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## **Module-3**

### **Lecture-13**

**Stability and Control - Discussion on Center of Pressure, Aerodynamic Center and Trim**

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## Center of pressure and Aerodynamic center

The resultant aerodynamic force and moment acting on body must have the same effect as the distributed load. The resultant moment will depend on where ever the resultant

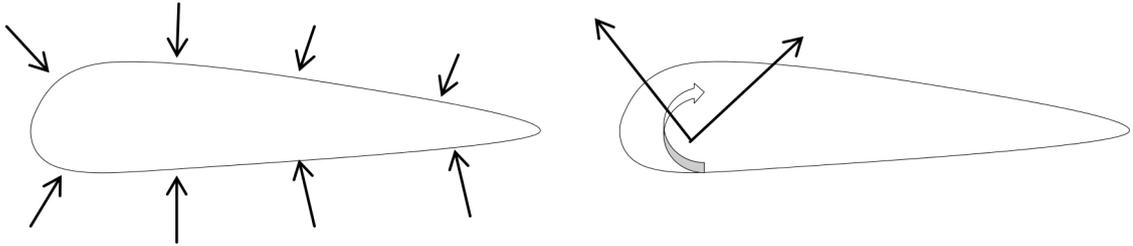


Figure 1: Resultant aerodynamic force and moment

force is placed on the body. For example, let  $x$  be the coordinate measured along the chord line of an airfoil, from the leading edge towards the trailing edge. The resultant

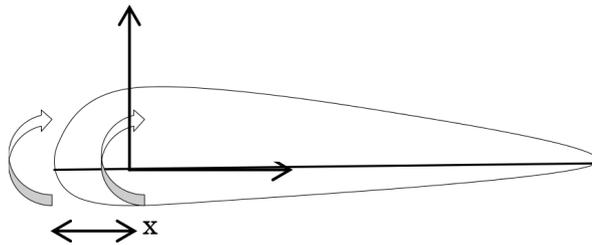


Figure 2: Various coordinate on chord line

moment about some arbitrary point on the chord line a distance  $x$  from the leading edge be  $M_x$ . Then

$$M_{LE} = M_x - xN$$

$$C_{m_{LE}} = C_{m_x} - \frac{x}{c}C_N$$

Two particular locations along the chord line are of special interest.

- $x_{cp}$  → **Center of pressure:** The point about which the resultant moment is zero.
- $x_{ac}$  → **Aerodynamic center:** The point about which the change in the resultant moment with respect to the angle of attack is zero.

## Center of Pressure ( $x_{cp}$ )

- By definition,

$$C_{m_{cp}} = 0$$

- For  $x = x_{cp}$ , this gives,

$$C_{m_{LE}} = C_{m_x} - \frac{x}{c}C_N = -\frac{x_{cp}}{c}C_N$$

$$\frac{x_{cp}}{c} = \frac{x}{c} - \frac{C_{m_x}}{C_N}$$

- Hence, the location of  $x_{cp}$  at any given angle of attack ( $\alpha$ ) can be determined from the normal force coefficient and moment coefficient about any point on the airfoil chord line.
- In general,  $x_{cp}$  may vary significantly with  $\alpha$ .

## Aerodynamic Center ( $x_{ac}$ )

- For  $x = x_{ac}$ , we have,

$$C_{m_{LE}} = C_{m_x} - \frac{x}{c}C_N = C_{m_{ac}} - \frac{x_{ac}}{c}C_N$$

$$C_{m_{ac}} = C_{m_x} + \left(\frac{x_{ac}}{c} - \frac{x}{c}\right)C_N$$

- From definition of aerodynamic center,

$$\frac{\partial C_{m_{ac}}}{\partial \alpha} = 0$$

$$\frac{\partial C_{m_{ac}}}{\partial \alpha} = \frac{\partial C_{m_x}}{\partial \alpha} + \left(\frac{x_{ac}}{c} - \frac{x}{c}\right) \frac{\partial C_N}{\partial \alpha} = 0$$

- Thus,

$$\frac{x_{ac}}{c} = \frac{x}{c} - \frac{\frac{\partial C_{m_x}}{\partial \alpha}}{\frac{\partial C_N}{\partial \alpha}}$$

- The location of the aerodynamic centre can be determined from the knowledge of how the normal force coefficient and moment coefficient about any point on the chord line vary with angle of attack.
- For most of the airfoils, the position of aerodynamic centre is very nearly constant at quarter chord.

