

Module-9: High temperature viscous flows and solution strategies

Lecture: 42: Equilibrium flows

42.1.High Temperature Flows

The governing equations for the flow in this realm is still the equation are the equations derived in lecture 2 to 6. However the major difference lies in the solution methodology due to presence of reactions. Moreover some practical experiences can help for making assumptions which might lead to simplifications for solving the governing equations. The presence of reaction is the major hurdle to define the solution strategy since at high temperature leads to dissociation or even ionization. The types of hypersonic flows can then be defined based on the reaction rates.

- 1.Frozen flow
- 2.Equilibrium flow
- 3.Non-equilibrium flow

Frozen flow

This is one of the extream condition of the chemically reacting hypersonic flows. The time required to complete the chemical reactions is extremly large due to exteamily low reactions rates. Lets consider T_f to be the time required for a fluid particle to pass unit distance with the hypersonic velocity and T_c to be the time required for completeion of a chemical reaction. If $T_c \gg T_f$ then the hypersonic flow can be assumed to be frozen flow. The major relief of this assumption is that the species continuity equation will be considered without any reaction rate and the flow is treated as the non-reacting mixture.

Equilibrium flow

This is other extreme condition of the chemically reacting hypersonic flows. If $T_f \ll T_c$ then the hypersonic flow can be assumed to be equilibrium flow. The major simplification of this assumption is that the equilibrium models can be used to solve the governing equations.

Non-equilibrium flow

This flow is more realistic where reaction time scale is of the same order as that of the flow time scale. Hence modeling or solving for the non-equilibrium flow becomes difficult.

42.1. Solution methodology for equilibrium flows

As mentioned earlier the equilibrium assumption is valid if the reaction time is very small as compared to the flow time scale. Hence the reaction can be assumed to be completed at all instances and flow can be assumed to be in chemical equilibrium. Let's consider hypersonic inviscid flow and corresponding implementation of equilibrium model. The algorithm or flowchart for solving the governing for perfect gas or equilibrium flow is as shown in **Fig. 42.1**.

The perfect gas model explained in the is valid for single species non-reacting hypersonic flow. However in the presence of reactions calculation of temperature from flow energy is not straight forward and should be done using the correlations given in the literature. One such widely used methodology, proposed by Tannehill and Mugg, (1974). The steps involved while solving is explained herewith.

