Module-5

Lecture-22

Lateral and Directional Aerodynamic Model

Aerodynamic model: lateral and directional

- First, there are two moment equations and one force equation and the moment equations are coupled kinematically through the product of inertia as well as aerodynamically.
- Second, the lateral mode (roll) has no inherent static stability; no aerodynamic restoring moment is generated directly by rolling. Rather, a secondary moment is generated through the directional axis due to sideslip and dihedral effect becomes the dominant factor.
- Third, the controls used to produce moments about either of the axes also produce moment about the other.
- Aileron deflection produces yawing moments and the rudder produces significant rolling moment.
- In spite of these three facts, it is still instructive to measure the static directional stability and the dihedral effect through steady state tests and to quantify the control authorities about the x and z axes with steady state maneuvers.
- These steady state test methods are discussed in the following sections. For this introduction to static lateral-directional flight test methods, it is sufficient to represent the equations of motion as described below
- We will be referring to the usual body axis system as presented in Figure 1 for future reference

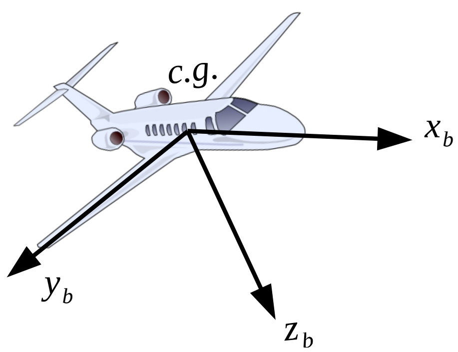

Figure 1: Body fixed axis system

- Force Equation

$$F_y = m(\dot{v} + ru - pw)$$

- Yawing Moment Equation

$$N = -I_{xz}\dot{p} + I_x\dot{r} + pq(I_y - I_z) + I_{xz}qr$$

- Rolling Moment Equation

$$l = I_x\dot{p} - I_{xz}\dot{r} + qr(I_x - I_y) - I_{xz}pq$$

Side Force (F_y)

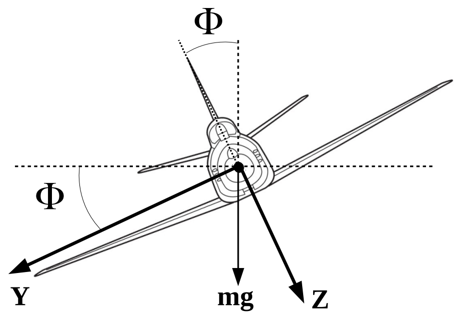

Figure 2: Airplane at positive bank (ϕ)

- The side force F_y, has aerodynamic and gravitational components:

$$F_y = (F_y)_{gravity} + (F_y)_{aerodynamics}$$

- Referring to above figure, we can express components of weight (mg) along body fixed y and z axis as:

$$(F_y)_{gravity} = mg\cos\theta\sin\phi$$

$$(F_z)_{gravity} = mg\cos\theta\cos\phi$$

where, θ and ϕ are the pitch and bank angle respectively.

- $F_{y_{aerodynamics}}$: The aerodynamic force along y direction can be expressed as function of roll rate p, yaw rate r, side slip angle β , aileron deflection δ_a, and rudder deflection δ_r. The following aerodynamic model is assumed:

$$F_{y_{aerodynamics}} = \frac{1}{2}\rho V_T^2 S\left[C_{y_p}\frac{pb}{2V_T} + C_{y_r}\frac{rb}{2V_T} + C_{y_\beta}\beta + C_{y_{\delta_a}}\delta_a + C_{y_{\delta_r}}\delta_r\right]$$

where,

$$V_T = \sqrt{u^2 + v^2 + w^2}$$

u, v, w are the components of total air relative velocity V_T along body fixed axes x, y, z respectively.

- Using,

$$C_y = \frac{F_y}{\frac{1}{2}\rho V_T^2 S}$$

we have:

$$F_y = \frac{1}{2}\rho VT^2 SC_y + mg\cos\theta\sin\phi = m(\dot{v} + ru - pw)$$

where,

$$C_y = C_{y_p}\frac{pb}{2V_T} + C_{y_r}\frac{rb}{2V_T} + C_{y_\beta}\beta + C_{y_{\delta_a}}\delta_a + C_{y_{\delta_r}}\delta_r$$

Yawing Moment (N)

- The yawing moment is also function of p, r, β, δ_a and δ_r and hence it can be experessed as:

$$N = \frac{1}{2}\rho V_T^2 SbC_n$$

where,

$$C_n = C_{n_p}\frac{pb}{2V_T} + C_{n_r}\frac{rb}{2V_T} + C_{n_\beta}\beta + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r$$

Rolling Moment (L)

- The rolling moment coefficient C_l, can also be expressed as:

$$C_l = C_{l_p}\frac{pb}{2V_T} + C_{l_r}\frac{rb}{2V_T} + C_{l_\beta}\beta + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r$$

Thus, Lateral-Directional stability can be studied by the help of following three equations:

$$\begin{aligned} F_y = \frac{1}{2}\rho V_T^2 S\left[C_{y_p}\frac{pb}{2V_T} + C_{y_r}\frac{rb}{2V_T} + C_{y_\beta}\beta + C_{y_{\delta_a}}\delta_a + C_{y_{\delta_r}}\delta_r\right] + mg\cos\theta\sin\phi \\ = m(\dot{v} + ru - pw) \end{aligned}$$

$$\begin{aligned} N = \frac{1}{2}\rho V_T^2 Sb\left[C_{n_p}\frac{pb}{2V_T} + C_{n_r}\frac{rb}{2V_T} + C_{n_\beta}\beta + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r\right] \\ = -I_{xz}\dot{p} + I_x\dot{r} + pq(I_y - I_z) + I_{xz}qr \end{aligned}$$

$$\begin{aligned} L = \frac{1}{2}\rho V_T^2 Sb\left[C_{l_p}\frac{pb}{2V_T} + C_{l_r}\frac{rb}{2V_T} + C_{l_\beta}\beta + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r\right] \\ = I_x\dot{p} - I_{xz}\dot{r} + qr(I_x - I_y) - I_{xz}pq \end{aligned}$$