Introduction

In the previous lectures we have introduced the concept of CCA model and then used those concepts to derive the expressions for effective axial modulus and Poisson’s ratio.

In this lecture, we continue with the CCA model to derive the expressions for effective axial shear modulus.

The Lecture Contains

- Effective Axial Shear Modulus
- Homework
- References

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Effective Axial Shear Modulus

The effective axial shear modulus is obtained by subjecting the concentric cylinders and equivalent homogeneous single cylinder to pure axial shear loading. Consider the concentric cylinders as shown in Figure 7.11. The outer surface of the cylinder in $x_1 - x_2$ plane is subjected to a displacement field such that the overall strain produced in this plane is equal to γ^0 , that is,

$$\gamma_2 = \gamma^0 \tag{7.240}$$

The displacement components on the boundary of the cylinder then becomes

$$u = 0, \qquad v = -\gamma^0 x_1 \sin\theta, \quad w = \gamma^0 x_1 \cos\theta \tag{7.241}$$

Let us assume that both fibre and matrix materials are transversely isotropic in nature. Further, assume that they experience only shear strains. Under these assumptions, it can be shown that the each component of the displacement in either of the phase is governed by Laplace equation. For the details of the derivation one can see work by Chou and Pagano [7]. For the present case of deformations, the strains are not the function of x. The displacement components in each constituent are then given with corresponding simplification in the general solution as

$$\begin{aligned} u^{(f)}(x,\theta,r) &= \left(A^f r + \frac{B^f}{r}\right)\cos\theta, & u^{(m)}(x,\theta,r) &= \left(A^m r + \frac{B^m}{r}\right)\cos\theta \\ v^{(f)}(x,\theta,r) &= -C^f x \sin\theta, & v^{(m)}(x,\theta,r) &= -C^m x \sin\theta \\ u^{(f)}(x,\theta,r) &= C^f x \cos\theta, & w^{(m)}(x,\theta,r) &= C^m x \cos\theta \end{aligned} \tag{7.242}$$

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Here, A^f, B^f, C^f, A^m, B^m and C^m are the unknown constants. Further, it should be noted that for the axisymmetric problem the displacement in fibre must be bounded. This poses a condition that the constant $B^f = 0$ as in Equation (7.178). The continuity of the displacement components at the interface may be written as

$$\begin{aligned} u^{(f)}(x,\theta,a) &= u^{(m)}(x,\theta,a) \\ v^{(f)}(x,\theta,a) &= u^{(m)}(x,\theta,a) \\ w^{(f)}(x,\theta,a) &= w^{(m)}(x,\theta,a) \end{aligned} \tag{7.243}$$

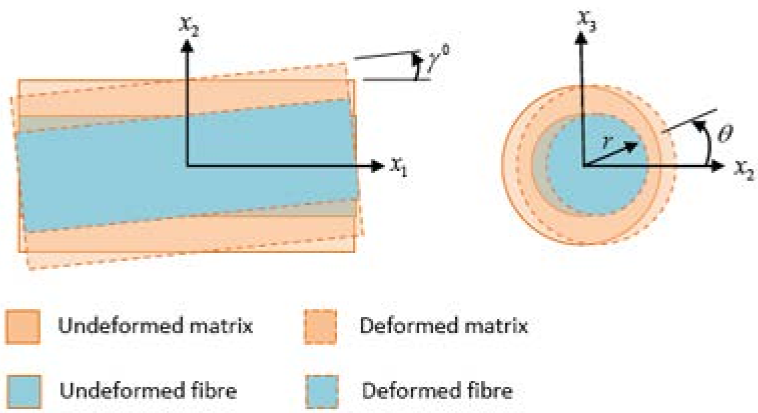

Figure 7.11: Undeformed and deformed concentric cylinders under shear γ^0

The first continuity condition of the above equation gives the relation

$$A^f a = A^m a + \frac{B^m}{a} \tag{7.244}$$

which is same as the first of Equation (7.180). The remaining two displacement continuity conditions give

