Module-3

Lecture-14

Static Stability - Wing contribution, Tail contribution and Static Margin

Wing Contribution

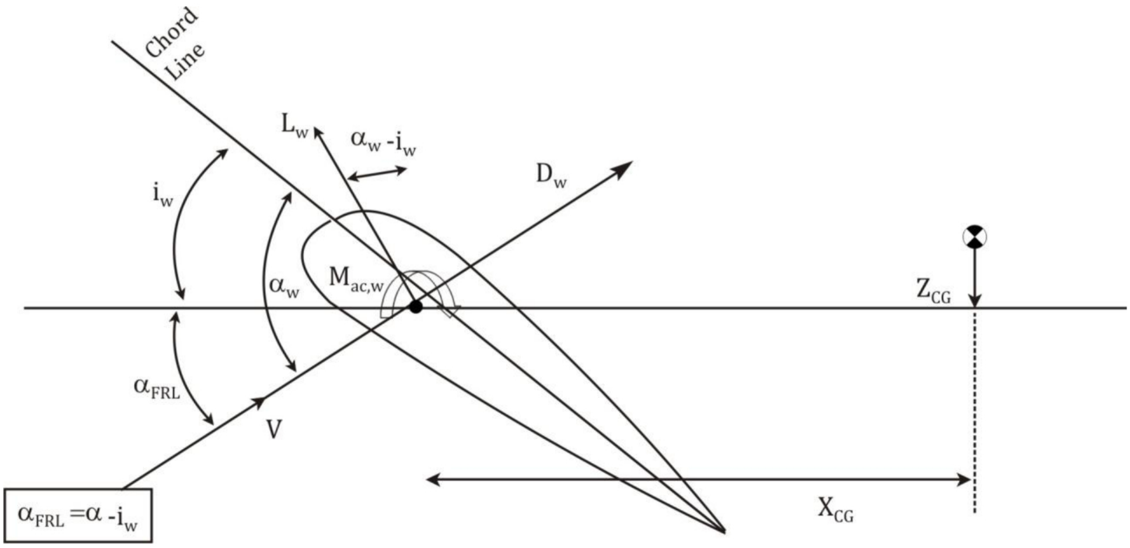

Figure 1: Line diagram showing wing contribution in stability

- Summing up moments about CG,

$$M_{cg} = L_w \cos(\alpha_w - i_w)[x_{cg} - x_{ac,w}] + D_w \sin(\alpha_w - i_w)[x_{cg} - x_{ac,w}]$$
$$+L_w \sin(\alpha_w - i_w)[z_{cg}] - D_w \cos(\alpha_w - i_w)[z_{cg}] + M_{o_{ac_w}}$$

- Divide by $\rho V^2 S\overline{c}/2$

$$C_{m_{cg}} = C_{L_w} \cos(\alpha_w - i_w)\frac{[x_{cg} - x_{ac,w}]}{\overline{c}} + C_{D_w} \sin(\alpha_w - i_w)\frac{[x_{cg} - x_{ac,w}]}{\overline{c}}$$
$$+C_{L_w} \sin(\alpha_w - i_w)\left[\frac{z_{cg}}{\overline{c}}\right] - C_{D_w} \cos(\alpha_w - i_w)\left[\frac{z_{cg}}{\overline{c}}\right] + C_{m_{ac_w}}$$

- Since,

$$\cos(\alpha_w - i_w) \approx 1$$

$$\sin(\alpha_w - i_w) \approx \alpha_w - i_w$$

$$C_L \gg C_D$$

$$z_{cg} \to 0$$

- Hence,

$$C_{m_{cg}} = C_{L_w}(\overline{x}_{cg} - \overline{x}_{ac,w}) + C_{m_{ac_w}}$$

$$C_{L_w} = C_{L_o} + C_{L_\alpha}\alpha$$

$$C_{m_{cg}} = C_{m_{ac_w}} + (C_{L_o} + C_{L\alpha}\alpha)(\overline{x}_{cg} - \overline{x}_{ac,w})$$

$$C_{m_{cg}} = C_{m_{ac_w}} + C_{L_o}(\overline{x}_{cg} - \overline{x}_{ac,w}) + C_{L_\alpha}\alpha(\overline{x}_{cg} - \overline{x}_{ac,w})$$

- This gives wing contribution as,

$$C_{m_{\alpha,w}} = C_{L_{\alpha,w}} \left(\overline{x}_{cg} - \overline{x}_{ac,w}\right)$$

$$C_{m_{o,w}} = C_{m_{ac,w}} + C_{L_{\circ}} \left(\overline{x}_{cg} - \overline{x}_{ac,w}\right)$$