**Note:**

- **Location of aerodynamic centre does not depend on magnitude of the aerodynamic coefficient. It depends on the derivative of the aerodynamic coefficient with respect to angle of attack.**

$$\frac{x_{ac}}{c} = \frac{x}{c} - \frac{\frac{\partial C_{m_x}}{\partial \alpha}}{\frac{\partial C_N}{\partial \alpha}}$$

- **In contrast, location of centre of pressure depends on the magnitude of aerodynamic coefficient.**

$$\frac{x_{cp}}{c} = \frac{x}{c} - \frac{C_{m_x}}{C_N}$$

## Stability and Trim

Knowing the airfoil terminology, we will now explore the requirements for trim and then examine the pitch stability of the equilibrium state.

### Assumptions

- Wing is symmetric in the span wise direction
- Motion of the wing through the air is in a direction normal to the span.
- This results in no side force, no rolling moment and no yawing moment.
- *C.G.*, *a.c.* are aligned with the thrust vector, which is aligned with the direction of flight.

For this symmetric flight condition, the aerodynamic forces acting on the wing can be resolved into a lift force  $L$ , a drag force  $D$  and a pitching moment about the aerodynamic centre of the wing  $m_{ac}$  shown in Figure 3.

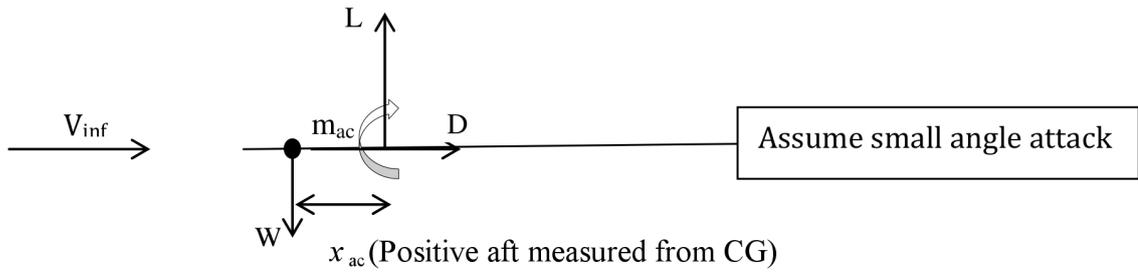


Figure 3: Relation between lift, drag and pitching moment

- For wing to be trimmed (i.e. equilibrium) the summation of forces in both the horizontal and vertical directions must be zero. This requires,

$$T = D$$

$$L = W$$

$$m_{cg} = 0$$

- From Figure 3, we can see that,

$$m_{cg} = m_{ac} - x_{ac}L$$

- At trim,

$$m = m_{ac} - x_{ac}L$$

$$\frac{1}{2}\rho V^2 S \bar{c} C_m = \frac{1}{2}\rho V^2 S \bar{c} \left( C_{m_{ac,w}} \frac{x_{ac}}{\bar{c}} C_L \right) = 0$$

$$C_m = C_{m_{ac,w}} - \bar{x}_{ac} C_L = 0$$

**Note:**

- For a given weight and airspeed, the lift coefficient is fixed by the trim requirement ( $L = W$ ).
- The moment coefficient about aerodynamic centre is fixed by the wing geometry  $C_{m_{ac}}$ . Thus, for a given geometry, weight and airspeed,  $x_{ac}$  is given by,

$$x_{ac} = \frac{C_{m_{ac}}}{C_L} \bar{c} \text{ for trim } C_m = 0$$

$$\left( \frac{x_{ac}}{\bar{c}} \right) = \bar{x}_{ac} = \left( \frac{C_{m_{ac}}}{C_L} \right)$$

$C_L$  is always positive

$C_{m_{ac}}$  is negative for cambered airfoil

- Thus,  $\bar{x}_{ac} < 0$

### Conclusion A

*For equilibrium (trim), the aerodynamic center of cambered wing must be forward of the center of gravity.*

Let us check if this equilibrium is a statically stable equilibrium or not?

