Module-5

Lecture-23

Directional Stability and Control

Directional stability

- Directional or weathercock stability is related to stability of the aircraft about z-axis.
- An airplane is said to possess static directional stability, if it has initial tendency to comeback to it's equilibrium condition when subjected to some form of yawing disturbance.

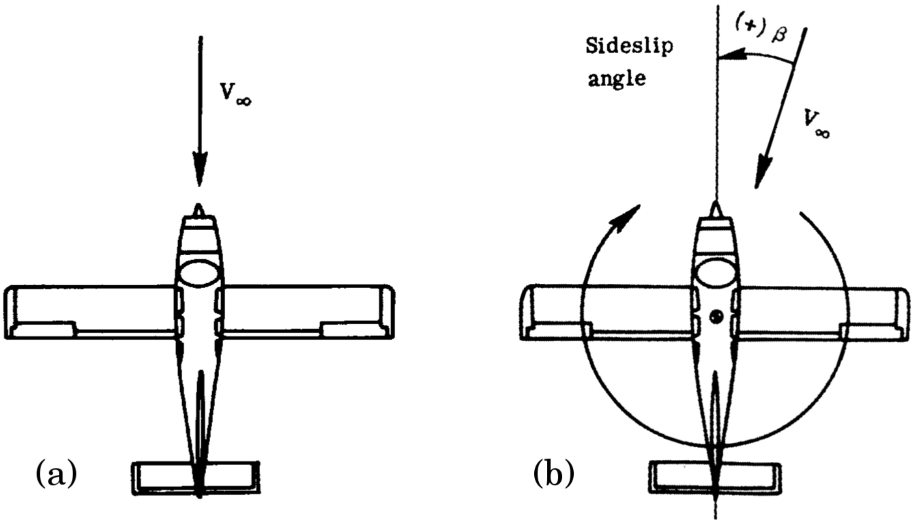

Figure 1: Static directional stability (a) Equilibrium condition of zero yaw (b) Sideslip disturbance

- From Figure 1, the airplane will have directional stability, if it generates positive yawing moment to counter positive yaw (β) disturbance. ($C_{n_\beta} > 0$)

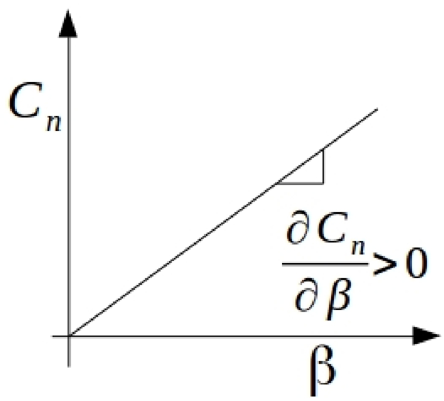

Figure 2: C_n vs β for directional stability

Contribution of airplane component toward C_{n_β}

- **Wing**: The contribution of the wing to directional static stability is quite small (for small angle of attack).
- **Fuselage/Engine/Nacells** usually create destabilizing contribution towards directional stability.

$$C_{n_{\beta,wing+fuselage}} = -K_1 K_2 \frac{S_f l_f}{S_w b} (\text{per degree})$$

where,

K_1	an emperical wing-body interference factor that is a function of the fuselage geometry
K_2	an emperical correlation factor that is a function of fuselage Reynolds number
S_f	projected side area of the fuselage
l_f	length of the fuselage

- Mostly wing-fuselage contribution to directional stability is destabilizing
- Vertical tail needs to be properly designed to ensure adequate directional stability
- Aircraft in positive side slip generates restoring moment through vertical tail. The restoring moment can be expressed as:

$$N = l_v C_{L_{\alpha v}} (\beta + \sigma) Q_v S_v$$

where,

l_v	tail arm length (w.r.t. $c.g$) from aerodynamic center of the vertical tail
$C_{L_{\alpha v}}$	lift curve slope of vertical tail
β	side slip angle
σ	sidewash angle

