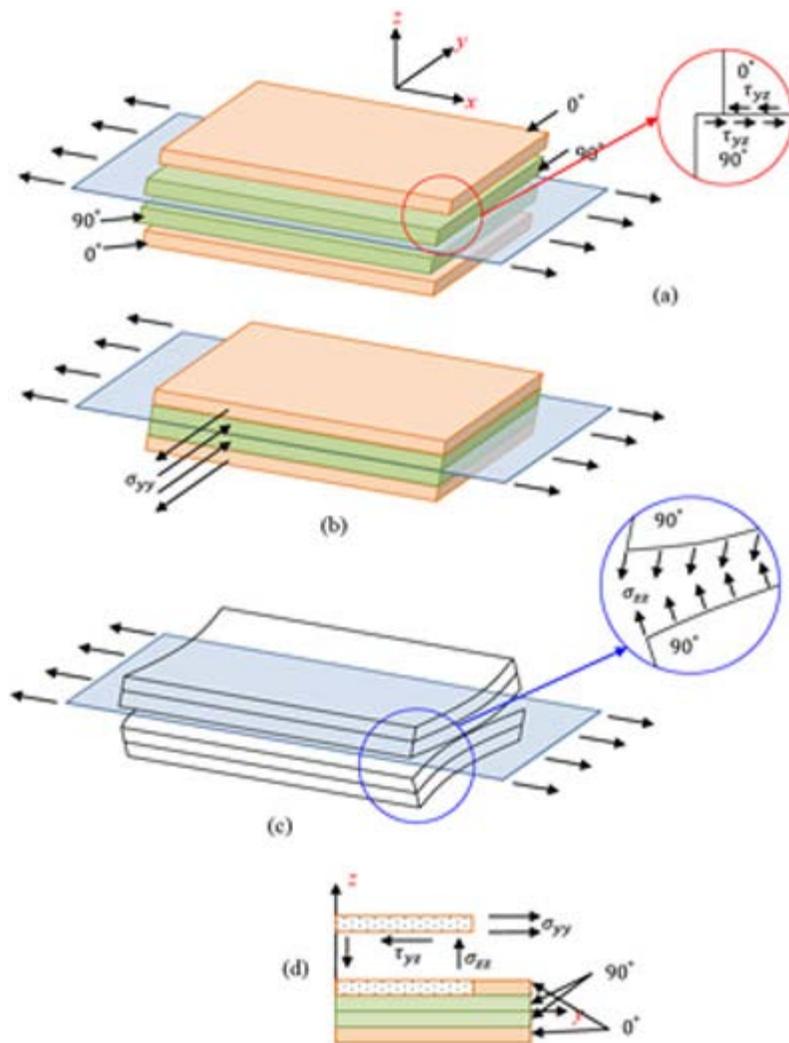


Figure 9.3: The free edge effect in $[0/90]_S$ laminate under axial extension

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Interlaminar stresses near the free edges can be controlled to an extent through the choice of materials, fibre orientations, stacking sequence, layer thickness and the use of functionally graded materials. However, the interlaminar stresses in the vicinity of free edges can completely be eliminated through the use of a homogeneous material, locally. The suppression of interlaminar stresses near the free edge by the technique of reinforcement is costly. This technique provides a restraint against the delamination due to interlaminar stress, but not a complete solution.

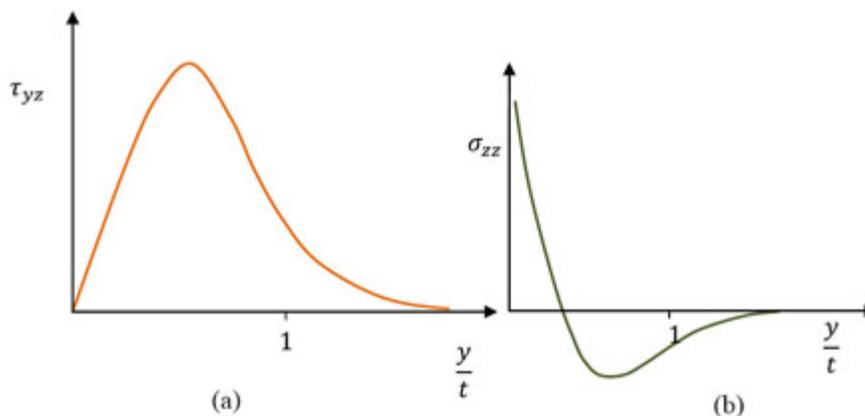


Figure 9.4: Variation of transverse stress across the width (a) τ_{yz} and (b)

σ_{zz}

Notch:

Notch in the laminates acts like an external crack giving rise to high three dimensional stress state in the vicinity of the notch. Hence, the notches should be avoided in the laminated structures.

Cut-out:

Cutouts are inevitable in structures. Cutouts are made to pass electric wires; fluid passage as in the wings, doors and windows in the fuselage of an air vehicle. These are, especially in aerospace vehicles, made also to reduce the weight of the component. The cutout boundaries act like free edges leading to significant transverse stresses. This is one of the most common site for onset of delamination. A laminate with cutout is shown in Figure 6.5(d).

Ply Drop/Termination:

The optimum design of composite structures in air vehicles is important. As a result of the optimization (e.g. weight minimization) process or sometimes purely due to geometric requirements/constraints, one or more of the plies have to be terminated (also known as “ply drop”) inside the laminate. The region of ply termination acts like a region of high stresses for neighbouring laminae which can be a reason for delamination of the plies adjacent to the ply drop region. A ply drop in laminate is shown in Figure 6.5(e).

Module 9: Design Considerations in Laminated Composites

Lecture 40: Design Considerations in Composites

Home Work:

1. What are the key points in the design considerations?
2. Explain in detail the development of interlaminar stresses in the free edge region.
3. What is positive and negative shear? Which one has detrimental effect?

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