Module-5

Lecture-24

Lateral Stability and Control

Lateral stability

An airplane is said to have roll (lateral) stability, if a restoring moment is generated when it is disturbed in bank orientation (ϕ).

- The restoring moment is function of side slip angle, β.
- The requirement for roll stability is that $C_{l_\beta} < 0$.
- The rolling moment created in airplane due to side slip angle also depends on
  - Wing dihedral
  - Wing sweep
  - Position of wing and fuselage
  - Vertical tail
- The major contributor to C_{l_β} is the wing dihedral angle, Γ.
- When an aircraft is disturbed from a wing-level attitude, it will begin to side slip.

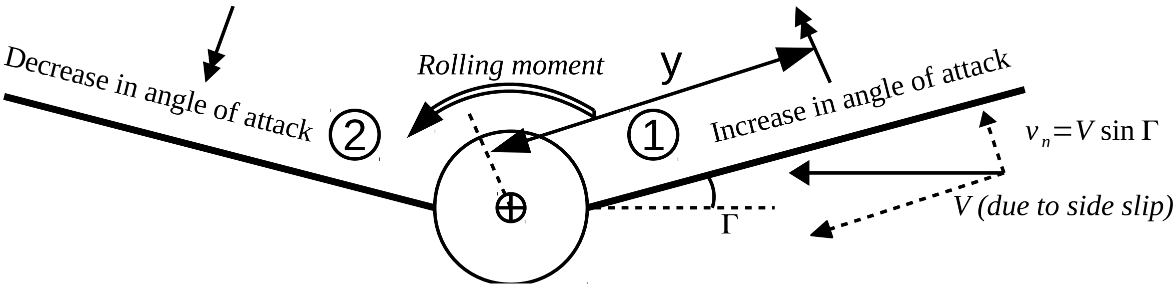

Figure 1: Wing-body dihedral effect

On wing 1,

$$\Delta\alpha = \frac{v_n}{u}$$
$$= \frac{V \sin \Gamma}{u}$$
$$\beta \cong \frac{v}{u}$$
$$\Delta\alpha = \beta\Gamma$$

On wing 2, angle of attack will decrease. Resulting in negative rolling moment to positive side slip angle.

$$C_{l_\beta} < 0$$

Rolling moment due to right wing

$$L_{w,R} = -\frac{1}{2}\rho V^2 C_{L_{\alpha,w}} \Delta\alpha \int_0^{\frac{b}{2}} c(y).y.dy$$

Rolling moment due to left wing

$$L_{w,L} = -\frac{1}{2}\rho V^2 C_{L_{\alpha,w}} \Delta\alpha \int_{-\frac{b}{2}}^{0} c(y).y.dy$$

Rolling moment (total)

$$L_w = -2\frac{1}{2}\rho V^2 C_{L_{\alpha,w}} \Delta\alpha \int_0^{\frac{b}{2}} cydy$$

$$\because \Delta\alpha = \beta\Gamma \text{ and } \bar{y} = \frac{2}{S_w}\int_0^{\frac{b}{2}} cydy$$

$$L_w = -2\frac{1}{2}\rho V^2 C_{L_{\alpha,w}} \Gamma\beta\frac{S}{2}\bar{y}$$

$$(C_{l,w})_\Gamma = \frac{L_w}{\frac{1}{2}\rho V^2 Sb} = -\Gamma\beta C_{L_{\alpha,w}}\frac{\bar{y}}{b}$$

$$C_{l_\beta} = -\Gamma C_{L_\alpha}\frac{\bar{y}}{b}$$

$$C_{l_{\beta,w}} < 0, \text{ Stabilizing for } \Gamma > 0 : \text{ Dihedral}$$

$$C_{l_{\beta,w}} > 0, \text{ Destabilizing for } \Gamma < 0 : \text{ Anhedral}$$

- C_{l_β}: Due to wing sweep

$$(L_w)_{R,\Lambda} = -C_L\frac{S}{2}\frac{1}{2}\rho V^2\bar{y}\cos^2(\Lambda - \beta)$$

$$(L_w)_{L,\Lambda} = C_L\frac{S}{2}\frac{1}{2}\rho V^2\bar{y}\cos^2(\Lambda + \beta)$$

Hence,

$$(L_w)_{\Lambda,Total} = -C_L\frac{S}{2}\frac{1}{2}\rho V^2\bar{y}\left\{\cos^2(\Lambda - \beta) - \cos^2(\Lambda + \beta)\right\}$$

