Module-6

Lecture-28

Flight Path Equations, Gravity Equations and Combined 6-DOF Model

Flight path equations

- Relationship between components of the linear velocities between the body fixed frame and earth fixed coordinate system.

- In X', Y', Z' frame (earth fixed)

$$\boldsymbol{V'} = \dot{X}'\hat{\boldsymbol{\imath}} + \dot{Y}'\hat{\boldsymbol{\jmath}} + \dot{Z}'\hat{\boldsymbol{k}}$$

- In X, Y, Z frame (body fixed)

$$\boldsymbol{V} = \dot{X}\hat{\boldsymbol{\imath}} + \dot{Y}\hat{\boldsymbol{\jmath}} + \dot{Z}\hat{\boldsymbol{k}}$$

- Since assumed frame (X_1, Y_1, Z_1) is parallel to (X', Y', Z') frame, so

$$U_1 = \dot{X}_1 = \dot{X}'$$

$$V_1 = \dot{Y}_1 = \dot{Y}'$$

$$W_1 = \dot{Z}_1 = \dot{Z}'$$

- Using transformation equation from reference frame (X_1, Y_1, Z_1) to reference frame (X_2, Y_2, Z_2)

$$\begin{Bmatrix} U_1 \\ V_1 \\ W_1 \end{Bmatrix} = \begin{bmatrix} \cos\Psi & -\sin\Psi & 0 \\ \sin\Psi & \cos\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} U_2 \\ V_2 \\ W_2 \end{Bmatrix} = R^{X_1Y_1Z_1 \to X_2Y_2Z_2} \begin{Bmatrix} U_2 \\ V_2 \\ W_2 \end{Bmatrix}$$

- Using transformation equation from reference frame (X_2, Y_2, Z_2) to reference frame (X_3, Y_3, Z_3)

$$\begin{Bmatrix} U_2 \\ V_2 \\ W_2 \end{Bmatrix} = \begin{bmatrix} \cos\Theta & 0 & \sin\Theta \\ 0 & 1 & 0 \\ -\sin\Theta & 0 & \cos\Theta \end{bmatrix} \begin{Bmatrix} U_3 \\ V_3 \\ W_3 \end{Bmatrix} = R^{X_2Y_2Z_2 \to X_3Y_3Z_3} \begin{Bmatrix} U_3 \\ V_3 \\ W_3 \end{Bmatrix}$$

- Using transformation equation from reference frame (X_3, Y_3, Z_3) to reference frame (X, Y, Z)

$$\begin{Bmatrix} U_3 \\ V_3 \\ W_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & -\sin\Phi \\ 0 & \sin\Phi & \cos\Phi \end{bmatrix} \begin{Bmatrix} U \\ V \\ W \end{Bmatrix} = R^{X_3Y_3Z_3 \to XYZ} \begin{Bmatrix} U \\ V \\ W \end{Bmatrix}$$

$$\begin{Bmatrix} \dot{X}' \\ \dot{Y}' \\ \dot{Z}' \end{Bmatrix} = \begin{Bmatrix} U_1 \\ V_1 \\ W_1 \end{Bmatrix} = R^{X_1Y_1Z_1 \to X_2Y_2Z_2}.R^{X_2Y_2Z_2 \to X_3Y_3Z_3}.R^{X_3Y_3Z_3 \to XYZ} \begin{Bmatrix} U \\ V \\ W \end{Bmatrix}$$

$$\begin{Bmatrix} \dot{X}' \\ \dot{Y}' \\ \dot{Z}' \end{Bmatrix} = R^{X_1Y_1Z_1 \to X_2Y_2Z_2}.R^{X_2Y_2Z_2 \to X_3Y_3Z_3}.R^{X_3Y_3Z_3 \to XYZ} \begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{Bmatrix}$$

Therefore

$$\begin{Bmatrix} \dot{X}' \\ \dot{Y}' \\ \dot{Z}' \end{Bmatrix} = \begin{Bmatrix} U_1 \\ V_1 \\ W_1 \end{Bmatrix} =$$

$$\begin{bmatrix} \cos\Psi & -\sin\Psi & 0 \\ \sin\Psi & \cos\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\Theta & 0 & \sin\Theta \\ 0 & 1 & 0 \\ -\sin\Theta & 0 & \cos\Theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & -\sin\Phi \\ 0 & \sin\Phi & \cos\Phi \end{bmatrix} \times \begin{Bmatrix} U \\ V \\ W \end{Bmatrix}$$

$$\begin{Bmatrix} \dot{X}' \\ \dot{Y}' \\ \dot{Z}' \end{Bmatrix} =$$

$$\begin{bmatrix} \cos\Psi\cos\Theta & -\sin\Psi\cos\Phi + \cos\Phi\sin\Theta\sin\Phi & \sin\Psi\sin\Phi + \cos\Psi\sin\Theta\cos\Phi \\ \sin\Psi\cos\Theta & \cos\Psi\cos\Phi + \sin\Phi\sin\Theta\sin\Phi & -\sin\Phi\cos\Psi + \sin\Psi\sin\Theta\cos\Phi \\ -\sin\Theta & \cos\Theta\sin\Phi & \cos\Theta\cos\Phi \end{bmatrix} \times \begin{Bmatrix} U \\ V \\ W \end{Bmatrix}$$

