Module-3

Lecture-15

Static Stability and Control - Elevator Control power, Elevator Angle to trim and Estimation of Stick Fixed Neutral Point

Longitudinal Control

Control of the pitch attitude of the airplane can be achieved by deflecting the elevator.

Elevator Effectiveness

When the elevator is deflected, it changes the lift and the pitching moment of the airplane. Change in lift for the airplane, ΔC_L;

$$\Delta C_L = C_{L_{\delta e}} \delta e$$

where,

$$C_{L_{\delta e}} = \frac{\partial C_L}{\partial \delta e}$$

$$(C_L)_{a/c} = C_{L_\alpha} \alpha + C_{L_{\delta e}} \delta e$$

Similarly change in the pitching moment, ΔC_m

$$\Delta C_m = C_{m_{\delta e}} \delta e$$

where,

$$C_{m_{\delta e}} = \frac{\partial C_m}{\partial \delta e}$$

$C_{m_{\delta e}}$: Elevator Control Power

$$(C_m)_{a/c} = C_{m_o} + C_{m_\alpha} \alpha + C_{m_{\delta e}} \delta e$$

The variation of C_m with δe in presented in Figure 1. Kindly note that Slope $\partial C_m / \partial \alpha$ remains same when elevator is deflected.

Expression for $C_{L_{\delta e}}$ and $C_{m_{\delta e}}$ The change in lift of the airplane due to deflecting the elevator is equal to the change in lift force acting on the tail

$$\Delta L = \Delta L_t = \Delta C_{L_t} q_t S_t$$

Let,

$$q_t \text{ - dynamic pressure at tail}$$

$$\Delta C_L = \frac{\Delta L_t}{\frac{1}{2} \rho V^2 S_w} = \frac{S_t}{S_w} \eta \frac{\partial C_{L_t}}{\partial \delta e} \delta e$$

$$\frac{\partial C_{L_t}}{\partial \delta e}\text{: Elevator Effectiveness}$$

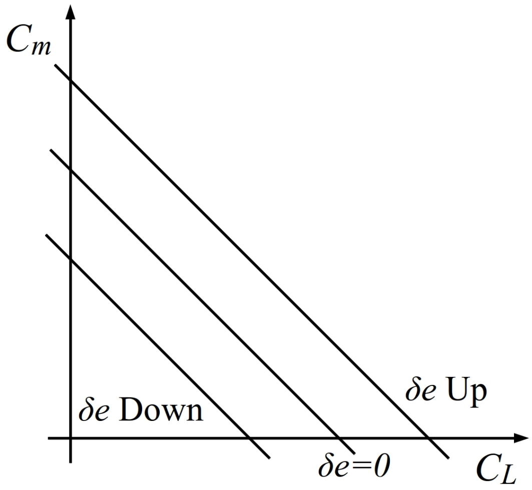

Figure 1: Graph showing relation between C_m and C_L or α

The elevator effectiveness is proportional to the size of the flap being used.

$$\frac{\partial C_{L_t}}{\partial \delta e} = \frac{\partial C_{L_t}}{\partial \alpha, t} . \frac{\partial \alpha, t}{\partial \delta e} = C_{L_{\alpha,t}} \tau$$

Typical value of τ lies between 0.2 to 0.6.

$$\Delta C_L = \frac{S_t}{S_w} \eta \frac{\partial C_{L_t}}{\partial \delta e} \delta e = \frac{S_t}{S_w} \eta C_{L_{\alpha,t}} \tau \delta e$$

$$\Rightarrow C_{L_{\delta e}} = \frac{S_t}{S_w} \eta C_{L_{\alpha,t}} \tau \tag{1}$$

The increment in pitching moment, ΔC_m

$$\Delta C_m = -\Delta C_L \frac{l_t}{\bar{c}} = -\frac{S_t}{S_w} \eta C_{L_{\alpha,t}} \tau \left(\frac{l_t}{\bar{c}} \right) \delta e$$

$$\Delta C_m = -\left(\frac{S_t l_t}{S_w \bar{c}} \right) \eta C_{L_{\alpha,t} \tau \delta e}$$

$$\Rightarrow C_{m_{\delta e}} = -V_H \eta C_{L_{\alpha,t}} \tau \tag{2}$$

This is known as elevator control power.

