Module 6: Failure and Damage

Lecture 21: Macroscopic Failure Theories

The Lecture Contains:

Previous Next

Introduction

In this lecture we will first present the issues or difficulties in failure theories for composite as compared to the homogeneous and isotropic materials. Then we will introduce some failure theories for unidirectional composites. We will conclude this lecture with some numerical examples.

Many theories are available for predicting failure of composites. These theories predict the failure of a lamina or laminate. Hence, these theories are called “lamina failure” or “laminate failure” theories. Further, these theories predict the very first failure in a lamina. Hence, these theories are popularly known as **“first-ply failure”** theories.

The macroscopic theories presented here are the early theories. From a designer point of view, any theory should be applicable at lamina, laminate as well as at component level. The aims of these theories were to give reasonably accurate prediction of failure as compared to experimental results and ease of implementation for analysis and design. Hence, some of theories were based on physical basis. Some of them were just extensions of theories for homogeneous; isotropic materials to composite materials. While most of the theories provided mere mathematical expressions such that it gave a best fit of the available experimental data.

All together, these theories are not good enough to predict the failure at all levels (like constituent, lamina and laminate). Further, none of them can be used for a general loading and any composite material as most of them were loadings and materials specific. At present, a significant progress has been made to address most of these issues.

Issues in Failure Theories for Unidirectional Composites:

The failure theories for unidirectional composites have some difficulties when they are extended from homogeneous, isotropic materials. In the following, we list some of the issues related to composite failure theories.

1. The composites are heterogeneous and orthotropic in nature. Hence, the effective properties in three directions need to be found.
2. The unidirectional laminae are orthotropic in nature. Hence, the strength parameters (like ultimate stress or strain) in three directions will be different.
3. In a given direction, the strength parameters will be different in tension and compression.
4. The strengths in normal direction and in shear directions are different.
5. For off axis laminae, the shear strength is different in positive and negative directions. If one is using the global coordinate system to decide the shear failure, then the positive and negative shear should be considered carefully. However, in principal material directions the positive and negative shear has no effect on shear strength.
6. The strength parameters are generally obtained experimentally in principal directions. Hence, the stresses or strains used in the failure theories should also be in principal directions. Thus, the transformation of stresses or strains from global coordinate system to principal material direction in each lamina is imperative.

7. Most of the theories do not give the mode of failure (like fibre breaking, matrix cracking, etc.). It just mentions that the lamina has failed. Further, they do not give propagation of damages until final failure.

8. The link between damage and first-ply failure is difficult to establish for failure theories.

***Note*:** (We refer to point 6 in the above.) One should not transform the strength parameters from principal coordinate system to global coordinate system to use it in a failure theory. This transformation from principal to global direction is not known. The strengths should be obtained by experiments on off-axis laminae.

Previous Next

Macroscopic Failure Theories:

In the following we will present some of the popular macroscopic failure theories used in design and analysis of composites.

In these theories we will use following quantities and symbols.

1. ***X*** denotes the ultimate normal stress magnitude in fibre direction (1-direction).
2. ***Y*** denotes the ultimate normal stress magnitude in in-plane transverse direction (2-direction).
3. ***Z*** denotes the ultimate normal stress magnitude in transverse direction (3-direction)
4. Subscript ***T*** and ***C*** denote tension and compression, respectively.
5. ***Q*, *R*** and ***S*** denote the ultimate shear stresses corresponding to 23, 13 and 12 planes.

We will see some definitions related to failure theories.

Strength Ratio (SR):

It is defined as the ratio of maximum load which can be applied such that a lamina does not fail to the actual load applied. **Thus,**

$$\mathrm{SR} = \frac{\text{Maximum load which can be applied}}{\text{Actual load applied}} \tag{6.1}$$

This concept can be extended to any failure theory. The strength ratio gives the factor by which the actual applied load can be increased or decreased upto a lamina failure. For example, if $\mathrm{SR} > 1$, it means that the lamina is safe and load applied can be increased by this factor and if $\mathrm{SR} < 1$, it means that the lamina is unsafe and the load applied must be decreased by this factor. It is needless to say that when $\mathrm{SR} = 1$ the condition corresponds to failure load.

