Module-1

Lecture-2

Standard Atmosphere

Standard Atmosphere

It is important to understand the definition of various altitudes that are usually used to analyze/compare the performance of flying vehicles in standard atmosphere.

The gravitational force experienced by any aircraft varies with altitude. Also, an aircraft experiences variation in aerodynamic forces with altitude. This is simply because of the fact that the atmospheric properties viz; Pressure, density and Temperature (P, ρ, T) also changes with altitude. Aerodynamic forces are strong function of these atmospheric properties (P, ρ, T). It is a necessity to specify the altitude that will help in postulating gravitational and aerodynamic forces explicitly.

Why do we need to define a standard atmosphere?

"Standard atmosphere is defined in order to relate flight tests, wind tunnel tests general airplane design and performance to a common reference".

Before proceeding further, let us define certain terms that are essential to understand and characterize standard atmosphere.

1. **Absolute altitude** (h_a) – The altitude as measured from the center of the earth
2. **Geometric altitude** (h_g) – The altitude as measured from the mean sea level
3. **Geo-potential altitude** (h) – The geometric altitude corrected for the gravity variation. We will discuss this later in detail.

From Figure 1, it can be concluded that the absolute altitude is the sum of geometric altitude and mean radius of the earth. Mathematically, this relationship can be numeralized

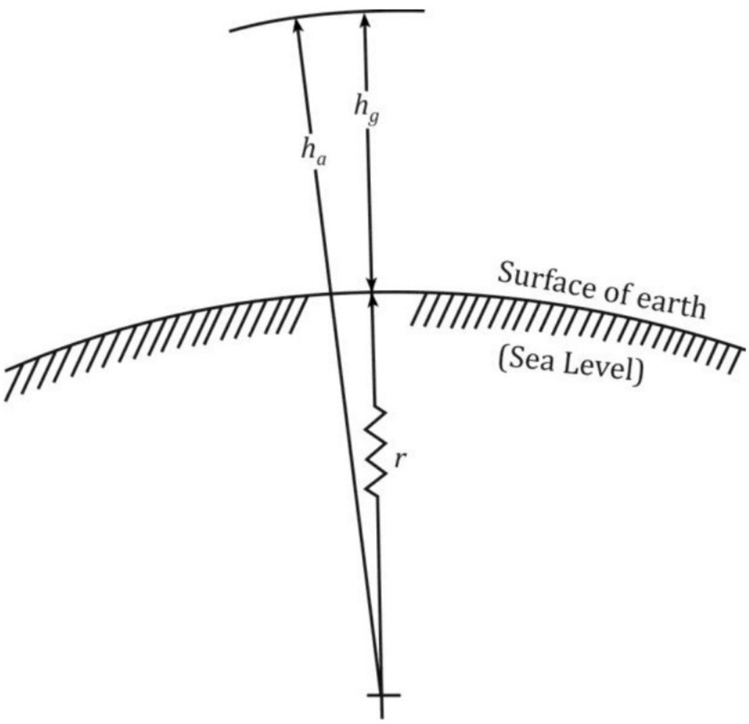

Figure 1: Schematic diagram representing geometric altitude and absolute altitude

as

$$h_a = h_g + r \tag{1}$$

where,

r is the mean radius of earth.

Acceleration due to gravity and altitude relationship

Now that various concepts about altitudes are familiarized, the variations of acceleration due to gravity with altitude needs to be understood. It can be obtained using Newton's universal law of gravitation.

1. If gravitational acceleration at the sea level is g_o and the local gravitational constant is g for a given absolute altitude h_a; then the relationship between g and g_o follows

$$g = g_o \left(\frac{r}{h_a}\right)^2 = g_o \left(\frac{r}{r + h_g}\right)^2 \tag{2}$$

Why are we discussing these altitudes and their dependencies?

"To express the thermodynamic properties of the atmosphere (P, ρ, T) as a function of altitude; these concepts are required.".