Wing and Tail Contribution

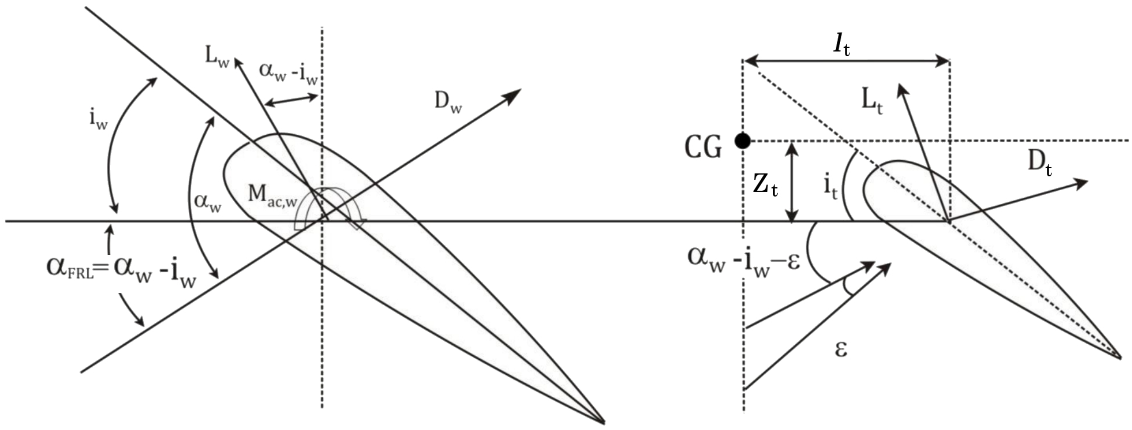

Figure 2: Line diagram showing wing and tail contribution in stability

Angle of attack at tail in presence of wing, (α_t) is given by:

$$\alpha_t = \alpha_w - i_w - \varepsilon + i_t \tag{1}$$

where,

ε: Downwash at tail

i_w: Wing setting angle

i_t Tail setting angle

Moment about *c.g.* due to lift and drag at tail.

- Kindly note that lift at tail will be perpendicular to the local velocity at tail (which is different from the velocity free stream). Similarly, drag experienced at tail will be along the local velocity at tail. The free stream velocity and local velocity directions differ by downwash angle ε. Hence,

$$M_{c.g.,t} = -l_t \left[L_t \cos\left(\alpha - i_w - \varepsilon\right) + D_t \sin\left(\alpha - i_w - \varepsilon\right)\right] + z_t L_t \sin\left(\alpha - i_w - \varepsilon\right)$$
$$- z_t D_t \cos\left(\alpha - i_w - \varepsilon\right) + C_{m_{a.c,t}}$$

- For small angle approximation along with

$$C_L \gg C_D$$

$$z_{cg} \to 0$$

$$C_{m_{a.c,t}} = 0 \ \because \text{Tail airfoil} \to \text{symmetric aerofoil}$$

$$\cos(\alpha_w - i_w - \varepsilon) \to 1$$

$$\sin(\alpha_w - i_w - \varepsilon) \to (\alpha_w - i_w - \varepsilon)$$

We have,

$$M_{c.g,t} = -L_t.l_t = -\frac{1}{2}\rho V_t^2 S_t C_{L_t} l_t$$

Note L_t at tail is proportional to dynamic pressure at tail and not free stream dynamic pressure.

$$C_{m_{c.g,t}} = \frac{M_{c.g,t}}{\left(\frac{1}{2}\rho V^2\right)_{freestream} \overline{C}_w} = -\eta \frac{S_t}{S_w}\frac{l_t}{C_w} C_{L_t}$$

$$C_{m_{c.g,t}} = -\eta \frac{S_t}{S_w}\frac{l_t}{C_w} C_{L_t} \text{ where, } \eta = \frac{\left(\frac{1}{2}\rho V^2\right)_t}{\left(\frac{1}{2}\rho V^2\right)_{fs}} \tag{2}$$

- For symmetric tail:

$$C_{L_t} = C_{L_{\alpha,t}} \alpha_t = C_{L_{\alpha,t}} \left[\alpha_w - i_w + i_t - \varepsilon\right]$$

ε: Downwash due to wing at tail

$$\varepsilon = \varepsilon_o + \frac{\partial \varepsilon}{\partial \alpha}\alpha_w$$