Gravity equations

$$\boldsymbol{g} = \hat{\boldsymbol{k'}} g = \hat{\boldsymbol{k_1}} g = \hat{\boldsymbol{k_2}} g = g_X \hat{\boldsymbol{\imath}} + g_Y \hat{\boldsymbol{\jmath}} + g_Z \hat{\boldsymbol{k}}$$

From previous discussions

$$\hat{\boldsymbol{k_2}} = -\sin\Theta \hat{\boldsymbol{\imath}} + \cos\Theta\sin\Phi \hat{\boldsymbol{\jmath}} + \cos\Theta\cos\Phi \hat{\boldsymbol{k}}$$

so

$$g\hat{\boldsymbol{k_2}} = g\left(-\sin\Theta \hat{\boldsymbol{\imath}} + \cos\Theta\sin\Phi \hat{\boldsymbol{\jmath}} + \cos\Theta\cos\Phi \hat{\boldsymbol{k}}\right) = g_X \hat{\boldsymbol{\imath}} + g_Y \hat{\boldsymbol{\jmath}} + g_Z \hat{\boldsymbol{k}}$$

$$
\begin{aligned}
g_X &= -g \sin\Theta \\
g_Y &= g \cos\Theta \sin\Phi \\
g_Z &= g \cos\Theta \cos\Phi
\end{aligned}
$$

Combined 6-DOF model

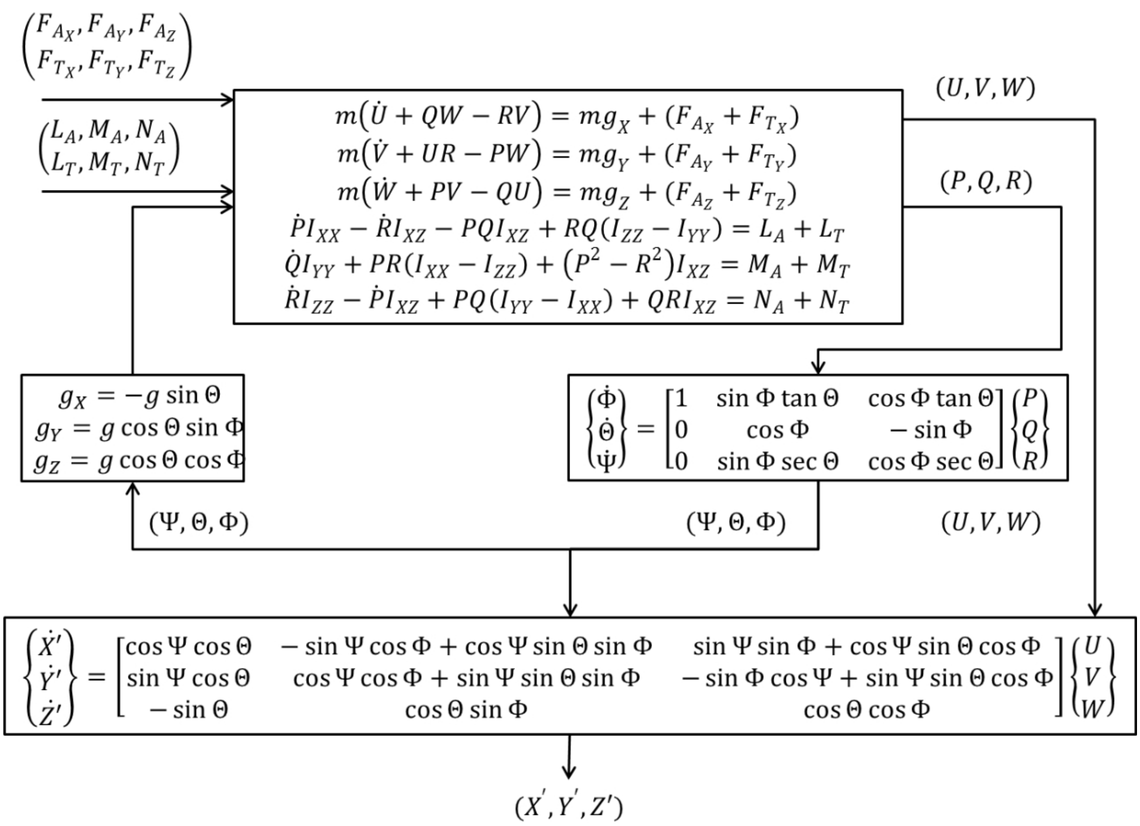

Figure 1: Block diagram showing the integration of the aircraft equation of motion