Failure Envelope:

The failure envelope is a surface formed by various combinations of normal and shear stresses (or strains) that can be applied to a lamina just before it fails. Thus, any state of stress (or strain) which lies inside the envelope is safe whereas the one which lies on or outside the envelope is unsafe.

1. Maximum Stress Theory:

This theory is a direct extension of maximum normal stress theory proposed by Rankiene [1] and maximum stress theory proposed by Tresca [2] for homogeneous, isotropic materials. In this theory the three normal and three shear stress components are compared with corresponding ultimate stresses. A given normal stress is compared with corresponding positive and negative, that is tensile and compressive ultimate stresses. The magnitude of shear stress is compared with corresponding ultimate shear stress.

Thus, the maximum stress theory results in the following expression for the safe condition.

For normal stresses,

$$\begin{aligned} X_C &< \sigma_1 < X_T \\ Y_C &< \sigma_2 < Y_T \\ Z_C &< \sigma_3 < Z_T \end{aligned} \qquad (6.2)$$

For shear stresses,

$$\begin{aligned} |\tau_{23}| &< Q & & & |\sigma_4| &< Q \\ |\tau_{13}| &< R & &\text{OR} & |\sigma_5| &< R \\ |\tau_{12}| &< S & & & |\sigma_6| &< S \end{aligned} \qquad (6.3)$$

Thus, according to this theory initiation of failure will correspond to one or more inequalities in Equations (6.2) and (6.3) become an equality. The maximum stress theory can be represented as intersecting planes in 3*D* stress space or intersecting lines in 2*D* stress space.

Previous Next

The $\sigma_1 - \sigma_2 - \sigma_3$ stress space is shown as intersecting planes in Figure 6.7. The region inside this space is regarded as safe, whereas any point on or outside the intersecting planes will be an unsafe or a failure point. A safe state of stress with normal stresses alone is shown inside the envelope.
A fully 3D state of stress will represent an envelope or surface in six dimensional stress space.

The maximum stress theory for planar state of stress is given for normal stresses as

$$X_C < \sigma_1 < X_T$$
$$Y_C < \sigma_2 < Y_T \qquad (6.4)$$

and for shear stress as

$$|\tau_{12}| < S \qquad (6.5)$$

Now, consider that an off axis lamina is subjected to an axial stress of σ_{xx}. Then, we can write the maximum stress theory for the planar state of stress for off axis lamina as follows.

Recalling the stress transformation for planar state of stress, we write the stress components in principal material directions as

$$\begin{aligned} \sigma_1 &= \sigma_{xx}\cos^2\theta \\ \sigma_2 &= \sigma_{xx}\sin^2\theta \\ \tau_{12} &= -\sigma_{xx}\sin\theta\cos\theta \end{aligned} \qquad (6.6)$$

Thus, the maximum stress theory for off-axis lamina loaded axially can be written as

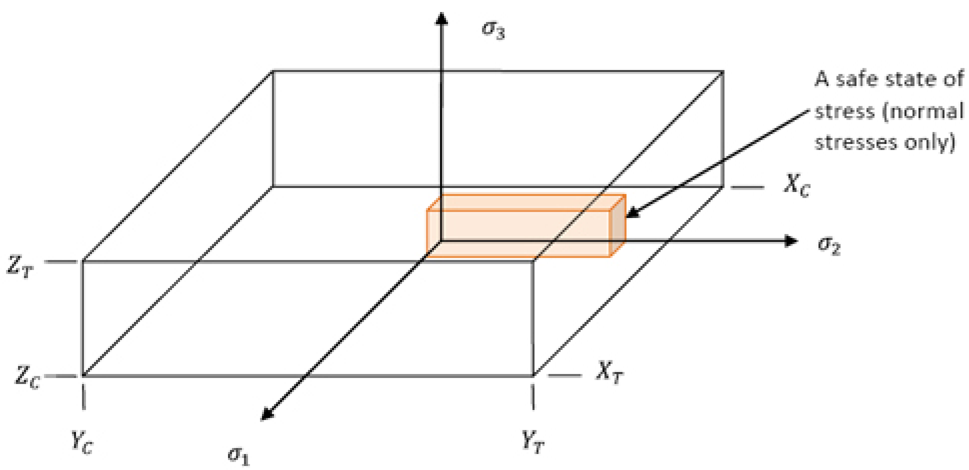

Figure 6.7: Failure envelope for normal stress space with an example safe stress state inside the envelope