- Yawing moment coefficient C_n

$$C_n = \frac{yN}{\frac{1}{2}\rho v^2 S_w b} = \frac{l_v S_v}{S_w b} C_{L_{\alpha v}} (\beta + \sigma) \frac{\left(\frac{1}{2}\rho v^2\right)_{vertical\ tail}}{\left(\frac{1}{2}\rho v^2\right)_{free\ stream}}$$

$$C_{n_\beta} = \frac{\partial C_n}{\partial \beta} = V_v \eta_v C_{L_{\alpha v}} \left(1 + \frac{d\sigma}{d\beta}\right)$$

where,

$$\eta_v = \frac{\left(\frac{1}{2}\rho v^2\right)_{vertical\ tail}}{\left(\frac{1}{2}\rho v^2\right)_{free\ stream}}$$

- A simple equation to estimate $d\sigma/d\beta$ (approximate value)

$$\eta_v\left(1+\frac{d\sigma}{d\beta}\right)=0.724+3.06\frac{S_v/S}{1+\cos\left(\Lambda_{c/4}\right)}+0.4\frac{Z_w}{d}+0.009AR_w$$

where,

S	Wing area
S_v	Vertical tail area including the submerged area to the fuselage senter-line
Z_w	Distance parallel to the z-axis, from wing root quarter chord to fuselage reference line
d	Maximum diameter of the fuselage
$\Lambda_{c/4}$	Quarter chord sweep angle of wing

Directional stability due to wing sweep

- The component of the free stream velocity normal to the quarter chord line primarily decides the aerodynamic forces.

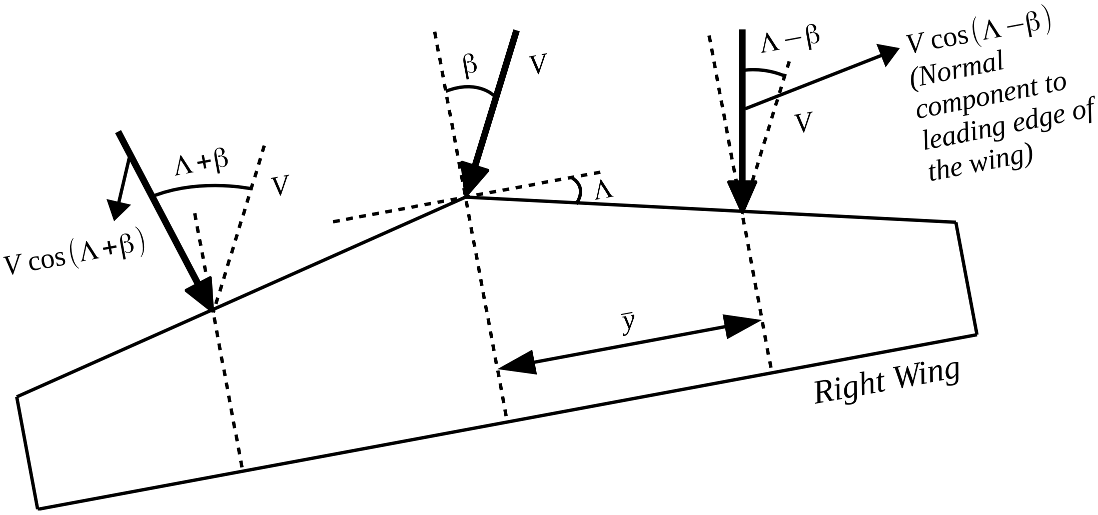

Figure 3: Effect of sweep on directional stability

- From Figure 3, the right wing will experience more dynamic pressure (as the velocity seen by wing leading edge is $V\cos(\Lambda-\beta)$) as compared to the left wing.

