Module-2

Lecture-5

Steady and level flight - Equations of motion, Drag polar and Thrust required

Cruise flight: equation of motion

Typically, cruise flight is also known as steady and level flight. Aircraft considered is subjected to aerodynamic, propulsive, and gravity forces. The general equations of motion of an airplane in flight (longitudinal motion). Summation of all forces in direction along the aircraft's air-relative velocity i.e. V (assuming mass of the aircraft is not changing) can be represented as given below:

$$\sum F_{||} = ma = m\frac{dv}{dt} \tag{1}$$

Next, the summation of all forces in the direction perpendicular to the aircraft's air-relative velocity, V , can similarily be represented by Equation 2.

$$\sum F_{\perp} = \frac{mV^2}{r} \tag{2}$$

From Figure 1, Using Equation 1 & 2, it is easy to express $\sum F_{||}$ and $\sum F_{\perp}$ as given in

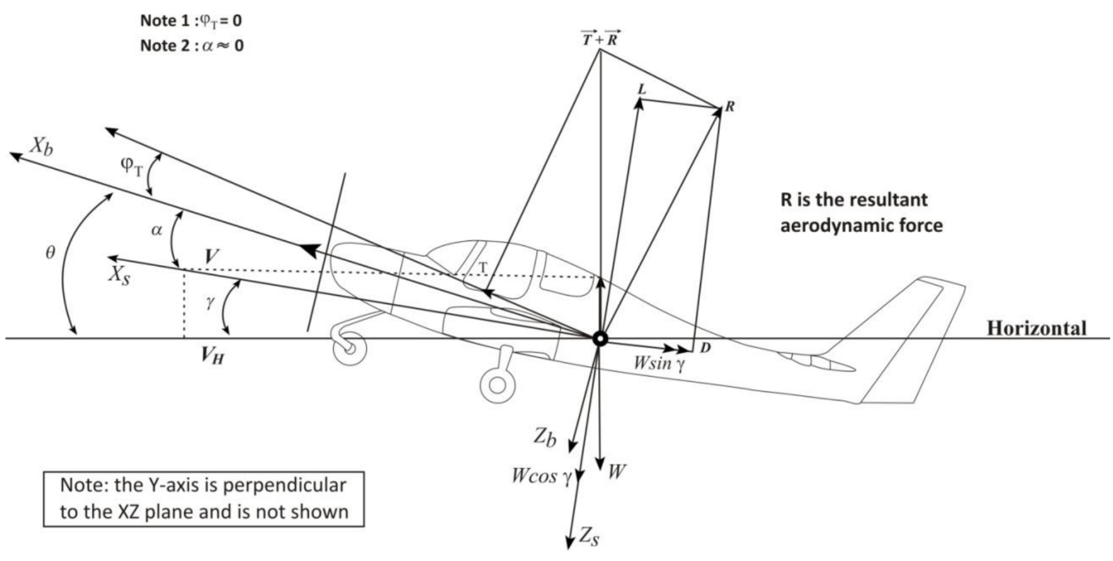

Figure 1: Aerodynamic forces on an aircraft

Equation 3 & 4

$$\sum F_{||} = T\cos(\alpha + \phi_T) - D - W\sin\gamma = m\frac{dV}{dt} \tag{3}$$

$$\sum F_{\perp} = T\sin\alpha + \phi_T + L - W\cos\gamma = \frac{mV^2}{r} \tag{4}$$

where,

$\sum F_{\|\|}$	summation of forces parallel to the aircraft air relative velocity, V
$\sum F_{\perp}$	summation of forces perpendicular to the aircraft air relative velocity, V
m	mass of the aircraft
$\frac{mV^2}{r}$	centripetal acceleration