$$X_C < \sigma_{xx}\, cos^2\theta < X_T$$
$$Y_C < \sigma_{xx}\, sin^2\theta < Y_T \qquad (6.7)$$
$$|-\sigma_{xx}\, sin\theta\, cos\theta| < S$$

Previous Next

2. Maximum Strain Theory:

Maximum strain theory is equivalent to maximum stress theory. This theory is based on maximum normal strain theory of St. Venant and the strain equivalent of maximum shear stress theory of Tresca for isotropic materials.

According to this theory, a lamina fails if either of the normal strain exceeds the maximum allowable strain in tension or compression or any of the shear strain exceeds the maximum allowable shear strain. The inequalities resulting are:

For normal strains,

$$\begin{aligned} \epsilon_1^C &< \epsilon_1 < \epsilon_1^T \\ \epsilon_2^C &< \epsilon_2 < \epsilon_2^T \\ \epsilon_3^C &< \epsilon_3 < \epsilon_3^T \end{aligned} \qquad (6.8)$$

For shear stresses,

$$\begin{aligned} |\gamma_{23}| &< \Gamma_{23} & & & |\epsilon_4| &< \Gamma_{23} \\ |\gamma_{13}| &< \Gamma_{13} & &\text{OR} & |\epsilon_5| &< \Gamma_{13} \\ |\gamma_{12}| &< \Gamma_{12} & & & |\epsilon_6| &< \Gamma_{12} \end{aligned} \qquad (6.9)$$

where, $\epsilon_1^T, \epsilon_2^T, \epsilon_2^T$ and $\epsilon_1^C, \epsilon_2^C, \epsilon_3^C$ are the ultimate normal strains in tension and compression, respectively. Further, $\Gamma_{23}, \Gamma_{13}, \Gamma_{12}$ are ultimate shear strains in 23, 13, 12 planes, respectively.
Thus, according to this theory initiation of failure will correspond to one or more inequalities in Equations (6.8) and (6.9) become equality.

The maximum strain theory for planar stress can be expressed as

$$\begin{aligned} \epsilon_1^C &< \epsilon_1 < \epsilon_1^T \\ \epsilon_2^C &< \epsilon_2 < \epsilon_2^T \end{aligned} \qquad (6.10)$$

and for shear strain as

$$|\gamma_{12}| < \Gamma_{12} \qquad (6.11)$$

The strains can be obtained from constitutive equation for strains in terms stresses as

$$\begin{aligned} \epsilon_1 &= \sigma_1 - \nu_{12}\ \sigma_2 \\ \epsilon_2 &= \sigma_2 - \nu_{21}\ \sigma_1 \\ \gamma_{12} &= \frac{\tau_{12}}{G_{12}} \end{aligned} \qquad (6.12)$$

These equations can be put in Equation (6.10) and Equation. (6.11). Further, for axial stress applied σ_{xx} we can write the stresses in principal directions as in Equation (6.6).

***Note*:** The maximum stress and maximum strain theories are similar. In both theories there is no interaction between various components of stress or strain. However, the two theories yield different results.

Previous Next

3. Tsai-Hill Theory:

This theory is an extension of distortional energy yield criterion of von-Mises [3] for isotropic materials as applied to anisotropic materials. It is known that total strain energy in a body is composed of two parts: One is distortion energy which cause change in shape and second one is dilation energy which causes the change in size or volume. In the von-Mises criterion it is assumed that the material fails when the maximum distortion energy of the body exceeds the distortion energy corresponding to yielding of the same material in tension.