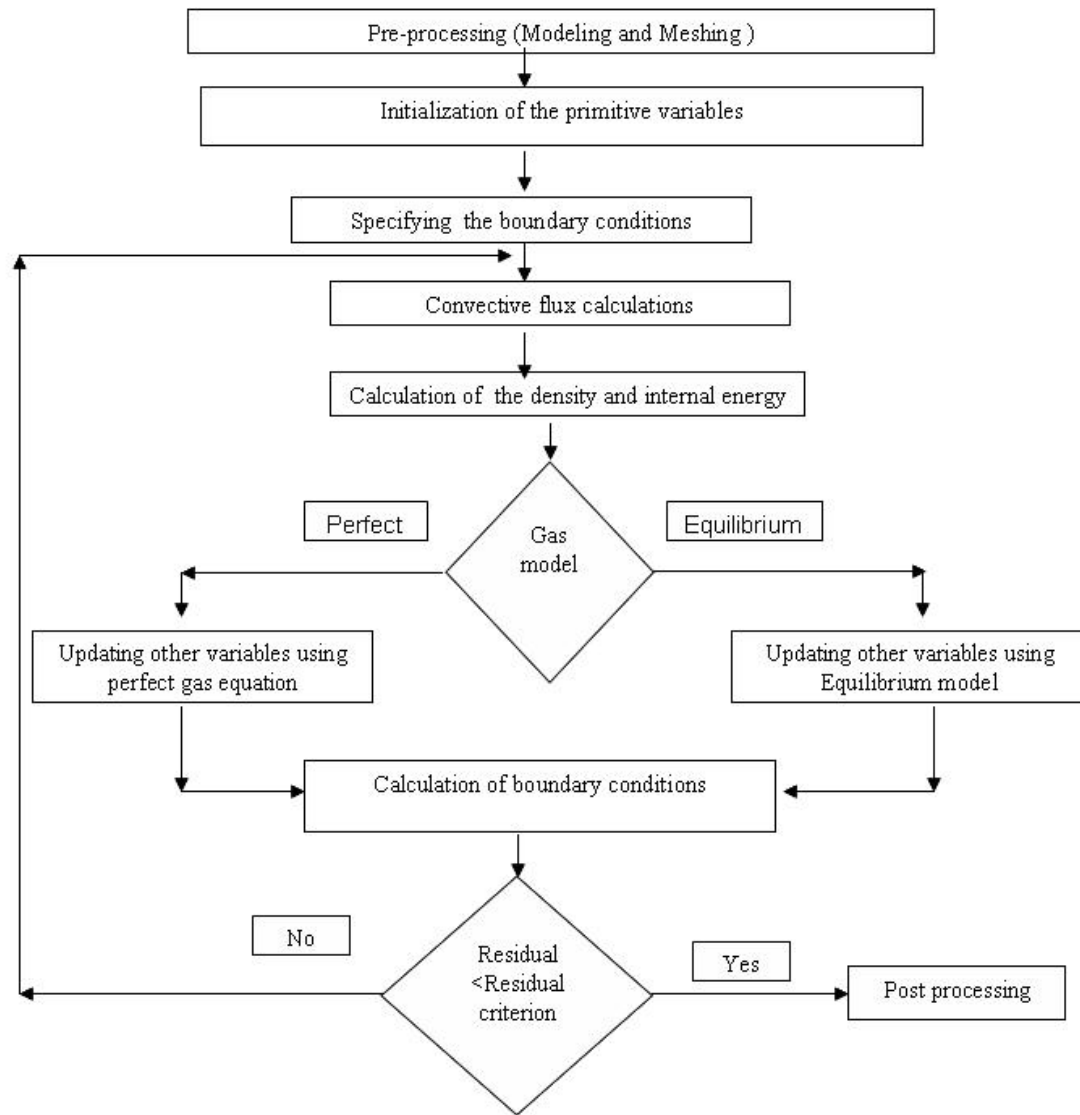


Fig. 42.1: Flowchart for solving the governing equations

1. An effective γ denoted as $\tilde{\gamma}$ is first calculated from following equation

$$\tilde{\gamma} = a_1 + a_2 Y_1 + a_3 Z_1 + a_4 Y_1 Z_1 + a_5 Y_1^2 + a_6 Z_1^2 + a_7 Y_1 Z_1^2 + a_8 Z_1^3 + \frac{a_9 + a_{10} Y_1 + a_{11} Z_1 + a_{12} Y_1 Z_1}{1 + \exp[(a_{13} + a_{14} Y_1)(Z_1 + a_{15} Y_1 + a_{16})]}$$

Where $Y_1 = \log(\rho/1.292)$ and $Z_1 = \log(e/78408.4)$. ρ is density in kg/m^3 and e is internal energy in m^2/s^2

2. Once $\tilde{\gamma}$ is calculated, then the equation of state is used to find the pressure.

$$p = \rho e^{\left(\tilde{\gamma}-1\right)}, \quad \text{pressure is in } N/m^2$$

3. Next the temperature is computed from following relation

$$\log\left(\frac{T}{151.78}\right) = b_1 + b_2 Y_2 + b_3 Z_2 + b_4 Y_2 Z_2 + b_5 Y_2^2 + b_6 Z_2^2 + b_7 Y_2 Z_2^2 + b_8 Z_2^3 \\ + \frac{b_9 + b_{10} Y_2 + b_{11} Z_2 + b_{12} Y_2 Z_2 + b_{13} Z_2^2}{1 + \exp\left[(b_{14} Y_2 + b_{15})(Z_2 + b_{16})\right]}$$

Where

4. Finally the exact expression for sound speed is calculated from

$$a = \left[e^{\left\{ k_1 + \left(\tilde{\gamma} - 1 \right) \left[\tilde{\gamma} + k_2 \left(\frac{\partial \tilde{\gamma}}{\partial \log_e e} \right)_{\rho} \right] + k_3 \left(\frac{\partial \tilde{\gamma}}{\partial \log_e \rho} \right)_e \right\}} \right]^{1/2}$$

The coefficients used in above equations are given by Tannehi and Mugg, (1974).

Figure 42.2 provides the difference between perfect gas model and equilibrium model for hypersonic flow over sphere. Hypersonic flow of $M=18$ is considered here passing over a 30 mm radius sphere is considered in this section. The freestream conditions considered as $p_{\infty} = 1197 Pa$ and $T_{\infty} = 226.5$. The decrease in shock stand-off distance in the presence of equilibrium model is clear from this figure. Increase in density due to prominence of the reactions is the major reason for decrease in shock stand-off distance.