Assuming that there is no change in the mass of the aircraft i.e. fuel consumption is negligible and assuming that the aircraft velocity remain constant i.e. no acceleration, Equation 3 & 4 can be simplified to as given in Equation 5 & 6

$$\sum F_{||} = T\cos(\alpha + \phi_T) - D - W\sin\gamma = 0 \tag{5}$$

$$\sum F_{\perp} = T\sin\alpha + \phi_T + L - W\cos\gamma = 0 \tag{6}$$

For very small values of α, ϕ_T; i.e.,

$$\alpha \approx 0$$

$$\phi_T \approx 0$$

Then we also have

$$\alpha + \phi_T \approx 0$$

Also note that $\gamma = 0$ for cruise. Equation 5 & 6 then can be simplified to

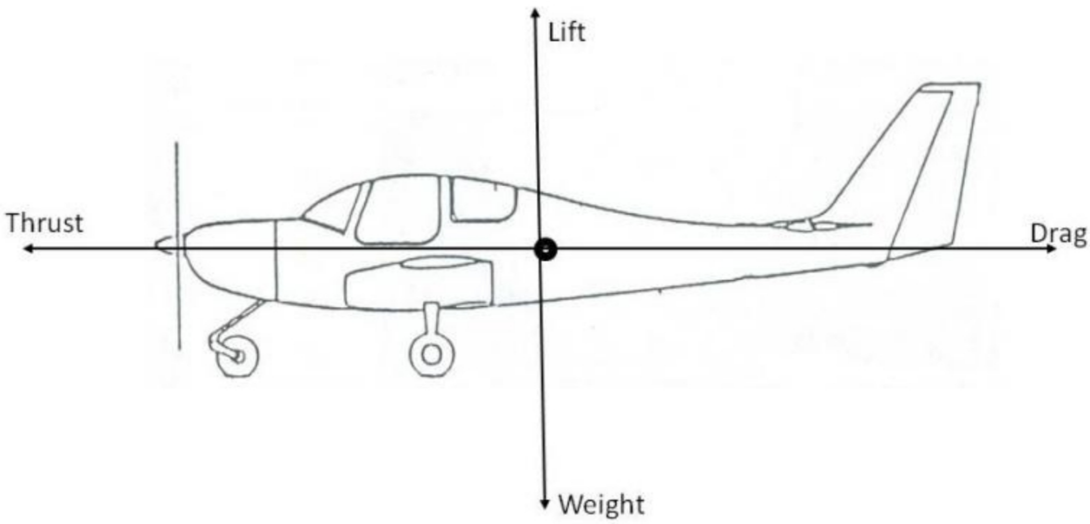

Figure 2: Forces in steady, symmetrical, straight and level flight

$$T - D = 0$$

$$L - W = 0$$

$$T = D \tag{7}$$

$$L = W \tag{8}$$

Figure 2 depicts the steady, symmetrical, straight line flight where Equation 7 & 8 are satisfied.

Note: Cruise flight is a steady (no acceleration), level flight ($\gamma \approx 0$, wings level) where the aerodynamic drag of the aircraft is balanced by the thrust delivered by engine, and the aerodynamic lift balances the weight of the aircraft

Cruise flight: thrust required & drag polar

Consider an airplane in steady, level flight, flying at a velocity V, trimmed at design C_L & C_D. Then the thrust to be delivered by engine to balance the drag can be modeled using Equation 9.

$$T_{req} = D = \frac{1}{2}\rho V^2 S C_D = \bar{q} S C_D \tag{9}$$

where, $\bar{q}$ is the free stream dynamic pressure.
By combining Equation 7 & 8, the thrust required for a cruise flight can be expressed as a function of aerodynamic efficiency C_L/C_D as show in Equation 10.

$$T_{req} = \frac{W}{\frac{L}{D}} = \frac{W}{\frac{C_L}{C_D}} \tag{10}$$

Expression given in Equation 10 suggests that for cruising with the minimum thrust required (which is to be delivered by engine); the aircraft needs to fly such that the ratio of C_L/C_D is at maximum. For a given **Reynolds number** and **Mach number**, C_L, C_D and C_L/C_D are functions of the angle of attack.

- Reynolds number is a dimensionless quantity used in fluid mechanics to predict similar flow pattern in different fluid flow situation. Similarly , Mach number is also a dimensionless quantity that is the ratio of speed of an object moving in a fluid to that of the speed of sound in the medium/fluid.

- The typical variations of C_L with C_D is depicted in Figure 3. The relationship between C_L and C_D is traditionally known as the **Drag Polar**, as given in Equation 11

$$C_D = C_{D_o} + K C_L^2 \tag{11}$$

A typical drag polar graph is depicted in Figure 3.

