# Module-4

## Lecture-21

## Maneuvering Flight: Stick free maneuvering point, Stick force Gradient

# Stick-free maneuvering point

- $$(\Delta C_m)_{maneuver} = \left[\frac{dC_m}{dC_L}\right]_{free} (C_L - C_{L_{trim}}) \tag{1}$$

  where,

  $$C_L = \frac{nW}{\frac{1}{2}\rho V^2 S}$$

  and $n$ is the load factor.

- This $(\Delta C_m)$ is required to be balanced by additional $\delta e$ deflection. So,

  $$(\Delta C_m)_{maneuver} = C_{m_{\delta e}} \Delta\delta e$$

- The equilibrium equation to calculate $\Delta\delta e$ is given below:

  $$C_{m_{\delta e}} \Delta\delta e + \left[\frac{dC_m}{dC_L}\right]_{free} (C_L - C_{L_{trim}}) = 0$$

  $$\Delta\delta e = -\left[\frac{dC_m}{dC_L}\right]_{free} \frac{(C_L - C_{L_{trim}})}{C_{m_{\delta e}}}$$

- This $\Delta\delta e$ deflection will create a hinge moment $\Delta C_h$ given by:

  $$\Delta C_h = C_{h_{\delta e}} \Delta\delta e$$

  $$= -\left[\frac{dC_m}{dC_L}\right]_{free} \frac{C_{h_{\delta e}}}{C_{m_{\delta e}}} (C_L - C_{L_{trim}})$$

- Stick froce to balance it, $\Delta F_s$

  $$\Delta F_s = -G S_e c_e \frac{1}{2} \rho V^2 C_h$$

  where, $G$ is gearing constant.

  $$\Delta F_s = -G S_e c_e \frac{1}{2} \rho V^2 \left[ -\left[\frac{dC_m}{dC_L}\right]_{free} \frac{C_{h_{\delta e}}}{C_{m_{\delta e}}} (C_L - C_{L_{trim}}) \right]$$

  $$\Delta F_s = G S_e c_e \frac{1}{2} \rho V^2 \left[ \left[\frac{dC_m}{dC_L}\right]_{free} \frac{C_{h_{\delta e}}}{C_{m_{\delta e}}} \left\{ \frac{nW}{\frac{1}{2}\rho V^2 S} - \frac{W}{\frac{1}{2}\rho V^2 S} \right\} \right]$$

  $$\Delta F_s = G S_e c_e \frac{W}{S} \frac{C_{h_{\delta e}}}{C_{m_{\delta e}}} \left[\frac{dC_m}{dC_L}\right]_{free} (n-1)$$

  $$\frac{dF_s}{dn} = G S_e c_e \frac{W}{S} \frac{C_{h_{\delta e}}}{C_{m_{\delta e}}} \left(\bar{x}_{cg} - \bar{n'}_m\right)$$

  where,

  $$\bar{n'}_m = \bar{n'}_o + \frac{C_{m_{\delta e}}}{\left(\frac{W}{S}\right) C_{h_{\delta e}}} \left[ \frac{\rho}{2} g l_t \left( C_{h_{\delta e}} - \frac{1.1 C_{h_{\delta e}}}{\tau} \right) \right]$$

- Recall: $n'_m$ (stick free) maneuvering point is that $cg$ location at which $dF_s/dn = 0$

***Note:***

- Stick force gradient is very important design parameter
- As $cg$ shifts aft; $F_s$ per $g$ **reduces**.
- Most aft $cg$ may be limited by it.
- Most forward $cg$ may be limited by maximum value of stick free gradient.