$$C^f = C^m = C \tag{7.245}$$

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The non-zero stresses resulting from the displacement field in Equation (7.242) are

$$\begin{aligned} \tau_{xr}^{(f)} &= G_{12}^{(f)}(A^f + C)\cos\theta, \quad & \tau_{xr}^{(m)} &= G^{(m)}\left(A^m + C - \frac{B^m}{r^2}\right)\cos\theta \\ \tau_{x\theta}^{(f)} &= -G_{12}^{(f)}(A^f + C)\sin\theta, \quad & \tau_{x\theta}^{(m)} &= -G^{(m)}\left(A^m + C + \frac{B^m}{r^2}\right)\cos\theta \end{aligned} \tag{7.246}$$

The continuity of the stresses in radial direction leads the continuity of the stress τ_{xr} at the interface. This condition becomes

$$G_{12}^{(f)}(A^f + C) = G^{(m)}\left(A^m + C - \frac{B^m}{a^2}\right) \tag{7.247}$$

Now, at the outer boundary of the concentric cylinders the displacements must match the following boundary conditions.

$$\begin{aligned} u^{(f)}(x,\theta,b) &= 0 = \left(A^m b + \frac{B^m}{b}\right)\cos\theta \\ v^{(f)}(x,\theta,b) &= -Cx\sin\theta = -\gamma^0 x\cos\theta \\ w^{(f)}(x,\theta,b) &= Cx\cos\theta = -\gamma^0 x\cos\theta \end{aligned} \tag{7.248}$$

Note that from the second and the third of the above condition, we get

$$C = \gamma^0 \tag{7.249}$$

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The constants A^f, C, A^m and B^m can be determined in terms of γ^0 by solving Equations (7.244), (7.247), the first of Equations(7.248) and (7.249). These are

$$A^f = \frac{-(a^2-b^2)G^{(m)}\left(G_{12}^{(f)}-G^{(m)}\right)}{a^2\left(G_{12}^{(f)}-G^{(m)}\right)-b^2\left(G_{12}^{(f)}+G^{(m)}\right)}\gamma^0, \quad A^m = \frac{-a^2G^{(m)}\left(G_{12}^{(f)}-G^{(m)}\right)}{a^2\left(G_{12}^{(f)}-G^{(m)}\right)-b^2\left(G_{12}^{(f)}+G^{(m)}\right)}\gamma^0$$
$$C=\gamma^0, \quad B^m = \frac{a^2b^2G^{(m)}\left(G_{12}^{(f)}-G^{(m)}\right)}{a^2\left(G_{12}^{(f)}-G^{(m)}\right)-b^2\left(G_{12}^{(f)}+G^{(m)}\right)}\gamma^0 \tag{7.250}$$

At last, at the outer boundary the shear stress τ_{xr} must match the shear stress τ_{12} in coordinate system. Thus, at $r=b$, the shear stress then becomes

$$\tau_{xr}=\tau_{12}=G^{(m)}\left(A^m+C-\frac{B^m}{b^2}\right) \tag{7.251}$$

The right hand side of above equation can be written as

$$\tau_{12}=G_{12}^*\gamma_{12}=G_{12}^*\gamma^0 \tag{7.252}$$

Thus, the equivalent axial shear modulus can be given combining Equation (7.251) and Equation (7.252). Then values of constants A^f, C, A^m and B^m are substituted from Equation (7.250). Thus, we get the result

$$\frac{G_{12}^*}{G^{(m)}}=\frac{G_{12}^{(f)}(1+V_f)+G^{(m)}(1-V_f)}{G_{12}^{(f)}(1-V_f)+G^{(m)}(1+V_f)} \tag{7.253}$$

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Module 7: Micromechanics

Lecture 32: CCA Model: Effective Axial Shear Modulus

Home Work:

1. Write a short note on the deformation or the loads to be imposed on the concentric cylinders to determine the effective axial shear modulus.
2. Derive the expression for the effective axial shear modulus of the composite using CCA model.

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