Module-7

Lecture-32

Test methods to determine dominant lateral-directional stability coefficients: Estimation of side slip coefficient $\left(C_{y_\beta}\right)$, Estimation of yaw moment coefficient $\left(C_{n_\beta}\right)$, Estimation of roll derivative $\left(C_{l_\beta}\right)$

Test methods to determine dominant lateral-directional stability coefficients

Steady Side Slip:

- The basic test method is the steady, state side slip - a classical, proven way to determine static directional stability and dihedral effect from single maneuver. It depends, though, on prior knowledge of the control derivatives like $C_{l_{\delta a}}$ and C_{n_r}.
- Like any other stability and control testing, the method also demands appropriate instrumentation, careful attention to trim, and precise control of airspeed and altitude.
- It may also be noted that these flight test techniques, though no longer widely used since parameter estimation technique have become popular, offers the advantage of simplicity and relatively uncomplicated instrumentation requirements.
- For our purpose, it also serves as a useful way to teach the fundamentals of such measurements.
- The classical steady, state test method is commonly used to obtain measures of C_{l_β} and C_{n_β} for all classes of airplanes.
- In this test, maneuver data is collected at various constant and stabilized heading. The pilot should choose a reference point on the distant visual horizon and use it to fly a stabilized heading for each individual point (side-slip angle).

- Rudder and aileron are applied, essentially simultaneously, to set up a stabilized side slip. This cross-controlled condition is an unnatural piloting technique; the rudder is normally used to maintain zero side slip in all maneuvers.
- Using rudder and aileron in opposite directions, particularly at large side slip angles, gives the pilot a sensation of "sliding sideways in the seat" due to the lateral force that are applied.
- These unusual sensations and unnatural control applications mean that the pilot must concentrate carefully on setting up the conditions and maintaining them.
- When equilibrium conditions have been attained, the data system should be turned on to record altitude, outside air temperature, airspeed, side slip angle, bank angle, control surface deflection, etc.

Estimation of side slip coefficient (C_{y_β})

- For steady side slip maneuver (level, $\theta = 0$), F_y, p, r and $\dot{v}$ are zero. The y force can be simplified as shown below:

$$0 = mg\sin\phi + \frac{1}{2}\rho V_T^2 S\left[C_{y_\beta}\beta + C_{y_{\delta a}}\delta a + C_{y_{\delta r}}\delta r\right]$$

further, for small bank angle, this simplifies to:

$$0 = mg\phi + \frac{1}{2}\rho V_T^2 S\left[C_{y_\beta}\beta + C_{y_{\delta a}}\delta a + C_{y_{\delta r}}\delta r\right]$$

- Dividing both side by $\frac{1}{2}\rho V_T^2 S$, we get:

$$0 = \frac{mg\phi}{\frac{1}{2}\rho V_T^2 S} + C_{y_\beta}\beta + C_{y_{\delta a}}\delta a + C_{y_{\delta r}}\delta r$$

- For small ϕ, $L = W$. i.e. $L = mg$ and noting:

$$\frac{mg}{\frac{1}{2}\rho V_T^2 S} = C_L = \frac{L}{\frac{1}{2}\rho V_T^2 S}$$

we get:

$$C_L\phi + C_{y_\beta}\beta + C_{y_{\delta a}}\delta a + C_{y_{\delta r}}\delta r = 0$$

- Generally, $C_{y_{\delta a}}$ is negligible and further if we neglect contribution to side force due to rudder, (please note rudder has dominating contribution to yawing moment), then side-force equation can be represented as:

$$C_L\phi + C_{y_\beta}\beta = 0 \tag{1}$$

or,

$$C_{y_\beta} = -C_L \left(\frac{\phi}{\beta}\right)$$

- The derivative C_{y_β} can be estimated with the help of measured values of C_L, ϕ and β. The lift coefficient C_L is obtained through $C_L = W/qS$ where $q = \rho V_T^2/2$

Flight test to estimate side slip coefficient (C_{y_β})

Step 1: Record the takeoff weight of the aircraft.

Step 2: Note down the velocity (v), altitude (h), bank angle (ϕ), and angle of side slip (β) during the steady side slip maneuver.