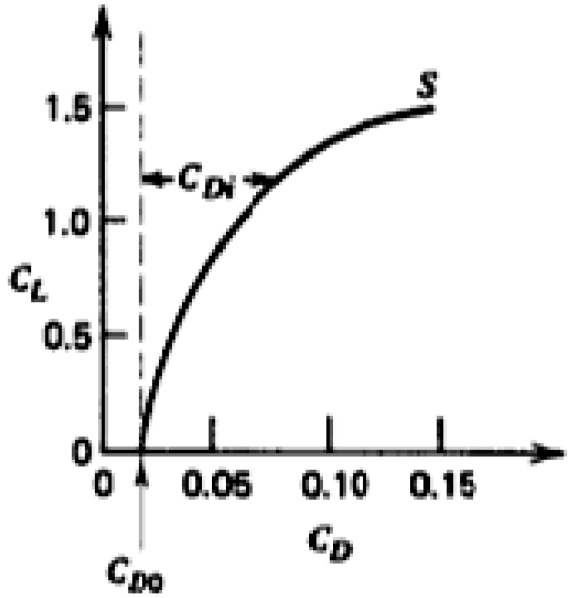

Figure 3: Variation of C_L with C_D

- Typical variations of C_L & C_D with respect to angle of attack is shown in Figure 4. Similarly, a typical variation of C_L/C_D with respect to angle of attack is presented in Figure 5.

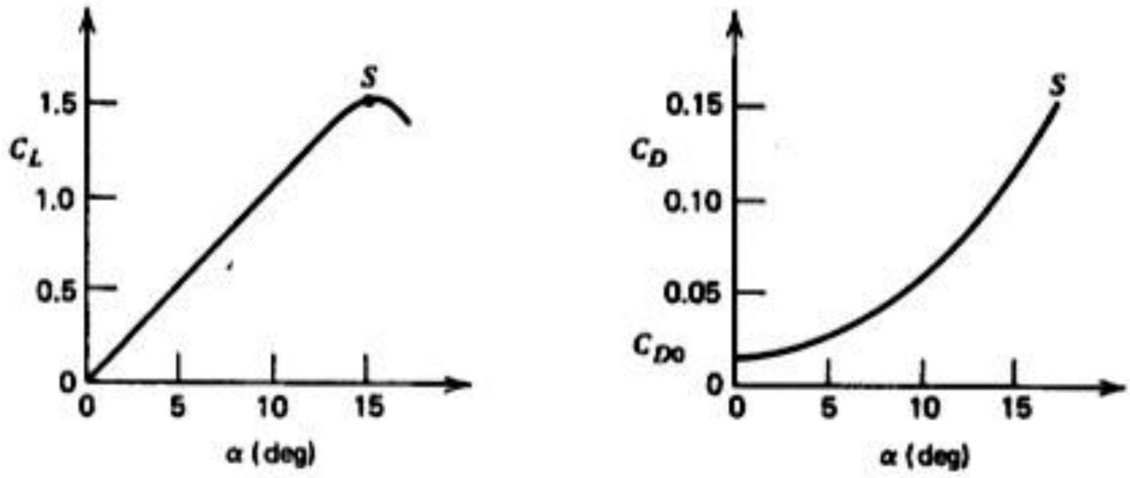

Figure 4: Variation of C_L & C_D with α

- As it can be seen from Figure 5, for a particular aircraft,$[C_L/C_D]_{max}$ could be achieved max only at a particular (fixed) angle of attack. We now try to find out what is that angle of attack for any given aircraft.
- For that we first need to have an insight through $[C_L/C_D]_{max}$. The maximum value of aerodynamic efficiency (E_{max}) and the value of C_L at which C_L/C_D is maximum

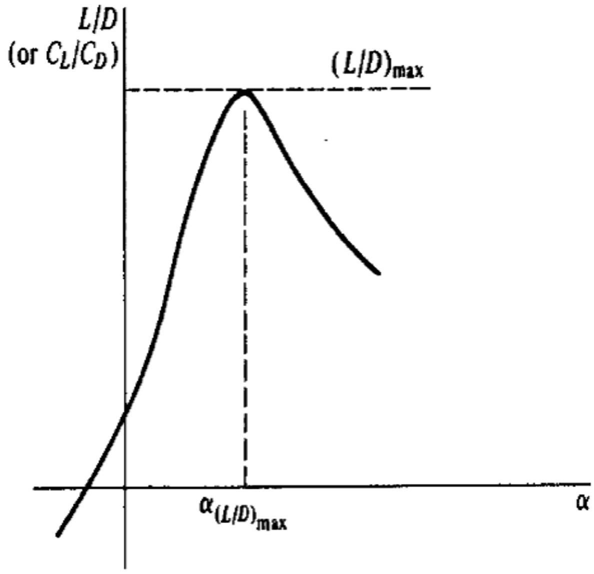

Figure 5: Variation of L/D or C_L/C_D with α

could be found out using the following procedure:

