Module-3

Lecture-17

Stick Free Stability and Control - Stick free Neutral Point, Stick force and Estimation of Stick free neutral Point.

Stick-Free Neutral Point

- This is the location of $\bar{x}_{cg}$ for which $(dC_m/dC_L)^{'} = 0$ which means,

$$\bar{n}'_o = \bar{x}_{ac,w} - \left[\frac{dC_m}{dC_L}\right]_{fs} + \frac{C'_{L_{\alpha_t}}}{C_{L_{\alpha_w}}} V_H \eta_t \left(1 - \frac{\partial e}{\partial \alpha}\right) \text{ - - - Stick free}$$

we know

$$\bar{n}_o = \bar{x}_{ac,w} - \left[\frac{dC_m}{dC_L}\right]_{fs} + \frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_w}}} V_H \eta_t \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) \text{ - - - Stick fixed}$$

since,

$$C'_{L_{\alpha_t}} = f C_{L_{\alpha_t}} \text{ and } f = \left(1 - \tau \frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}}\right)$$

We can show that,

$$\bar{n}_o - \bar{n}'_o = \left(\frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_w}}}\right) V_H \eta_t \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) \tau \frac{C_{h_{\alpha_t}}}{C_{h_{\delta_t}}}$$

As,

$$C_{h_{\alpha_t}} < 0$$

$$C_{L_{\alpha_t}} > 0$$

$$C_{L_{\alpha_w}} > 0$$

$$C_{h_{\delta_e}} < 0$$

Therefore,

$$\bar{n}_o - \bar{n}'_o > 0$$

- Therefore, it can easily be understood that stick free neutral point is always ahead of stick fixed neutral point as represented in the given Figure 1

- The static margin will be $(\bar{n}'_o - \bar{x}_{cg})$

$$\left(\frac{dC_m}{dC_L}\right)^{'} = \bar{x}_{cg} - \bar{n}'_o$$

Therefore,

$$\bar{n}'_o - \bar{x}_{cg} = -\left(\frac{dC_m}{dC_L}\right)^{'}$$

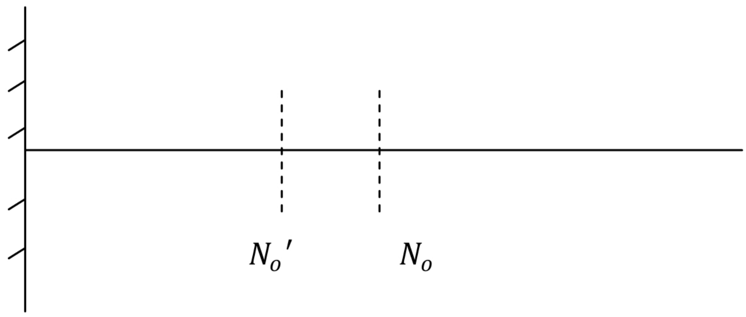

Figure 1: Location of stick fixed and stick free neutral point

Stick Force

- At $C_{L_{trim}}$, net stick force is zero since $C_h = 0$ (Hinge moment at elevator) as pilot's hands are desired to be free. Thus, at trim,

$$(\delta e)_{float} = -\left(\frac{C_{h_{\alpha_t}}}{C_{h_{\delta e}}}\right)\alpha_t$$

- Suppose we want to change from $C_{L_{trim}}$ to C_L, The pilot has to apply force to bring δe to desired position. At $C_{L_{trim}}$, δe floats at such an angle that net $C_{m_{cg}}$. When new C_L is aimed, δe again takes new float position.

$$\Delta C_m = \left[\frac{dC_m}{dC_L}\right]_{free}\{C_L - C_{L_{trim}}\}$$

- To achieve equilibrium, elevator has to be moved to balance this increase in $C_{m_{cg}}$

$$(\Delta C_m)_{\delta e} + (\Delta C_m)_{C_L} = 0$$

$$C_{m_{\delta e}}\Delta\delta e + \left[\frac{dC_m}{dC_L}\right]_{free}\{C_L - C_{L_{trim}}\} = 0$$

$$\Delta\delta e = -\left[\frac{dC_m}{dC_L}\right]_{free}\left\{\frac{C_L - C_{L_{trim}}}{C_{m_{\delta e}}}\right\}$$

- Hinge moment due to this additional $\Delta\delta e$

$$\Delta C_h = C_{h_{\delta e}}\Delta\delta e$$

$$\Delta C_h = -C_{h_{\delta e}}\left[\frac{dC_m}{dC_L}\right]_{free}\left\{\frac{C_L - C_{L_{trim}}}{C_{m_{\delta e}}}\right\}$$

- This need to be balanced by stick force, F_s

$$F_s = H_e/l_g$$

where, l_g is the length of the effective lever arm associated with the mechanism.

$$F_s \propto H_e$$

$$F_s = -G\frac{1}{2}\rho V^2 S_e c_e C_{h_{\delta e}} \left[\left[-\frac{dC_m}{dC_L}\right]'_{free} \left\{\frac{C_L - C_{L_{trim}}}{C_{m_{\delta e}}}\right\}\right]$$

we know that

$$C_L = \frac{2W/S}{\rho V^2} \text{ and } C_{L_{trim}} = \frac{2W/S}{\rho V_{trim}^2}$$

Hence,

$$F_s \propto \left[-\frac{dC_m}{dC_L}\right]'_{free} \left(1 - \frac{V^2}{V_{trim}^2}\right) G S_e c_e \frac{W}{S} \frac{C_{h_{\delta e}}}{C_{m_{\delta e}}}$$

$$\frac{dF_s}{dV} \propto -\frac{2V}{V_{trim}^2}\left[-\frac{dC_m}{dC_L}\right]'_{free}$$

At $V = V_{trim}$

$$\frac{dF_s}{dV} \propto -\frac{2}{V_{trim}}\left[-\frac{dC_m}{dC_L}\right]'_{free}$$

- $dF_s/dV < 0$ implies we need to pull to increase C_L and decrease speed and still maintain cruise flight.

Flight test to estimate stick-free neutral point

Step 1: Cruise the aircraft at different speed.

Step 2: Note down the stick force F_s required for each trim.

Step 3: Using equation:

$$\frac{\frac{dF_s}{q}}{dC_L} \propto \left[\frac{dC_m}{dC_L}\right]_{free} = \left[\bar{x}_{cg} - \bar{n}_o^{'}\right]$$

Plot F_s/q vs C_L for various $\bar{x}_{cg}$ locations.

Step 4: Plot $(dF_s/q)/dC_L$ vs $\bar{x}_{cg}$

Step 5: Extrapolate to get $\bar{n}_o^{'}$ $(\bar{x}_{cg})$ at which $(dF_s/q)/dC_L$