Hill [4] extended the von-Mises distortion energy criterion of isotropic materials to anisotropic materials. Later Tsai [5, 6] extended this criterion for anisotropic materials to a unidirectional lamina. Hence, the theory is called Tsai-Hill theory.

According to this theory the failure takes place when the stress state is such that

$$f(\sigma_{ij}) = F(\sigma_2 - \sigma_3)^2 + G(\sigma_3 - \sigma_1)^2 + H(\sigma_1 - \sigma_2)^2 + 2L\,\sigma_4^2 + 2M\,\sigma_5^2 + 2N\,\sigma_6^2 = 1 \quad (6.13)$$

which upon simplifications can be written as

$$\begin{aligned}&(G+H)\,\sigma_1^2 + (F+H)\,\sigma_2^2 + (F+G)\,\sigma_3^2 - 2H\,\sigma_1\,\sigma_2 - 2G\,\sigma_1\,\sigma_3 - 2F\,\sigma_2\,\sigma_3\\ &+2L\,\sigma_4^2 + 2M\,\sigma_5^2 + 2N\,\sigma_6^2 = 1\end{aligned} \quad (6.14)$$

where, *F*, *G*, *H*, *L*, *M* and *N* are the material strength parameters. Thus, any state of stress which lies inside this envelope is safe and the one which lies on or outside the envelope is unsafe.

The strength parameters correspond to failure stresses in one dimensional loading. These can be obtained by a set of thought experiments. For example, consider that for the pure shear loading in 2-3 plane, that is with $\sigma_4 = \tau_{23} \neq 0$, with corresponding shear strength *Q* and all other stress components are zero, the Equation (6.14) becomes

$$2L = \frac{1}{Q^2} \quad (6.15)$$

Similarly, for the other two shear stress components, we can get

$$\begin{aligned}2M &= \frac{1}{R^2}\\ 2N &= \frac{1}{S^2}\end{aligned} \quad (6.16)$$

Now the strength parameters *F*, *G* and *H* are obtained by three states of stress. The state of stress $\sigma_1 \neq 0$ with corresponding strength *X* and all other stress components being zero, in Equation (6.14) leads to

$$G + H = \frac{1}{X^2} \quad (6.17)$$

Now the conditions $\sigma_2 \neq 0$ and $\sigma_3 \neq 0$ (and other stress components being zero) in Equation (6.14) result in

$$F + H = \frac{1}{Y^2}$$
$$F + G = \frac{1}{Z^2} \qquad (6.18)$$

solving the simultaneous equations in Equation (6.17) and Equation (6.18), we get the required strength parameters as

$$\begin{aligned} 2H &= \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2} \\ 2G &= \frac{1}{X^2} - \frac{1}{Y^2} + \frac{1}{Z^2} \\ 2F &= -\frac{1}{X^2} + \frac{1}{Y^2} + \frac{1}{Z^2} \end{aligned} \qquad (6.19)$$

Thus, Equation (6.14) becomes

$$\begin{aligned} &\frac{\sigma_1^2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\sigma_3^2}{Z^2} - \sigma_1\sigma_2\left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}\right) - \sigma_1\sigma_3\left(\frac{1}{X^2} - \frac{1}{Y^2} + \frac{1}{Z^2}\right) \\ &-\sigma_2\sigma_3\left(-\frac{1}{X^2} + \frac{1}{Y^2} + \frac{1}{Z^2}\right) + \frac{\sigma_4^2}{Q^2} + \frac{\sigma_5^2}{R^2} + \frac{\sigma_6^2}{S^2} = 1 \end{aligned} \qquad (6.20)$$

This is Tsai-Hill theory for 3*D* state of stress. Note that this is quadratic in stress terms with no linear terms.

