Introduction

We have seen in the previous lecture the improvements over Tsai-Hill theory by Hofftman theory. Further, we have seen in detail a tensor polynomial theory proposed by Tsai and Wu. In this lecture we will see the criteria proposed by Hashin [1] which give various modes of failure. Then we will see how all mentioned theories or criteria can be expressed in terms of a tensor polynomial.

We will conclude this lecture with some numerical examples.

The Lecture Contains :

6. Hashin Criteria:

In the Tsai-Wu theory, the determination of constants like $F_{12}, F_{16},$ etc. requires a combined state of stress. These tests are very complicated and expensive. Further, Tsai-Wu theory is mathematically a smooth failure criterion which holds true for fibrous composite as well as an hexagonal crystal with only difference in the values of coefficients. These two materials have same material symmetry. However, their modes of failure are totally different.

In case of composites, the modes of failure can be fibre rupture in tension or buckling in compression, matrix may fail in transverse tension or compression. The Tsai-Wu theory predicts the ultimate failure very closely with experimental results, but the distinction between various modes of failure is not possible.

In the following we will see the mode dependent criteria proposed by Hashin [1].

Tensile Fibre Mode:

If $\sigma_1 > 0$

$$\left(\frac{\sigma_1}{X_T}\right)^2 + \frac{\sigma_5^2+\sigma_6^2}{S^2} = \begin{cases} \geq 1 & Failure \\ < 1 & No\ failure \end{cases} \qquad (6.65)$$

Compressive Fibre Mode:

If $\sigma_1 < 0$

$$\frac{\sigma_1}{X_C} = \begin{cases} \geq 1 & Failure \\ < 1 & No\ failure \end{cases} \qquad (6.66)$$

Tensile Matrix Mode:

If $\sigma_2 + \sigma_3 > 0$

$$\left(\frac{1}{Y_T}\right)^2 (\sigma_2+\sigma_3)^2 + \frac{1}{Q^2}(\sigma_4^2 - \sigma_2\sigma_3) + \frac{1}{S^2}(\sigma_5^2+\sigma_6^2) = \begin{cases} \geq 1 & Failure \\ < 1 & No\ failure \end{cases} \qquad (6.67)$$

Matrix Compressive Mode:

If $\sigma_2 + \sigma_3 < 0$

$$\frac{1}{Y_C}\left[\left(\frac{Y_C}{2Q}\right)^2 - 1\right](\sigma_2+\sigma_3) + \frac{1}{4Q^2}(\sigma_2+\sigma_3)^2 + \frac{1}{Q^2}(\sigma_4^2 - \sigma_2\sigma_3) + \frac{1}{S^2}(\sigma_5^2+\sigma_6^2) = \begin{cases} \geq 1 & Failure \\ < 1 & No\ failure \end{cases} \qquad (6.68)$$

***Interlaminar Tensile Failure*:**

If $\sigma_3 > 0$

$$\frac{\sigma_3}{Z_T} = \begin{cases} \geq 1 & Failure \\ < 1 & No\ failure \end{cases} \tag{6.69}$$

***Interlaminar Compressive Failure*:**

If $\sigma_3 < 0$

$$\frac{\sigma_3}{Z_C} = \begin{cases} \geq 1 & Failure \\ < 1 & No\ failure \end{cases} \tag{6.70}$$

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7) Plane Stress Hashin Criteria:

Hashin criteria in plane stress state is given below.

Tensile Fibre Mode:

If $\sigma_1 > 0$

$$\left(\frac{\sigma_1}{X_T}\right)^2 + \left(\frac{\sigma_6}{S}\right)^2 = \begin{cases} \geq 1 & Failure \\ < 1 & No\,failure \end{cases} \quad (6.71)$$

Compressive Fibre Mode:

If $\sigma_1 < 0$

$$\frac{\sigma_1}{X_C} = \begin{cases} \geq 1 & Failure \\ < 1 & No\,failure \end{cases} \quad (6.72)$$

Tensile Matrix Mode:

If $\sigma_2 > 0$

$$\left(\frac{\sigma_2}{Y_T}\right)^2 + \left(\frac{\sigma_6}{S}\right)^2 = \begin{cases} \geq 1 & Failure \\ < 1 & No\,failure \end{cases} \quad (6.73)$$

Matrix Compressive Mode:

If $\sigma_2 < 0$

$$\frac{\sigma_2}{Y_C}\left[\left(\frac{Y_C}{2Q}\right)^2 - 1\right] + \left(\frac{\sigma_2}{2Q}\right)^2 + \left(\frac{\sigma_6}{S}\right)^2 = \begin{cases} \geq 1 & Failure \\ < 1 & No\,failure \end{cases} \quad (6.74)$$

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Various Criteria in Tensor Polynomial Form:

In this section we express the criteria/theory we have seen in tensor polynomial form as expressed by Equation (6.35).