Elevator angle to trim

An aircraft is said to be trimmed if the net forces and moments acting on the airplane are zero.

$$C_m = C_{m_o} + C_{m_\alpha}\alpha + C_{m_{\delta e}}\delta e$$

At trim, $C_m = 0$

$$0 = C_{m_o} + C_{m_\alpha}\alpha_{trim} + C_{m_{\delta e}}\delta e_{trim}$$

The lift coefficient at trim is $C_{L_{trim}}$

$$C_{L_{trim}} = C_{L_\alpha}\alpha_{trim} + C_{L_{\delta e}}\left(\delta e\right)_{trim}$$

$$\alpha_{trim} = \frac{\left[C_{L_{trim}} - C_{L_{\delta e}}\left(\delta e\right)_{trim}\right]}{C_{L_\alpha}} \tag{3}$$

Combining α_{trim} and $C_{L_{trim}}$ equations,

We have

$$\delta_{e_{trim}} = \left[\frac{C_{m_o}C_{L_\alpha} + C_{m_\alpha}C_{L_{trim}}}{C_{m_{\delta e}}C_{L_\alpha} - C_{m_\alpha}C_{L_{\delta e}}}\right]$$

or

$$\delta e_{trim} = -\frac{C_{m_o}}{C_{m_{\delta e}} - \frac{C_{m_\alpha}}{C_{L_\alpha}}C_{L_{\delta e}}} - \frac{\frac{C_{m_\alpha}}{C_{L_\alpha}}C_{L_{trim}}}{C_{m_{\delta e}} - \frac{C_{m_\alpha}}{C_{L_\alpha}}C_{L_{\delta e}}}$$

Assuming,

$$\frac{C_{m_\alpha}}{C_{L_\alpha}}C_{L_{\delta e}} \ll C_{m_{\delta e}}$$

We have,

$$\delta e_{trim} = -\frac{C_{m_o}}{C_{m_{\delta e}}} - \frac{\frac{C_{m_\alpha}}{C_{L_\alpha}}C_{L_{trim}}}{C_{m_{\delta e}}}$$

$$\frac{C_{m_\alpha}}{C_{L_\alpha}} = \frac{\partial C_m}{\partial C_L}$$

so

$$\delta e_{trim} = -\frac{C_{m_o}}{C_{m_{\delta e}}} - \left(\frac{\frac{\partial C_m}{\partial C_L}}{C_{m_{\delta e}}}\right)C_{L_{trim}} \tag{4}$$

$$\left(\delta e\right)_{trim} = \delta e_o + \left(\frac{\partial \delta e}{\partial C_L}\right)_{trim} C_{L_{trim}}$$

where

$$\delta e_o = -\frac{C_{m_o}}{C_{m_{\delta e}}}$$

$$\frac{\partial \delta e}{\partial C_{L_{trim}}} = -\frac{\frac{\partial C_m}{\partial C_L}}{C_{m_{\delta e}}}$$

$$\delta e_{trim} = \delta e_o + \left(\frac{\partial \delta e}{\partial C_{L_{trim}}}\right) C_{L_{trim}} \tag{5}$$

This equation can be used to estimate the value of elevator deflection required to trim a given aircraft at a particular $C_{L_{trim}}$

Estimation of Neutral Point (Stick Fixed)

$$\delta e_{trim} = \delta e_o + \left(\frac{\partial \delta e}{\partial C_{L_{trim}}}\right) C_{L_{trim}}$$

$$\frac{\partial \delta e_{trim}}{\partial C_{L_{trim}}} = -\frac{\frac{\partial C_m}{\partial C_L}}{C_{m_{\delta e}}}$$

Neutral point is the $c.g.$ location at which $\partial C_m/\partial C_L = 0$

Therefore, at neutral point,

$$\frac{\partial \delta e_{trim}}{\partial C_{L_{trim}}} = 0$$

Flight test to estimate Stick Fixed Neutral Point

Above equation can be used to estimate stick fixed neutral point.

- Fly at different center of gravity configuration and execute cruise
- Estimate corresponding
  $$C_{Ltrim} = \frac{2W/S}{\frac{1}{2}\rho V^2}$$
  and record δe
- Plot δe_{trim} v/s $C_{L_{trim}}$
- Cross plot $[\partial \delta e/\partial C_L]_{trim}$ v/s $\bar{x}_{cg}$ to get neutral point.