ε_o = Downwash at $\alpha = 0$ (for cambered wing)

$$\frac{\partial \varepsilon}{\partial \alpha} \approx \frac{2C_{L_{\alpha,w}}}{\pi AR_w} \qquad \left(\begin{array}{c}\text{Assuming } e = 1 \\ \because \text{Elliptic lift distribution}\end{array}\right)$$

$$\frac{\partial \varepsilon}{\partial \alpha} = \frac{2C_{L_{\alpha,w}}}{\pi AR_w}$$

$$C_{L_t} = C_{L_{\alpha,t}} \left\{\alpha_w - i_w + i_t - \varepsilon_o - \frac{\partial \varepsilon}{\partial \alpha_w}\alpha_w\right\}$$

$$C_{m_{c.g,t}} = -\eta \frac{S_t}{S_w}\frac{l_t}{c_w} C_{L_t} = -V_H \eta_t C_{L_t}$$

where, V_H: tail volume ratio (typical value 0.5 to 1.0)

$$C_{m_{c.g,t}} = -\eta_t V_H C_{L_{\alpha,t}} \left\{\alpha_w - i_w + i_t - \varepsilon_o - \left(\frac{\partial \varepsilon}{\partial \alpha}\right)\alpha_w\right\}$$

$$= -\eta V_H C_{L_{\alpha,t}} \left\{i_w + i_t - \varepsilon_o\right\} - \eta V_H C_{L_{\alpha,t}} \left\{\alpha_w - \left(\frac{\partial \varepsilon}{\partial \alpha}\right)\alpha_w\right\}$$

$$C_{m_{c.g,t}} = \eta V_H C_{L_{\alpha,t}} \{i_w + \varepsilon_o - i_t\} - \eta V_H C_{L_{\alpha,t}} \left\{1 - \left(\frac{\partial \varepsilon}{\partial \alpha}\right)\right\} \alpha_w \tag{3}$$

$$C_{m_{c.g,t}} = C_{m_o} + \frac{\partial C_m}{\partial \alpha}\alpha$$

$$C_{m_{o,t}} = \eta V_H C_{L_{\alpha,t}} \{i_w + \varepsilon_o - i_t\}$$

$$\left[\frac{\partial C_m}{\partial \alpha}\right]_{c.g,t} = -\eta V_H C_{L_{\alpha,t}} \left\{1 - \frac{\partial \varepsilon}{\partial \alpha}\right\}$$

Adding the effects of wing, tail & fuselage ($\alpha_w = \alpha_{a/c}$):

$$\left(C_{m_{cg}}\right)_{a/c} = C_{m_{o_w}} + C_{m_{o_t}} + C_{m_{o,fs}} + \left(\frac{\partial C_m}{\partial \alpha}\right)_w \alpha + \left(\frac{\partial C_m}{\partial \alpha}\right)_t \alpha + \left(\frac{\partial C_m}{\partial \alpha}\right)_{fs} \alpha$$

$$\left(C_{m_{cg}}\right)_{a/c} = C_{m_{o_w}} + C_{m_{o_t}} + C_{m_{o_{fs}}} + C_{L_\alpha}\left(\overline{x}_{cg} - \overline{x}_{ac,w}\right)\alpha - \eta V_H C_{L_{\alpha,t}} \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\alpha + C_{m_{\alpha_{fs}}}\alpha$$

where,

$$C_{m_{o_w}} = C_{m_{ac_w}} + C_{L_o}\left(\overline{x}_{cg} - \overline{x}_{ac,w}\right)$$

$$C_{m_{o_t}} = V_H C_{L_{\alpha,t}}\left(\varepsilon_o + i_w - i_t\right)$$

From the expression for $\left(C_{m_{cg}}\right)_{a/c}$,

$$\left(C_{m_\alpha}\right)_{a/c} = C_{L_{\alpha_w}}\left(\overline{x}_{cg} - \overline{x}_{ac,w}\right) - V_H \eta_t C_{L_{\alpha,t}}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) + C_{m_{\alpha_{fs}}}$$

Definition of Neutral Point (Stick Fixed): It is that *c.g.* location at which $\partial C_m/\partial \alpha$ or $\partial C_m/\partial C_L$ vanishes ($= 0$) $\rightarrow$ Neutrally Stable. Hence,

$$0 = C_{L_{\alpha_w}}\left(\overline{x}_{cg} - \overline{x}_{ac,w}\right) - V_H \eta_t C_{L_{\alpha,t}}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) + C_{m_{\alpha_{fs}}}$$

Which gives,

$$\overline{x}_{np} = \overline{x}_{ac,w} - \frac{C_{m_{\alpha_{fs}}}}{C_{L_{\alpha_w}}} + \eta_t V_H \frac{C_{L_{\alpha,t}}}{C_{L_{\alpha,t}}}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) = \overline{N}_\circ$$

$\overline{x}_{np} = \overline{N}_o$ Stick Fixed Neutral Point.