- Yawing moment due to drag force on right wing will be

$$N_{wR}=\frac{1}{2}\rho V^2 SC_D\left(\frac{S_w}{2}\right)\bar{y}\cos^2(\Lambda-\beta)$$

- Yawing moment due to drag force on left wing will be

$$N_{wL} = \frac{1}{2}\rho V^2 S C_D \left(\frac{S_w}{2}\right) \bar{y} \cos^2(\Lambda + \beta)$$

- Differential yawing moment is

$$\begin{aligned} N_w &= \frac{1}{2}\rho V^2 C_D \left(\frac{S_w}{2}\right) \bar{y} \left\{\cos^2(\Lambda - \beta) - \cos^2(\Lambda + \beta)\right\} \\ &= \frac{1}{2}\rho V^2 C_D \left(\frac{S_w}{2}\right) \bar{y} \left\{4 \cos\Lambda \sin\Lambda \cos\beta \sin\beta\right\} \end{aligned}$$

For small β; $\sin\beta = \beta$, $\cos\beta = 1$

$$N_w = C_D S_w \bar{y} \frac{1}{2}\rho V^2 \beta \sin 2\Lambda$$

$$(C_n)_w = \frac{N_w}{\frac{1}{2}\rho V^2 S b} = C_D \frac{\bar{y}}{b} \beta \sin 2\Lambda$$

$$\left(C_{n_\beta}\right)_w = C_D \frac{\bar{y}}{b} \sin 2\Lambda > 0$$

Directional Control

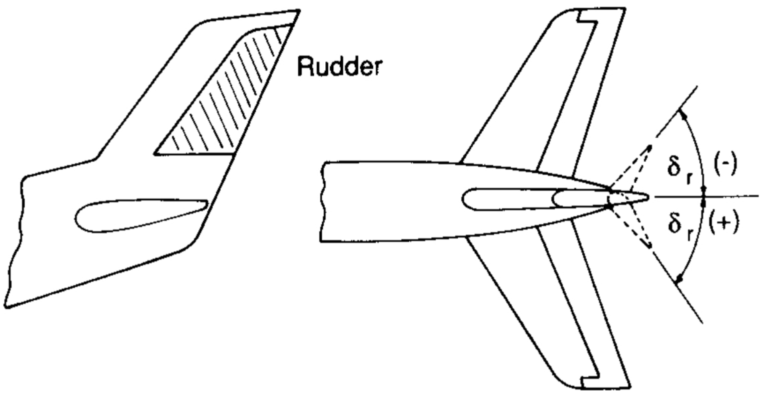

Figure 4: Sign convention of rudder deflection

- Directional control is primarily achieved by rudder, located on the vertical tail.
- Yawing moment produced by the rudder deflection depends on the change in side-force on the vertical tail due to deflection of the rudder.
- For a positive rudder deflection, a positive side force (Y_v) is generated by vertical tail. This side force creates a negative yawing moment.

$$N = -l_v Y_v$$

$$Y_v = C_{L_{\alpha_v}} Q_v S_v$$

$$C_n = \frac{N}{\frac{1}{2}\rho V^2 S_w} = -\eta_v \frac{l_v S_v}{S_w b} \frac{dC_{L_v}}{d\delta r} \delta r$$

$$C_n = -\eta_v V_v \frac{dC_{L_v}}{d\delta r} \delta r \ ; \ \left\{ V_v = \frac{l_v S_v}{S_w b} \right\}$$

$$C_n = C_{n_{\delta r}} \delta r$$

$$C_{n_{\delta r}} = -\eta_v V_v \frac{dC_{L_v}}{d\delta r}$$

$$\frac{dC_{L_v}}{d\delta r} = \frac{dC_{L_v}}{d\alpha_v} \frac{d\alpha_v}{d\delta r} = C_{L_{\alpha_v}} \tau$$

τ is the flap effectiveness parameter which depends on ratio of rudder area to vertical fin area

- The sign of $C_{n_{\delta r}}$ is negative