Previous Next

Now consider a transversely isotropic material with inplane stresses as the significant stresses. For this planar state of stress, we have $\sigma_3 = \sigma_4 = \sigma_5 = 0$ and remaining stress components are non zero. In this case the failure envelope becomes a three dimensional space. Thus, the failure condition in Equation (6.14) becomes

$$(G+H)\,\sigma_1^2 + (F+H)\,\sigma_2^2 - 2H\,\sigma_1\,\sigma_2 + 2N\,\sigma_6^2 = 1 \qquad (6.21)$$

Now, using the strength parameters from Equation (6.17), Equation (6.18) and Equation (6.19), we get

$$\frac{\sigma_1^2}{X^2} + \frac{\sigma_2^2}{Y^2} - \sigma_1\sigma_2\left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}\right) + \frac{\sigma_6^2}{S^2} = 1 \qquad (6.22)$$

For transverse isotropy, we also have $Y = Z$. Thus, the above equation is rearranged as

$$\frac{\sigma_1^2}{X^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\sigma_6^2}{S^2} = 1 \qquad (6.23)$$

The above equation gives the Tsai-Hill criterion for failure for planar state of stress. From Tsai-Hill theory it is clear that it does not differentiate between tension and compression strengths for normal stresses. Infact, Tsai-Hill theory assumes same strengths in tension and compression. However, this situation does not occur in case of shear stresses. Thus, for normal stresses the theory represents a severe limitation that the sign of the normal stresses should be known a priori and the appropriate strength value should be used for normal stresses in the failure theory.

It should be noted that unlike maximum stress theory or maximum strain theory Tsai-Hill theory considers the interaction between three lamina strength parameters or interaction between stress components.

Further, it should be noted that Tsai-Hill theory is a unified theory and does not give the mode of failure like the maximum stress and maximum strain theory. However, one can make a guess of failure mode by calculating the quantities $\frac{\sigma_1^2}{X^2}$, $\frac{\sigma_2^2}{Y^2}$ and $\frac{\sigma_6^2}{S^2}$. The maximum of these three values can be said to give the mode of failure.

***Note*:** The right hand side of Equation (6.20) or (6.23) is called as **"failure index"**.

Previous Next

Examples

In the following examples the strength parameters for various composites used are given (properties from Soden et al [9]). In these examples AS4/3501-6 Epoxy material is used.

Table 6.1: Strength parameters for AS4/3501-6 Epoxy [9]

Composite	X_T MPa	X_C MPa	Y_T MPa	Y_C MPa	S MPa
AS4/3501-6 Epoxy	1950	1480	48	200	79
T300/BSL914C Epoxy	1500	900	27	200	80
E-glass 21xK43 Gevetex/ LY556/HT907/DY063 Epoxy	1140	570	35	114	72
Silenka E-glass 1200tex/ MY750/HY917/DY063 Epoxy	1280	800	40	145	73

Table 6.2: Ultimate strains for AS4/3501-6 Epoxy [9]

Composite	ϵ_1^T %	ϵ_1^C %	ϵ_2^T %	ϵ_2^C %	Γ_{12} %
AS4/3501-6 Epoxy	1.38	1.175	0.436	2.0	2.0
T300/BSL914C Epoxy	1.087	0.652	0.245	1.818	4.0
E-glass 21xK43 Gevetex/ LY556/HT907/DY063 Epoxy	2.132	1.065	0.197	0.644	3.8
Silenka E-glass 1200tex/ MY750/HY917/DY063 Epoxy	2.807	1.754	0.246	1.2	4.0

Example 6.1: A ply of 60° fibre orientation is in the planar state of stress. The strains are $\{\epsilon\}_{xy} = 10^{-2}\{1.2 \quad 0.2 \quad 0.1\}^T$. Check that whether lamina will fail if a) maximum stress theory b) maximum strain theory and c) Tsai-Hill theory is used.