Geo-potential and geometric altitudes

1. The hydrostatic equation of an infinitesimal fluid is given by

$$dP = -\rho g dh_g \tag{3}$$

   where, P - hydostatic pressure (Pa),
   ρ - fluid density kg/m^3 and
   g - acceleration (m/s^2) due to gravity corresponding to the geometric altitude h_g

2. In order to obtain the hydrostatic pressure (P) at a particular geometric altitude (h_g), the above expression has to be integrated. Density and acceleration due to gravity, are functions of altitude makes integration a bit more complex/difficult.

3. In order to simplify this integration, the concept of geo-potential altitude (h) has been introduced. We will consider this concept next.

Geo-potential altitude (h)

1. It is a fictitious altitude corrected for the gravity variation, which is typically used to ease the integration process (Equation 3). In simple terms, it can be called as "gravity adjusted height". The adjustment uses Earth's mean sea level as reference.

2. Now we can rewrite the hydrostatic equation, by replacing the geometric altitude with geo-potential altitude (h).

$$dP = -\rho g_o dh \tag{4}$$

   where, g_o is the acceleration due to gravity at mean sea level.

3. Using the two hydrostatic equations, viz, Equation 3 & Equation 4, we can derive the relationship between geometric and geo-potential altitude

$$1 = \frac{g_o}{g}\frac{dh}{dh_g}$$

$$dh = \frac{g}{g_o} dh_g \tag{5}$$

$$dh = \frac{r^2}{(r+h_g)^2} dh_g \tag{6}$$

4. Integrating dh from sea level up to any given value of h (different from h_g) at a given point in atmosphere where geometric altitude is h_g, we get

$$\int_0^h dh = \int_0^{h_g} \frac{r^2}{(r+h_g)^2} dh_g$$

$$= \int_0^{h_g} \frac{dh_g}{(r+h_g)^2}$$

$$h = r^2 \left(\frac{-1}{r+h_g} \right)_0^{h_g}$$

$$= r^2 \left(\frac{-1}{r+h_g} + \frac{1}{r} \right)$$

$$= r^2 \left(\frac{-r+r+h_g}{r(r+h_g)} \right)$$

Thus,

$$h = \left(\frac{r}{r+h_g} \right) h_g \tag{7}$$

Definition of standard atmosphere

1. By now we studied about the different types of altitudes. So now we will proceed further to discuss about the standard atmosphere. From early on, researchers conducted experiments with sounding rockets and hot air balloons to study about the variation of temperature with altitude.

2. Typical pattern of variation of temperature to altitude is shown in Figure 2. One can easily notice that there are some vertical lines (known as constant temperature or isothermal regions) and inclined lines (known as gradient regions).