1) Maximum Stress Theory:

The maximum stress theory can be given in tensor polynomial form as

$$\begin{aligned}&(\sigma_1+X_T)(\sigma_1+X_C)(\sigma_2+Y_T)(\sigma_2+Y_C)(\sigma_3+Z_T)(\sigma_3+Z_C)\\&(\sigma_4+Q)(\sigma_4-Q)(\sigma_5+R)(\sigma_4-R)(\sigma_6+S)(\sigma_6-S)=0\end{aligned} \tag{6.75}$$

Thus, the strength parameters corresponding to tensor polynomial criterion as in Equation (6.35) for maximum stress theory are:

$$\begin{aligned}&F_1=\frac{1}{X_T}+\frac{1}{X_C}, && F_2=\frac{1}{Y_T}+\frac{1}{Y_C}, && F_3=\frac{1}{Z_T}+\frac{1}{Z_C}\\&F_{11}=-\frac{1}{X_TX_C}, && F_{22}=-\frac{1}{Y_TY_C}, && F_{33}=-\frac{1}{Z_TZ_C}\\&F_{44}=\frac{1}{Q^2}, && F_{55}=\frac{1}{R^2}, && F_{66}=\frac{1}{S^2},\\&F_{12}=-\frac{F_1F_2}{2}, && F_{13}=-\frac{F_1F_3}{2}, && F_{23}=-\frac{F_2F_3}{2},\end{aligned} \tag{6.76}$$

The remaining strength parameters are zero. In above equation the higher order terms are neglected.

2) Tsai-Hill Theory:

Comparing Equation (6.20) with Equation (6.35), we get

$$F_i=0$$

$$\begin{aligned}&F_{11}=\frac{1}{X^2}, && F_{22}=\frac{1}{Y^2}, && F_{33}=\frac{1}{Z^2}\\&F_{44}=\frac{1}{Q^2}, && F_{55}=\frac{1}{R^2}, && F_{66}=\frac{1}{S^2}\\&F_{12}=-\frac{1}{2}\left(\frac{1}{X^2}+\frac{1}{Y^2}-\frac{1}{Z^2}\right), && F_{13}=-\frac{1}{2}\left(\frac{1}{X^2}-\frac{1}{Y^2}+\frac{1}{Z^2}\right), && F_{23}=-\frac{1}{2}\left(-\frac{1}{X^2}+\frac{1}{Y^2}+\frac{1}{Z^2}\right)\end{aligned} \tag{6.77}$$

***3) Hoffman Theory*:**

Comparing Equation (6.33) and Equation (6.35), we get

$$\begin{aligned}&F_1=\frac{1}{X_T}+\frac{1}{X_C}, && F_2=\frac{1}{Y_T}+\frac{1}{Y_C}, && F_3=\frac{1}{Z_T}+\frac{1}{Z_C}\\&F_{11}=-\frac{1}{X_TX_C}, && F_{22}=-\frac{1}{Y_TY_C}, && F_{33}=-\frac{1}{Z_TZ_C}\\&F_{44}=\frac{1}{Q^2}, && F_{55}=\frac{1}{R^2}, && F_{66}=\frac{1}{S^2}\end{aligned}$$

$$
\begin{aligned}
F_{12} &= \frac{1}{2}\left[-\frac{1}{X_C\,X_T} - \frac{1}{Y_C\,Y_T} + \frac{1}{Z_C\,Z_T}\right] \\
F_{13} &= \frac{1}{2}\left[-\frac{1}{X_C\,X_T} + \frac{1}{Y_C\,Y_T} - \frac{1}{Z_C\,Z_T}\right] \\
F_{23} &= \frac{1}{2}\left[\frac{1}{X_C\,X_T} - \frac{1}{Y_C\,Y_T} - \frac{1}{Z_C\,Z_T}\right]
\end{aligned}
\qquad (6.78)
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Examples:

Example 6.5**:** For the details in Example 6.1, check the modes of failure using Hashin's criteria.

Example 6.6**:** For the details in Example 6.2, check the modes of failure using Hashin's criteria.

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Module 6: Failure and Damage

Lecture 23: Macroscopic Failure Theories

Home Work:

1. Explain Hashin's criteria for three dimensional and planar state of stress.
2. Explain connection of various failure theories with respect to tensor polynomial criterion.
3. Verify the strength parameters of all the theories studied according tensor polynomial criterion.

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