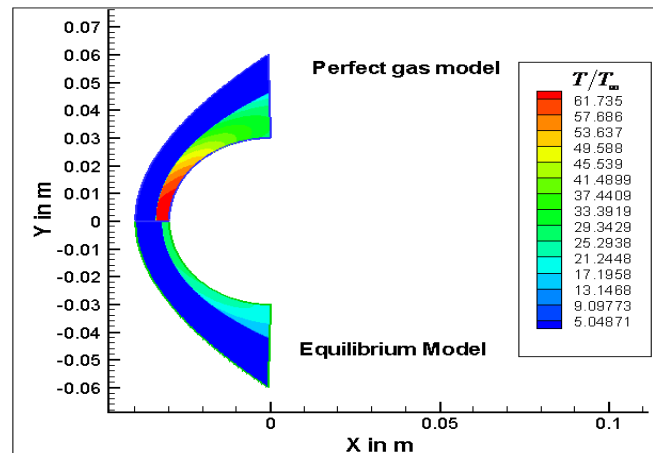


Fig. 42.2: Hypersonic flow over sphere

References:

Tannehil, J.C. and Mugg, P.H., Improved curve fits for the thermodynamic properties of equilibrium air suitable for numerical computation using time-dependent or shock-capturing methods. NASA CR-2470, Oct 1974.

Lecture: 43: Special topics in hypersonics

43.1.High Temperature Viscous Flows

The flowchart given in **Fig.42.1** is valid for inviscid and viscous flows as well. The flux calculation involved in the inviscid flows is the only inviscid fluxes or convective fluxes of mixture mass, momentum, energy and of individual species comprising the mixture. The boundary conditions to be applied at the wall are the adiabatic free-slip wall. The adiabatic wall leads no energy exchange to and from the wall while the free-slip wall boundary condition asserts on the body profile to be a streamline. The viscous high temperature hypersonic flow on the other hand should be carefully handled by calculating viscous or diffusive fluxes and no-slip wall boundary conditions at the wall. The calculation involved for evaluating the viscous fluxes over here in the domain in turn means the calculation of shear stress, heat flux and mass diffusion. The no-slip wall boundary condition leads to zero velocity at the wall for the fluid. This boundary condition is most preferable for non-ablative walls. If ablation takes place at the wall then the net mass transfer should be used as the boundary condition for normal mass flux at the wall. The boundary condition for energy equation is the isothermal or adiabatic wall. Isothermal wall internally specifies infinite thermal conductivity of the wall material which leads to no change in wall temperature. Adiabatic wall temperature in other words means zero thermal conductivity of wall material which leads to no heat transfer to the wall. Therefore the wall heat flux should be equated to zero for adiabatic wall boundary condition. However this heat flux for high temperature flows is comprised of heat transfer by conduction and heat transfer by mass diffusion. Hence the summation of heat transfer by conduction and mass diffusion should be equated to zero for adiabatic wall boundary condition. However the wall can practically take place in reaction by acting as the catalyst. If wall acts as perfect catalyst then the concentration of all the species at the wall should be considered at local chemical equilibrium at corresponding temperature and pressure. However if the wall is a non-catalytic wall then the concentration gradient at the wall should be equated to zero. Moreover in reality, the wall acts as partially catalytic wall. In this boundary condition, the mass of the fluid participating in the reaction is the diffused mass. Hence the mass of a particular

species lost or generated or lost by reaction at the wall can be equated to the diffused mass flux of the same species.

The simulation or computations of viscous flows leads to two extra non-dimensional parameters viz. Lewis number and Eckert number. These numbers are defined as,

$$\text{Lewis Number} = \frac{\text{Energy transport by mass diffusion}}{\text{Energy transport by mass conduction}}$$

$$\text{Eckert Number} = \frac{\text{Kinetic Energy of the flow}}{\text{Thermal Energy of the flow}}$$

Reynolds number, Prandtl number, Mach number and specific heat ratio also appear in the non-dimensional viscous high temperature hypersonic flow formulation. However the importance of the Mach number and Reynolds number gets reduced at high enthalpy or high temperature conditions.

43.2. Rarefied Flows

The flow of fluid considered and associated calculations by experimental measurements or computations are based on the assumption of continuum of fluid. However the actual aircraft far above from the sea level experiences the a different flow regime termed as rarefied flow.

