Module-7

Lecture-33

Test methods to determine dominant lateral-directional stability coefficients: Steady Coordinated turn, Estimation of roll derivative (C_{l_r}), Estimation of yaw moment coefficient (C_{n_r})

Steady coordinated turn (without side slip, $\beta = 0$)

- The aim of this experiment is to determine the lateral and directional control angles required for trim under steady coordinated turn. This study also enables the estimation of some of the directional and lateral static stability derivatives of the airplane (in approximate manner).
- Watch: `https://www.youtube.com/watch?v=Cpviulm7VNw`
- The side force equation F_y, for steady coordinated turn is as follows:

$$F_y = \frac{1}{2}\rho V_T^2 S \left[C_{y_p}\frac{pb}{2V_T} + C_{y_r}\frac{rb}{2V_T} + C_{y_\beta}\beta + C_{y_{\delta a}}\delta a + C_{y_{\delta r}}\delta r \right] + mg\cos\theta\sin\phi$$
$$= m(\dot{v} + ru - pw)$$

- For steady coordinated level turn with small bank angle, $\dot{v} = 0$, $p = 0$, $r \neq 0$ and $\beta = 0$ (no slip). Thus the above force equation gets simplified to:

$$mg\phi + \frac{1}{2}\rho V_T^2 S \left[C_{y_r}\frac{rb}{2V_T} + C_{y_{\delta a}}\delta a + C_{y_{\delta r}}\delta r \right] = mrV_T$$

note, $u \cong V_T$

- Neglecting C_{y_r}, $C_{y_{\delta a}}$ and $C_{y_{\delta r}}$ contribution (generally small), we have:

$$mg\phi = mV_T r$$

$$-V_T r + g\phi = 0$$

- This equation indicates a relationship between the rate of turn and the bank angle at a given flight speed to maintain a steady coordinated turn (no side slip).

Estimation of roll derivative (C_{l_r})

- Rolling Moment Equation:

$$C_l = C_{l_p}\frac{pb}{2V_T} + C_{l_r}\frac{rb}{2V_T} + C_{l_\beta}\beta + C_{l_{\delta a}}\delta a + C_{l_{\delta r}}\delta r$$

- For steady coordinated turn (static case) $C_l = 0$, $p = 0$, $\beta = 0$. This implies,

$$C_{l_r}\frac{rb}{2V_T} + C_{l_{\delta a}}\delta a + C_{l_{\delta r}}\delta r = 0$$

- Given the value of $C_{l_{\delta a}}$ and $C_{l_{\delta r}}$, the drivative C_{l_r} can be estimated.

- Neglecting $C_{l_{\delta r}}$ contribution, we have:

$$C_{l_r}\frac{rb}{2V_T} + C_{l_{\delta a}}\delta a = 0$$

- This rolling moment equation indicates that the aileron is applied solely to counteract the rolling moment due to yaw which generally tends to bring the lower wing further down. Also

$$r = \frac{g\phi}{u} = \frac{g\phi}{V_T}$$

since, $u \cong V_T$. Substituting this into

$$C_{l_r}\frac{rb}{2V_T} + C_{l_{\delta a}}\delta a = 0$$

we have:

$$C_{l_r}\left(\frac{g\phi}{V_T}\right)\frac{b}{2V_T} + C_{l_{\delta a}}\delta a = 0$$

$$C_{l_r}\left(\frac{g\phi}{2}\right)\frac{b}{V_T^2} = -C_{l_{\delta a}}\delta a$$

$$\frac{\delta a}{\phi} \propto \frac{1}{V_T^2}$$

Flight test to estimate roll derivative (C_{l_r})

Step 1: Record the takeoff weight of the aircraft.

Step 2: Note down the velocity (v), altitude (h), aileron deflection (δa), and roll angle (ϕ) during the steady coordinated turn maneuver.

Step 3: Now given the value of $C_{l_{\delta a}}$ the other derivative C_{l_r} can be estimated with the help of the following expression.

$$C_{l_r}\left(\frac{g\phi}{2}\right)\frac{b}{V_T^2} = -C_{l_{\delta a}}\delta a$$

Step 4: Also plot δa vs ϕ.

Estimation of yawing moment coefficient (C_{n_r})

- Yawing Moment Equation:

$$C_n = C_{n_p}\frac{pb}{2V_T} + C_{n_r}\frac{rb}{2V_T} + C_{n_\beta}\beta + C_{n_{\delta a}}\delta a + C_{n_{\delta r}}\delta r$$

under the steady coordinated turn maneuver (no side slip,$p = 0$ and $\beta = 0$), it reduces to:

$$C_{n_r}\frac{rb}{2V_T} + C_{n_{\delta a}}\delta a + C_{n_{\delta r}}\delta r = 0$$

- This equation indicates that the rudder is held to counteract the moments due to yaw damping C_{n_r} and the aileron yaw $C_{n_{\delta a}}$.

- If $C_{n_{\delta a}}$ is neglected then,

$$C_{n_r}\frac{rb}{2V_T} = -C_{n_{\delta r}}\delta r$$

$$\frac{r}{\delta r} = -\frac{C_{n_{\delta r}}}{C_{n_r}}\left(\frac{2V_T}{b}\right)$$

- Generally $C_{n_{\delta r}}$, C_{n_r} are negative. Thus, one is expected to see $r/\delta r$ to be negative for a stable aircraft. Also knowing the numerical values of $C_{n_{\delta r}}$, r, δr,V_T and b, the dutch roll damping C_{n_r} can be estimated.

Flight test to estimate yawing moment coefficient (C_{n_r})

Step 1: Record the takeoff weight of the aircraft.

Step 2: Note down the velocity (v), altitude (h), rudder deflection (δr), and roll angle (ϕ) during the steady coordinated turn maneuver.

Step 3: Now given the value of $C_{n_{\delta r}}$ the other derivative C_{n_r} can be estimated with the help of the following expression.

$$\frac{r}{\delta r} = -\frac{C_{n_{\delta r}}}{C_{n_r}}\left(\frac{2V_T}{b}\right)$$

Step 4: Also Plot δr vs r. The slope will be negative.

Note:

Since $C_{n_{\delta r}}$ ***is negative and*** C_{n_r} ***is negative*** $-C_{n_{\delta r}}/C_{n_r}$ ***will be negative.***