Step 3: Calculate lift coefficient (C_L) for the experiment using the expression:

$$C_L = \frac{W}{\frac{1}{2}\rho V_T^2 S}$$

Step 4: Estimate C_{y_β} using the relation:

$$C_{y_\beta} = -C_L \left(\frac{\phi}{\beta}\right)$$

Estimation of yaw moment coefficient (C_{n_β})

- Yawing Moment Equation:

$$C_n = C_{n_p}\frac{pb}{2V_T} + C_{n_r}\frac{rb}{2V_T} + C_{n_\beta}\beta + C_{n_{\delta a}}\delta a + C_{n_{\delta r}}\delta r$$

- For steady state side slip (static case) $C_n = 0$, $p = 0$ $r = 0$.

$$C_{n_\beta}\beta + C_{n_{\delta a}}\delta a + C_{n_{\delta r}}\delta r = 0$$

- This equation shows that the rudder is required to counteract the weather-cock stability C_{n_β} and the aileron yaw $C_{n_{\delta a}}$.

- It may be noted that, the value of C_{n_β} can be estimated using recorded values of δa, δr and β, provided the values of $C_{n_{\delta a}}$ and $C_{n_{\delta r}}$ are known a priori. Further, If we neglect $C_{n_{\delta a}}$, then one can show that

$$\frac{\beta}{\delta r} = -\frac{C_{n_{\delta r}}}{C_{n_\beta}}$$

where, $C_{n_{\delta r}}$ is negative and C_{n_β} is positive by definition for a stable airplane.

- Thus for an aircraft having directional stability, the slope of the plot of β v/s δr will be positive.

Flight test to estimate yawing moment coefficient (C_{n_β})

Step 1: Record the takeoff weight of the aircraft.

Step 2: Note down the velocity (v), altitude (h), rudder deflection (δr), and angle of side slip (β) during the steady side slip maneuver.

Step 3: Now given the value of $C_{n_{\delta r}}$ the other derivative C_{n_β} can be estimated with the help of the expression.

$$\frac{\beta}{\delta r} = -\frac{C_{n_{\delta r}}}{C_{n_\beta}}$$

Step 4: Plot β vs. δr, we will find the slope of the plot will be positive.

Note:

Since $C_{n_{\delta r}}$ is negative and $C_{n_{beta}}$ is positive $-C_{n_{\delta r}}/C_{n_\beta}$ will be positive.

Estiamtion of roll derivative (C_{l_β})

- Rolling moment equation:

$$C_l = C_{l_p}\frac{pb}{2V_T} + C_{l_r}\frac{rb}{2V_T} + C_{l_\beta}\beta + C_{l_{\delta a}}\delta a + C_{l_{\delta r}}\delta r$$

- For steady-state (static case) $C_l = 0$, $p = 0$, $r = 0$

$$C_{l_\beta}\beta + C_{l_{\delta a}}\delta a + C_{l_{\delta r}}\delta r = 0$$

- Knowing the value of $C_{l_{\delta r}}$ and $C_{l_{\delta a}}$, C_{l_β} can be estimated using the above relation. Usually, $C_{l_{\delta r}}$ is small (not always, specially modern aircraft), and if we neglect its contribution , then above equation further simplifies to:

$$C_{l_\beta}\beta + C_{l_{\delta a}}\delta a$$

or,

$$\frac{\beta}{\delta a} = -\frac{C_{l_{\delta a}}}{C_{l_\beta}}$$

- This equation shows that the aileron is deflected to counteract the dihedral effect.

- Further, an equation can be generated to obtain the relationship between side slip angle and aileron deflection required to execute steady side slip maneuver, provided the value $C_{l_{\delta a}}$ and C_{l_β} are known.

Flight test to estimate rolling moment coefficient (C_{l_β})

Step 1: Record the takeoff weight of the aircraft.

Step 2: Note down the velocity (v), altitude (h), aileron deflection (δa), and angle of side slip (β) during the steady side slip maneuver.

Step 3: Now given the value of $C_{l_{\delta a}}$ the other derivative C_{l_β} can be estimated with the help of the expression.

$$\frac{\beta}{\delta a} = -\frac{C_{l_{\delta a}}}{C_{l_\beta}}$$

Step 4: Plot β vs δa, we will find the slope of the plot will be negative.

Note:

Since $C_{l_{\delta a}}$ is negative and C_{l_β} is negative $-C_{l_{\delta a}}/C_{l_\beta}$ will be negative.