At higher altitudes, density of air decreases and continuum assumption remains no longer valid after certain distance. At these heights, the characteristic length scale of the flow either becomes comparable or lesser than the mean free path of air. Hence such flows should be treated separately and the concerned subject is called as rarefied gas dynamics or free molecular gas dynamics. This subject deals with the special treatment undertaken to deal the non-continuum flows and associated solution strategies.

The important parameter which is essential to be evaluated for better understanding of the flow is Knudsen number. It is the defined as the ratio of the molecular mean free path and the characteristic length scale of the flow given as,

$$\text{Kn} = \frac{\lambda}{L}$$

Here ‘ λ ’ is molecular mean free path or the average distance travelled by the molecule between two successive collisions. The characteristic length scale (‘ L ’) of can either be the specific dimension of the spacecraft or length of gradients of macroscopic magnitudes which is necessarily the characteristic length scale defined by the hypersonic flow. Hence

$$L = \frac{\rho}{\partial \rho / \partial x}$$

$$L = \frac{\partial x}{\partial \rho / \rho}$$

Thus the characteristic length scale can be seen as the ratio of the elemental distance to the density strain over that length. Here density is considered for calculating the characteristic length scale, however one can make a choice of velocity, temperature etc. as per the application specifications.

The governing N-S equations are valid for computations of the flow field if $\text{Kn} < 0.3$, since derivation of these equations is based on continuum assumption. We can still continue the calculations for $0.3 < \text{Kn} < 1$ with free-slip boundary condition for N-S equations. However for high Knudsen numbers, Kinetic Theory of Gases should be incorporated due to free molecular flows. Hence the flow regimes are,

Slip Flow regime: $\text{Kn} < 0.3$

Transitional regime: $0.3 < \text{Kn} < 1$

Free molecular regime: $\text{Kn} > 1$

The kinetic theory of gases considers the discrete molecular motions and their interactions in the analysis of gas flow problems. At high Knudsen number flow conditions, the gas is highly rarefied. Hence the collisions of the molecule become more important than the intermolecular collisions. Therefore the intermolecular collisions can be ignored. Apart from this, the molecules travel larger distance after

reflection from the solid surface before colliding with another molecule. This is the reason to term such flows as free molecular flows.

43.2. Applications considering the rarefaction effect

There are various application in space engineering as well as in other engineering disciplines where the rarefied gas dynamics becomes essential for design of system.

- This subject is primarily important for high altitude flights and space missions. The consideration of the rarefaction helps to evaluate the aerodynamic parameters precisely.
- Using these considerations, thickness of shock waves can also be evaluated for high altitude flight or celestial shock waves.
- Design of vacuum systems also need the knowledge of rarefied gas dynamics since increase in the vacuum level decreases the characteristic length scale of the flow and increases the mean free path of the molecules. Hence the flow problem needs special treatment
- The requirement of continuum break down or knowledge of rarefied gas dynamics also becomes essential for designing objects of micro-scale or nano-scales viz. MEMS, study of aerosol particles etc. This treatment gets considered at normal density conditions for these applications due to decrease in characteristic length scale of the object.

43.3. Radiative Hypersonic Flows.

In the case of very high Mach number and high enthalpy flows, the transfer of heat from the flow the spacecraft takes place by mixed way where convection and radiation become important modes of heat transfer. Hence the usual calculations of shock and associated effects deviate largely from the reality in the presence of radiation, The re-entry of Apollo vehicle experienced Mach number of 32.5. For this situation, the predicted temperature in the shock layer is around 58,300K from conventional normal shock relations for perfect gas. However the actual temperature encountered by the vehicle is around 11,600K which is about 20% of the calculated value. Effect of dissociation, ionisation and radiation deviate the prediction from reality. At such elevated temperature, effect of heat loss by radiation from the fluid element to others and at the same time heat gain from other elements by the radiation

becomes essential to get accounted. Therefore the fluid flow does not remain adiabatic in the presence of volumetric heat addition by radiation. Hence solution methodology gets further complicated due to radiation. Hence two types of gases are defined for simplicity. In a type gas is assumed to be transparent gas where gas does not absorb any radiation. In this type of gas the self emitted radiation from the high temperature gas are assumed to be escaped to the surrounding. However, self absorbing gas is the reality. In this type, the gas not only emits but also absorbs the radiation. The transparent gas assumption helps to simplify the incorporation of radiation effect by retaining the nature and solution procedure of the governing equation. However the self absorbing gas assumption or rather reality disturbs the solution method by changing the nature of the governing equation to elliptic. Hence the flow field gets largely altered in the presence of radiative heat transfer.