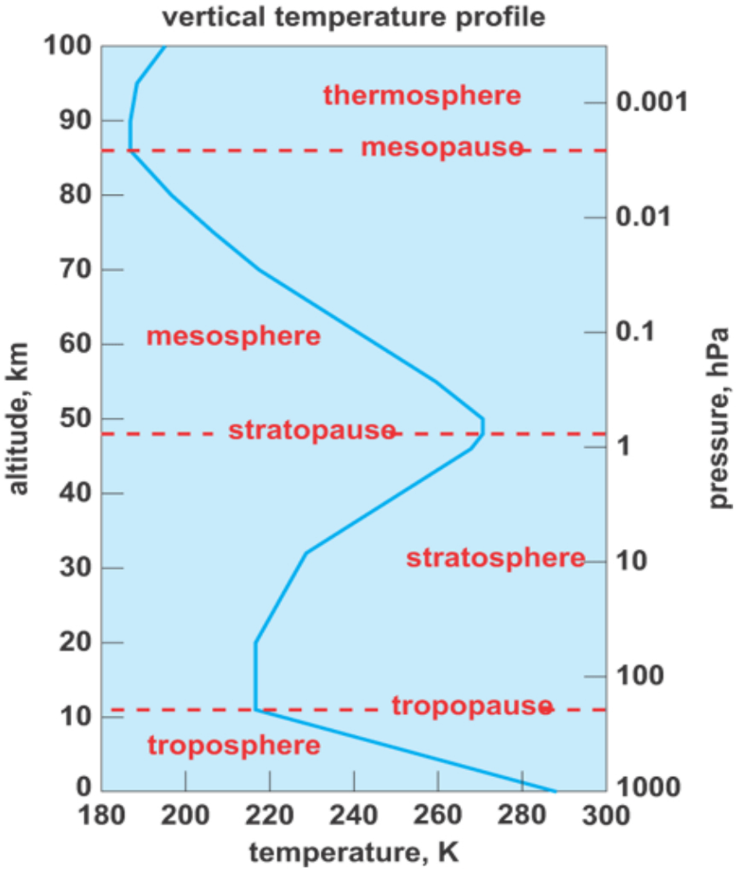

Figure 2: Plot representing variation of temperature with altitude

Modelling the atmosphere

The plot of variation of temperature with altitude (Figure 2) was obtained from using experimental data, However, a similar experiment about the thermodynamic properties of interest to us (pressure and density) is not available. Hence, we proceed to derive the pressure and density variation with altitude with the help of the temperature vs altitude plot shown in Figure 2.

1. The modified hydrostatic equation is

$$dP = -\rho g_o dh \tag{8}$$

2. Divide by the equation of state, ($P = \rho RT$), we have

$$\frac{dP}{P} = -\frac{\rho g_o dh}{\rho RT} = -\frac{g_o}{RT}dh \tag{9}$$

3. First we will consider the isothermal region where the temperature remain relatively constant. Temperature, pressure and density at the base of isothermal region are

T_1, P_1, and ρ_1. Substituting the parameters in Equation 8, we get :

$$\int_{P_1}^{P} \frac{dP}{P} = -\frac{g_o}{RT} \int_{h_1}^{h} dh \tag{10}$$

$$ln\frac{P}{P_1} = -\frac{g_o}{RT}(h - h_1)$$

Equilently we can write,

$$\frac{P}{P_1} = e^{-[g_o/(RT)](h-h_1)} \tag{11}$$

4. Again using the equation of state,

$$\frac{P}{P_1} = \frac{\rho T}{\rho_1 T_1} = \frac{\rho}{\rho_1} \tag{12}$$

Thus,

$$\frac{\rho}{\rho_1} = e^{-[g_o/(RT)](h-h_1)} \tag{13}$$

5. Now consider the gradient region, and the temperature variation can be written as,

$$\frac{T - T_1}{h - h_1} = \frac{dT}{dh} \equiv \alpha \tag{14}$$

where, α is specified constant known as Lapse rate. Thus,

$$dh = \frac{1}{\alpha} dT \tag{15}$$

6. Substituting the value of dh from Equation 15 in Equation 9, we get,

$$\frac{dP}{P} = -\frac{g_o}{\alpha R}\frac{dT}{T} \tag{16}$$

7. Integrating Equation 16 between the base and some given altitude h, yields,

$$\int_{P_1}^{P} \frac{dP}{P} = -\frac{g_o}{\alpha R} \int_{T_1}^{T} \frac{dT}{T}$$

$$ln\frac{P}{P_1} = -\frac{g_o}{\alpha R} ln\frac{T}{T_1}$$

$$\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_o}{\alpha R}} \tag{17}$$

8. Using equation of state $P = \rho RT$, we can write,

$$\frac{P}{P_1} = \frac{\rho T}{\rho_1 T_1} \tag{18}$$

9. Hence, Equation 17 can be re written using Equation 18 to,

$$\frac{\rho T}{\rho_1 T_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_o}{\alpha R}}$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_o}{\alpha R}-1}$$

or,

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-\left(\frac{g_o}{\alpha R}+1\right)} \tag{19}$$

The computation of pressure and density at different layer using standard expressions, listed in Table 1.

Table 1: Gradient & isothermal layer equation

Variables	**Gradient layer**	**Isothermal layer**
Pressure	$\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_o}{\alpha R}}$	$\frac{P}{P_1} = e^{-[g_o/(RT)](h-h_1)}$
Density	$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-\left(\frac{g_o}{\alpha R}+1\right)}$	$\frac{\rho}{\rho_1} = e^{-[g_o/(RT)](h-h_1)}$

In a normal day, the standard atmosphere will always be reliable, since the assumptions used to develop the mathematical models are not violated. But on a non-standard day, this model cannot be completely relied upon. Hence, it is necessary to define two more new altitudes based on the standard atmosphere model, which will be discussed in the next section of this course.