Solution:

The strains given in global direction need to be transformed in principal directions. The state of strain and stress given here is planar. Hence, we need to use the planar transformations. For $\theta = 60^\circ$, we have

$$[T_2] = \begin{bmatrix} 0.250 & 0.750 & 0.433 \\ 0.750 & 0.250 & -0.433 \\ -0.866 & 0.866 & -0.500 \end{bmatrix} \text{ and } [Q] = \begin{bmatrix} 126.87 & 3.10 & 0 \\ 3.10 & 11.08 & 0 \\ 0 & 0 & 6.6 \end{bmatrix} 10^3\ MPa$$

First, we find the strains and stresses in principal material directions as:

$$\begin{aligned}\{\epsilon\}_{123} &= [T_2]\{\epsilon\}_{xyz}\\ &= \begin{bmatrix} 0.250 & 0.750 & 0.433\\ 0.750 & 0.250 & -0.433\\ -0.866 & 0.866 & -0.500\end{bmatrix}\begin{Bmatrix}1.2\\0.2\\0.1\end{Bmatrix}10^{-2} = \begin{Bmatrix}4.9\\9.1\\-9.2\end{Bmatrix}10^{-3}\\ \{\sigma\}_{123} &= [Q]\{\epsilon\}_{123}\\ &= \begin{bmatrix}126.87 & 3.10 & 0\\ 3.10 & 11.08 & 0\\ 0 & 0 & 6.6\end{bmatrix}10^{3}\begin{Bmatrix}4.9\\9.1\\-9.2\end{Bmatrix}10^{-3} = \begin{Bmatrix}653.96\\115.72\\-60.46\end{Bmatrix} MPa\end{aligned}$$

Maximum Stress Theory:

From stresses we see that $\sigma_1 < X_T$, $\sigma_2 > Y_T$ and $\sigma_6 < S$. Hence, according to this theory the lamina fails as σ_2 has exceeded Y_T.

Maximum Strain Theory:

From the strains in principal direction we see that $\epsilon_1 < \epsilon_1^T$, $\epsilon_2 > \epsilon_2^T$ and $\epsilon_6 < \Gamma_{12}$. Thus, according to this theory the ϵ_2 strain component has exceeded the limiting value ϵ_2^T. Hence, this lamina fails according to this theory.

Tsai-Hill Theory:

For planar stress the Tsai-Hill theory is

$$\frac{\sigma_1^2}{X^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\sigma_6^2}{S^2} = 1$$

Now we have to check whether left hand side exceeds unity or not. Here, the normal stresses are positive (tensile) hence we use tensile strength parameters in respective mode.

$$LHS = \frac{\sigma_1^2}{X_T^2} - \frac{\sigma_1\sigma_2}{X_T^2} + \frac{\sigma_2^2}{Y_T^2} + \frac{\sigma_6^2}{S^2}$$

Putting the values of stresses and strength parameters

$$\begin{aligned}LHS &= \frac{(653.96)^2}{(1950)^2} - \frac{(653.96)(115.72)}{(1950)^2} + \frac{(115.72)^2}{(48)^2} + \frac{(-60.46)^2}{(79)^2}\\ &= 0.11 - 0.02 + 5.81 + 0.59\\ &= 6.49 > 1\end{aligned}$$

The failure index is more than unity. Hence, according to this theory lamina will fail. Further, it can be seen that the contribution to failure index due to σ_2 is significant compared to other terms. Hence, the major mode of failure is tension in 2-direction of lamina.

Previous Next

Example 6.2: Find the maximum value of $P > 0$ if a state of stress of $\sigma_{xx} = 2P, \sigma_{yy} = -3P,$ and $\tau_{xy} = 4P,$ is applied to the 60^0 lamina using a) maximum stress theory b) maximum strain theory and c) Tsai-Hill theory.

Solution:

The strain transformation matrix and reduced stiffness matrix are as given above. The stress transformation matrix $[T_1]$ is given as

$$[T_1] = \begin{bmatrix} 0.250 & 0.750 & 0.866 \\ 0.750 & 0.250 & -0.866 \\ -0.433 & 0.433 & -0.500 \end{bmatrix}$$