$\bar{y}$ is the location of resultant lift on the wing half

Assume β to be small; $\cos\beta \to 1$ and $\sin\beta \to \beta$

$$(L_w)_{\Gamma,Total} = -C_L\frac{1}{2}\rho V_\infty^2\bar{y}\beta.\sin 2\Gamma$$

$$(C_{l_w})_\Gamma = -C_L\frac{\bar{y}}{b}\beta\sin 2\Gamma$$

$$C_{l_\beta} = -C_L\frac{\bar{y}}{b}\sin 2\Gamma$$

$$C_{l_\beta} < 0 \Rightarrow \text{ Stabilizing}$$

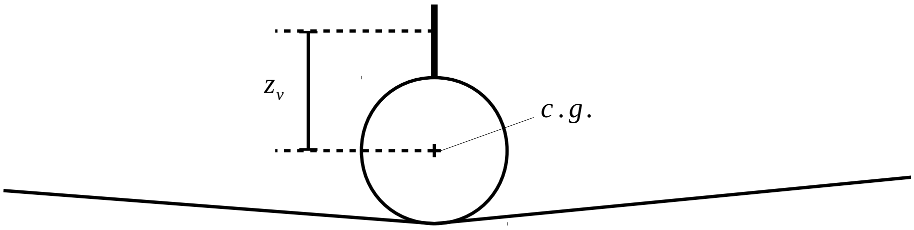

Figure 2: Effect of vertical tail on lateral stability

- C_{l_β}: Due to vertical tail

  The rolling moment due to vertical tail when aircraft is side slipping can be written as

$$l = -\frac{1}{2}\rho V^2 \eta_v S_v \left(\frac{dC_L}{d\alpha}\right)_v \beta . z_v$$

  hence,

$$\left(C_{l_\beta}\right)_v = -\eta_v \frac{S_v}{S_w}\frac{z_v}{b} C_{L_{\alpha,v}}$$

Roll control

- It is achieved by differential deflection of small flaps called ailerons.
- The basic principle lies on the fact that due to differential deflection of ailerons, the lift distribution over the wing becomes unequal, causing a rolling moment.
- An approximate expression for roll control power can be obtained using simple strip integration method.

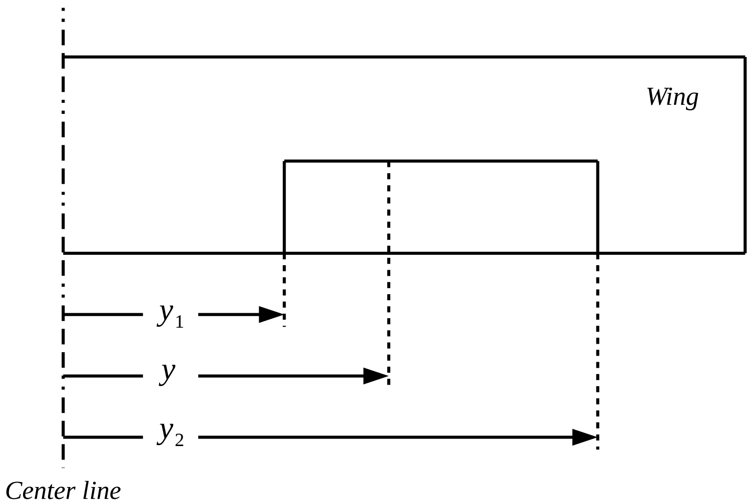

Figure 3: Aileron element for integration

$$\Delta(\text{Rolling moment due to } \delta a) = \Delta l$$

$$= (\Delta \text{Lift}).y$$

$$\Delta C_l = \frac{\Delta L}{qSb} = \frac{C_l Q c y dy}{QSb} = \frac{C_l c y dy}{Sb}$$

$$C_l = C_{L_\alpha}.\frac{d\alpha}{d\delta a}.\delta a = C_{L_\alpha}.\tau.\delta a$$

Integrating over the region containing the aileron yields

$$C_l = \frac{2C_{L_{\alpha,w}} \tau \delta a}{Sb} \int_{y_1}^{y_2} c y dy$$

$$C_{l_{\delta a}} = \frac{2C_{L_{\alpha,w}} \tau}{Sb} \int_{y_1}^{y_2} c y dy$$