$$E = \frac{L}{D} = \frac{C_L}{C_D} = \frac{C_L}{C_{D_o} + KC_L^2}$$

$$\left[\frac{\partial E}{\partial C_L}\right]_{E_{max}} = \frac{\partial}{\partial C_L}\left[\frac{C_L}{C_{D_o} + KC_L^2}\right] = 0$$

$$\Rightarrow \frac{[C_{D_o} + KC_L^2] - C_L\,[0 + 2KC_L]}{[C_{D_o} + KC_L^2]^2} = 0$$

$$\Rightarrow C_{D_o} - KC_L^2 = 0$$

thus,

$$C_{L_{E_{max}}} = \sqrt{\frac{C_{D_o}}{K}} \quad C_{D_{E_{max}}} = 2C_{D_o} \tag{12}$$

where, $C_{L_{E_{max}}} = C_L$ at $\left[\frac{C_L}{C_D}\right]_{max}$

$$E_{max} = \frac{1}{2\sqrt{C_{D_o}K}} = \left[\frac{C_L}{C_D}\right]_{max} \tag{13}$$

- The above expressions in Equation 12 and 13 suggest how to estimate the value for $[C_L/C_D]_{max}$ and the value of coefficient of lift at $[C_L/C_D]_{max}$. That is, estimate $C_{L_{E_{max}}}$ by using known values of C_{D_o} & K.

- From the lift curve slope i.e. C_L vs α curve of the aircraft, we can determine that at what angle of attack (α) the $C_{L,a/c}$ has the value same as we obtained for $C_{L_{E_{max}}}$ (i.e. $\sqrt{C_{D_o}/K}$). This is the angle of attack at which the aircraft should fly to ensure $[C_L/C_D]_{max}$ i.e. maximum aerodynamic efficiency.

Example:

Now, if an aircraft with wing loading W/S is cruising at a fixed altitude and at a particular to ensure C_L/C_D is maximum then what should be the velocity of the aircraft to maintain cruise flight?

Solution:

To solve this question, One should clearly understand the requirement for cruise condition. For an aircraft to cruise at a given altitude, the weight of the aircraft should be balanced by the lift generated by the aircraft. Thus, for a given C_L, the velocity of the aircraft should be sufficient enough to generate enough lift to balance its weight. *To identify the velocity that will generate enough lift to balance the aircraft's weight can be obtained using the following steps.* To estimate this, consider;

$$L = \frac{1}{2}\rho V^2 S C_L = W$$

Here the C_L is for maximum aerodynamic efficiency, i.e. $C_{L_{E_{max}}}$ so,

$$W = \frac{1}{2}\rho V^2 S C_{L_{E_{max}}}$$

$$\Rightarrow V = \sqrt{\frac{2\frac{W}{S}}{\rho C_{L_{E_{max}}}}}$$

as

$$C_{L_{E_{max}}} = \sqrt{\frac{C_{D_o}}{K}}$$

$$V_{minT_{req}} = \sqrt{\frac{2\frac{W}{S}}{\rho\sqrt{\frac{C_{D_o}}{K}}}} \tag{14}$$

So this is the velocity with which the aircraft should fly at a particular for $[C_L/C_D]_{max}$ (i.e. α for $C_{L_{E_{max}}}$) to maintain cruise.

How can a pilot take advantage of the cruise at maximum efficiency?

For an aircraft to cruise at a particular altitude (for thrust required minimum), the pilot should be requested to trim the aircraft at a velocity as given by Equation 14.To calculate that speed apriori , using Equation 14 for a given aircraft, the following parameters are necessary:

1. Wing loading W/S
2. C_{D_o} from Drag polar
3. K from drag polar or could be calculated for low subsonic aircraft using $K = 1/\pi ARe$

Thus, the desired speed can easily be computed by substituting parameters W/S, C_{D_o} & K in Equation 14.