Stick Fixed: elevator is fixed not allowed to move or float.

Static margin (SM)

$$\boxed{\text{SM}=\overline{x}_{np}-\overline{x}_{cg}=\overline{N}_{\circ}-\overline{x}_{cg}}$$

Typically transport aircraft SM $\approx$ 5% to 15% of the mean aerodynamic chord.

Approximate expression for SM relating $\partial C_m/\partial C_L$:

$$\overline{x}_{np}=\overline{x}_{ac,w}-\frac{C_{m_{\alpha_{fs}}}}{C_{L_{\alpha_w}}}+\eta_t V_H\frac{C_{L_{\alpha,t}}}{C_{L_{\alpha,t}}}\left(1-\frac{\partial\varepsilon}{\partial\alpha}\right)$$

$$\left(C_{m_\alpha}\right)_{a/c}=C_{L_{\alpha_w}}\left(\overline{x}_{cg}-\overline{x}_{ac,w}\right)-V_H\eta_t C_{L_{\alpha,t}}\left(1-\frac{\partial\varepsilon}{\partial\alpha}\right)+C_{m_{\alpha_{fs}}} \tag{4}$$

Dividing LHS & RHS by $C_{L_{\alpha_w}}$

$$\frac{C_{m_\alpha}}{C_{L_{\alpha_w}}}=\overline{x}_{cg}-\overline{x}_{ac,w}-V_H\eta\frac{C_{L_{\alpha,t}}}{C_{L_{\alpha_w}}}\left(1-\frac{\partial\varepsilon}{\partial\alpha}\right)+\frac{C_{m_{\alpha_{fs}}}}{C_{L_{\alpha_w}}}$$

Under the assumption $C_{L_{\alpha,w}}\approx C_{L_{\alpha,a/c}}$ (approximately)

$$\approx\left[\frac{\partial C_m}{\partial C_L}\right]_{a/c}\cong\overline{x}_{cg}-\overline{x}_{ac,w}-V_H\eta\frac{C_{L_{\alpha,t}}}{C_{L_{\alpha_w}}}\left(1-\frac{\partial\varepsilon}{\partial\alpha}\right)+\frac{C_{m_{\alpha_{fs}}}}{C_{L_{\alpha_w}}}$$

$$\left[\frac{\partial C_m}{\partial C_L}\right]_{a/c}\cong\overline{x}_{cg}-\overline{x}_{ac,w}-V_H\eta\frac{C_{L_{\alpha,t}}}{C_{L_{\alpha_w}}}\left(1-\frac{\partial\varepsilon}{\partial\alpha}\right)+\left[\frac{\partial C_m}{\partial C_L}\right]_{fs}$$

According to the definition of neutral point, it is the $\overline{x}_{cg}$ at which $\partial C_m/\partial\alpha=0$ or equivalently $\partial C_m/\partial C_L=0$

$$0\approx\overline{x}_{cg}-\overline{x}_{ac,w}-V_H\eta\frac{C_{L_{\alpha,t}}}{C_{L_{\alpha_w}}}\left(1-\frac{\partial\varepsilon}{\partial\alpha}\right)+\left(\frac{\partial C_m}{\partial C_L}\right)_{fs}$$

$$\overline{x}_{cg}\to\overline{x}_{np}$$

$$\overline{x}_{np}=\overline{x}_{ac,w}+V_H\eta\frac{C_{L_{\alpha,t}}}{C_{L_{\alpha_w}}}\left(1-\frac{\partial\varepsilon}{\partial\alpha}\right)-\left(\frac{\partial C_m}{\partial C_L}\right)_{fs}$$

$$\left[\frac{\partial C_m}{\partial C_L}\right]_{a/c}\cong\overline{x}_{cg}-\left\{\overline{x}_{ac,w}+V_H\eta\frac{C_{L_{\alpha,t}}}{C_{L_{\alpha_w}}}\left(1-\frac{\partial\varepsilon}{\partial\alpha}\right)-\left[\frac{\partial C_m}{\partial C_L}\right]_{fs}\right\}$$

$$-\left[\frac{\partial C_m}{\partial C_L}\right]_{a/c}\cong\overline{N}_{\circ}-\overline{x}_{cg}=\text{Static Margin} \tag{5}$$

Typical values: 5% to 15% based on mean aerodynamic chord.