The stresses in principal direction are

$$\begin{aligned} \{\sigma\}_{123} &= [T_1]\{\sigma\}_{xyz} \\ &= \begin{bmatrix} 0.250 & 0.750 & 0.866 \\ 0.750 & 0.250 & -0.866 \\ -0.433 & 0.433 & -0.500 \end{bmatrix} \begin{Bmatrix} 2 \\ -3 \\ 4 \end{Bmatrix} P = \begin{Bmatrix} 1.7141 \\ -2.7141 \\ -4.1651 \end{Bmatrix} P\ MPa \end{aligned}$$

The strains in principal direction are

$$\begin{aligned} \{\epsilon\}_{123} &= [Q]^{-1}\{\sigma\}_{123} \\ &= \begin{bmatrix} 0.0079 & -0.0022 & 0 \\ -0.0022 & 0.0909 & 0 \\ 0 & 0 & 0.1515 \end{bmatrix} 10^{-3} \begin{Bmatrix} 1.7141 \\ -2.7141 \\ -4.1651 \end{Bmatrix} P = \begin{Bmatrix} 0.0196 \\ -0.2505 \\ -0.6311 \end{Bmatrix} 10^{-3} P \end{aligned}$$

Maximum Stress Theory:

Using the inequalities for this theory, we have

$$\begin{aligned} -1480 < 1.7141\,P < 1950 \quad &or \quad -863.43 < P < 1137.6 \\ -200 < -2.7141\,P < 48 \quad &or \quad -17.68 < P < 73.68 \\ -79 < -4.1651\,P < 79 \quad &or \quad -18.96 < P < 18.96 \end{aligned}$$

Thus, we see that for $P = 18.96$ will cause failure as τ_{12} will exceed the limiting value.

Maximum Strain Theory:

Using the inequalities for this theory, we get

$$\begin{aligned} -0.01175 < 0.0196 \times 10^{-3}P < 0.0138 \quad &or \quad -600 < P < 704.08 \\ -0.02 < -0.2505 \times 10^{-3}P < 0.00436 \quad &or \quad -17.41 < P < 7.984 \\ -0.02 < -0.6311 \times 10^{-3}P < 0.02 \quad &or \quad -31.69 < P < 31.69 \end{aligned}$$

Thus, we see that for $P = 7.984$ will cause failure as σ_2 will exceed the limiting value in tension.

Tsai-Hill Theory:

The expression of failure envelope of this theory for planar stresses becomes

$$\frac{\sigma_1^2}{X_T^2}-\frac{\sigma_1\sigma_2}{X_T^2}+\frac{\sigma_2^2}{Y_C^2}+\frac{\sigma_6^2}{S^2}=1$$

$$\begin{aligned}LHS &= \frac{(1.7141\,P)^2}{(1950)^2}-\frac{(1.7141\,P)(-2.7141\,P)}{(1950)^2}+\frac{(-2.7141\,P)^2}{(-200)^2}+\frac{(-4.1651\,P)^2}{(79)^2}\\ &= (7.72687\times10^{-7}+1.22347\times10^{-6}+1.84158\times10^{-4}+2.77964\times10^{-3})\,P^2\\ &= 0.0029657\,P^2=1\end{aligned}$$

This gives $P = 18.36$. For this value of P the lamina fails by Tsai-Hill theory.

Previous Next

Home Work:

1. What are issues in failure theories for composites as compared to theories for homogeneous and isotropic materials?
2. Explain in detail the following failure theories.
    a. Maximum stress theory
    b. Maximum strain theory
    c. Tsai-Hill theory
3. What are the differences between maximum stress (or strain) and Tsai-Hill theory?
4. The strains $\{\epsilon\}_{xy} = 10^{-2}\{0.2 \quad 1.2 \quad 0.1\}^T$ are acting on $45°$ ply. Check weather this ply of AS4/3501-6 Epoxy material will fail or not using a) Maximum stress b) maximum strain and c) Tsai-Hill theory.
5. For $45°$ ply the state of stress acting on it is $\{\sigma\}_{xy} = \{0 \quad 0 \quad \tau\}^T$. Find the value of $\tau > 0$ for which this ply of AS4/3501-6 Epoxy material will fail by a) Maximum stress b) maximum strain and c) Tsai-Hill theory.

Previous Next

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Previous Next