Mathematical Methods in Engineering and Science

IIT Kanpur

Assignment Policy of the Course

- Assignments of this course are of the traditional problem-solving type. Not only the result, but also the approach will be considered for evaluation.

- Submissions are invariably to be in the PDF format. PDF version of scanned copy of neat handwritten work is acceptable. Brevity and to the point solutions will be rewarded. Illegible writing, sloppy work and beating around the bush will be penalized.

- For many segments of the course, a programming background will be useful, though it is not essential.

- For some problems in the assignments, programming will be needed to get the complete solution. But, the programme does not need to be submitted.
  Those students who do not have a programming background may conduct the intial few (2 to 5) steps or iterations manually and indicate through what repititive and automated procedure a programme would need to proceed in order to get the final complete solution. Such solutions will be considered for partial credit.

- There may also be some problems, for which programming may not be needed as such, but may be useful in solving the problem systematically. Students having a programming background are encouraged to take advantage of this.

MMES (2017) Assignment 7

(Full marks = 100)

1. Interpolate the function $p(x) = \frac{1}{1+x^2}$ with a single 16th degree polynomial on the interval $[-5, 5]$ with 17 equally spaced samples, and estimate the maximum interpolation error. Compare this function approximation against the interpolation with a 10th degree polynomial by means of 11 samples.

2. Function $f(t)$ is being approximated in the interval $[0, 1]$ by a cubic interpolation formula in terms of the boundary conditions as

$$f(t) = [f(0) \quad f(1) \quad f'(0) \quad f'(1)] \ \mathbf{W} \ [1 \quad t \quad t^2 \quad t^3]^T \ .$$

   Determine the matrix $\mathbf{W}$.

3. Schedule the motion of a railroad car on a 20 km track for 40 minutes, if it is to start from rest, terminate at rest, have acceleration continuity everywhere along the track and pass stations at 2 km and 15 km from the starting point at time 6 min and 28 min, respectively, from the starting time. Use two options: a single Hermite polynomial and a (cubic) spline.

4. For a smooth function $f(x)$ over $[x_0, x_2]$, we have $f(x_0) = f_0$, $f(x_1) = f_1$ and $f(x_2) = f_2$ where $x_1 = x_0 + h$ and $x_2 = x_0 + 2h$. We want to develop a formula for the integral $\int_{x_0}^{x_2} f(x)dx$.

   (a) With the substitution $x = x_1 + sh$, develop function $p(s)$ such that $p(s) = f(x(s))$ at the known points. Develop the quadratic interpolation of $p(s)$ in its domain.

   (b) Convert the required integral to one with s as the variable of integration. Evaluate this integral using the above interpolation.

   (c) Now, expand $f(x_0)$ and $f(x_2)$ as Taylor's series about x_1 and estimate the error in the above formula by comparing it with the integral of Taylor's series of $f(x)$ about x_1.

5. Using the central difference formula and Richardson extrapolation, evaluate the derivative of

$$\phi(x) = \frac{\sin(e^{\sqrt{7x+2}} + \tan^{-1}\sqrt{1+3x^3})}{\cos^2(2x+3)}$$

   at $x = 0$, up to six places of decimal.

6. For the integrals (a) $\int_1^2 \frac{1}{x}dx$ and (b) $\int_0^1 \frac{\sin x}{x}dx$, obtain estimates I_4, I_8 and I_{16} by trapezoidal rule with four, eight and sixteen subintervals, respectively. Then, compute the Romberg integral I_R by extrapolation in each case.

7. Find the volume of intersection between the sphere $x^2 + y^2 + z^2 = 25$ and the ellipsoid $\frac{x^2}{100} + \frac{y^2}{16} + \frac{z^2}{4} = 1$.

8. Use Euler's, Improved Euler's and fourth order Runge-Kutta methods, with step size $h = 0.1$ in all cases, to solve the initial value problem $y' = (x + y - 1)^2$, $y(0) = 0$ for $0 \le x \le 2$ and compare the results with $y = \tan(x - \pi/4) - x + 1$, the correct (analytical) solution.

9. Using fourth order Runge-Kutta method, evaluate the solution of

$$\frac{d^2y}{dt^2} - y(1-y)\frac{dy}{dt} + y = 0, \quad y(0) = 1,\ y'(0) = 1,$$

up to $t = 5$, with step size 0.1.

10. Using the fourth order Runge-Kutta method with adaptive step size, solve the IVP,

$$\ddot{x} = x^2 - y + e^t,\ \ddot{y} = x - y^2 - e^t, \quad x(0) = \dot{x}(0) = 0,\ y(0) = 1,\ \dot{y}(0) = -2,$$

for $t \in [0, 2]$ correct up to three places of decimal.