## Static Stability

- For static stability, a small increase in angle of attack must produce a negative pitching moment about the center of gravity, to decrease the angle of attack back towards trim.
- Conversely, a small decrease in angle of attack must produce a positive pitching moment to increase the angle of attack to restore the trim.
- Thus, “the pitching moment about CG must vary with angle of attack such that any change in angle of attack produces a **change of opposite sign** in the pitching moment about center of gravity.”

$$\frac{\partial C_m}{\partial \alpha} = \frac{1}{2} \rho V^2 S \bar{c} \frac{\partial M}{\partial \alpha} < 0$$

- Thus for static stability,

$$\frac{\partial C_m}{\partial \alpha} < 0 \text{ *This is also called pitch stiffness*}$$

$$C_m = C_{m_{ac,w}} - \bar{x}_{ac} C_L = 0 \text{ *At trim/equilibrium*}$$

$$\frac{\partial C_m}{\partial \alpha} = \frac{\partial C_{m_{ac,w}}}{\partial \alpha} - \frac{x_{ac}}{c} \frac{\partial C_L}{\partial \alpha} < 0 \text{ *Static stability requirement*}$$

- From definition of aerodynamic center,

$$\frac{\partial C_{m_{ac,w}}}{\partial \alpha} = 0$$

this gives,

$$- \frac{x_{ac}}{c} \frac{\partial C_L}{\partial \alpha} < 0$$

but

$$\frac{\partial C_L}{\partial \alpha} > 0 \Rightarrow \frac{x_{ac}}{c} > 0$$

This is for  $\alpha$  less than  $\alpha_{stall}$

## **Conclusion B**

$x_{ac}/c > 0$  implies, for static stability, the aerodynamic centre must be aft of the centre of gravity.

- From conclusion A, for trim,  $ac$  must be ahead CG (cambered)
- From conclusion B, for stability,  $ac$  must be behind/aft of CG.

*Conclusion A and B are opposite. Thus a simple cambered wing is not statically stable in free flight*

- For trim,

$$x_{ac} = \left( \frac{C_{mac}}{C_L} \right) \bar{c}$$

- For stability,

$$- \frac{x_{ac}}{c} \frac{\partial C_L}{\partial \alpha} < 0$$

- Substitute  $x_{ac}$  as in above equation

$$- \frac{C_{mac}}{C_L} \frac{\partial C_L}{\partial \alpha} < 0$$

- For trim,  $\partial C_L / \partial \alpha > 0$  for  $\alpha$  below  $\alpha_{stall}$  as lift coefficient must be positive to support the weight.
- Thus to get stable trim  $C_{mac}$  must be positive.
- If stable trim is to be maintained, a single wing with no tail must always produce a positive pitching moment coefficient about aerodynamic centre. We know that,
  - Symmetric airfoil produces  $C_{mac} = 0$
  - Cambered airfoil produces  $C_{mac} < 0$
- To produce  $C_{mac}$ , airfoil section must have negative camber over atleast some section of the chord.

### **Note:**

- **An airfoil with negative camber throughout the chord is inefficient in producing positive lift and has a low maximum lift coefficient.**

- A better choice is an airfoil that has negative camber over only some portion of the chord near the trailing edge i.e. Reflex airfoil. See Figure 4
- It is possible to design an aircraft consisting of only a single flying wing with no tail, so that stable trim flight can be achieved.
- However, such designs are not preferred as this is prone to poor handling qualities (damping is less). A better option usually is to combine a wing with a conventional tail.

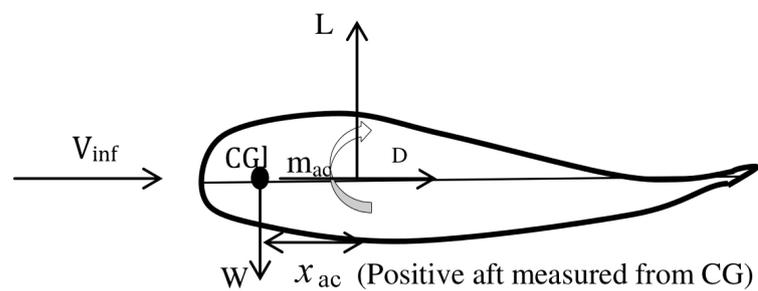


Figure 4: Schematic diagram of a reflexed airfoil