Mathematical Methods in Engineering and Science

IIT Kanpur

Assignment Policy of the Course

- Assignments of this course are of the traditional problem-solving type. Not only the result, but also the approach will be considered for evaluation.
- Submissions are invariably to be in the PDF format. PDF version of scanned copy of neat handwritten work is acceptable. Brevity and to the point solutions will be rewarded. Illegible writing, sloppy work and beating around the bush will be penalized.
- For many segments of the course, a programming background will be useful, though it is not essential.
- For some problems in the assignments, programming will be needed to get the complete solution. But, the programme does not need to be submitted.
  Those students who do not have a programming background may conduct the intial few (2 to 5) steps or iterations manually and indicate through what repititive and automated procedure a programme would need to proceed in order to get the final complete solution. Such solutions will be considered for partial credit.
- There may also be some problems, for which programming may not be needed as such, but may be useful in solving the problem systematically. Students having a programming background are encouraged to take advantage of this.

MMES (2017) Assignment 6

(Full marks = 100)

1. Bracket a minimum of the function $2e^x - x^3 - 8x \sin x$.

2. Find a minimum value of the function $p(x) = 2x^4 - 20x^3 + 75x^2 - 125x$ in the bracket $[0, 4]$ by the golden section search and regula falsi methods, and compare their performance.

3. For minimizing the function $f(\mathbf{x}) = (x_1 - x_2)^2 + (1 - x_1)^2 + (2x_1 - x_2 - x_3)^2$, consider the origin as the starting solution, i.e. $\mathbf{x}_0 = [0 \quad 0 \quad 0]^T$.

   (a) Evaluate the function $f(\mathbf{x}_0)$ and gradient $\mathbf{g}(\mathbf{x}_0)$ at this point. Using the negative gradient as the search direction, define a new function $\phi(\alpha) = f(\mathbf{x}_0 - \alpha \mathbf{g}(\mathbf{x}_0))$.

   (b) Find out the minimizer α_0 of $\phi(\alpha)$ and update $\mathbf{x}_1 = \mathbf{x}_0 - \alpha_0 \mathbf{g}(\mathbf{x}_0)$.

   (c) Similarly, carry out two more such iterations, i.e. find out $f(\mathbf{x}_k)$, $\mathbf{g}(\mathbf{x}_k)$, α_k and $\mathbf{x}_{k+1}$ for $k = 1, 2$.

   (d) Tabulate the results and analyze them in terms of function value as well as *distance* from the actual minimum point.

4. Use Nelder and Mead's simplex search method, starting from the origin, to find the minimum point of the function

$$f(x, y, z) = 2x^2 + xy + y^2 + yz + z^2 - 6x - 7y - 8z + 9.$$

5. For minimizing the function $f(\mathbf{x}) = (x_1^2 - x_2)^2 + (1 - x_1)^2$, perform one iteration of Newton's method from the starting point $[2 \quad 2]^T$ and compare this step with the direction of the steepest descent method, regarding approach towards the optimum.

6. Solve the following systems of equations by formulating them as optimization problems:
   (a) $x^2 - 5xy + y^3 = 2$, $x + 3y = 6$ and (b) $ze^x - x^2 = 10y$, $x^2 z = 0.5$, $x + z = 1$.

7. Find constants a_1, a_2, a_3, a_4 and λ for least square fit of the following tabulated data in the form $a_1 + a_2 x + a_3 x^2 + a_4 e^{\lambda x}$.

x	0	1	2	3	4	5	6	7	8
y	20	52	69	76	74	67	55	38	17

   [*Hint:* You may attempt it as a five-variable least square problem or as a single-variable optimization problem with a linear least square problem involved in the function evaluation.]

8. In three-dimensional space, we have a line segment with known end-points A (a_1, a_2, a_3) and B (b_1, b_2, b_3). Similarly, we have a triangle with known vertices P (p_1, p_2, p_3), Q (q_1, q_2, q_3) and R (r_1, r_2, r_3). Formulate the problem of finding the closest distance between the line segment and the triangle as an optimization problem. Develop the KKT conditions for the problem. If a given pair of points (on the line segment and on the triangle) together satisfies the KKT conditions, can we say that this pair gives a local minimum for the distance?

9. For the problem

$$\text{minimize} \quad f(\mathbf{x}) = (x_1 - 3)^2 + (x_2 - 3)^2 \quad \text{subject to} \quad 2x_1 + x_2 \leq 2;$$

develop the dual function, maximize it and find the corresponding point in $\mathbf{x}$-space.

10. A chain is suspended from two thin hooks that are 160 cm apart on a horizontal line. The chain consists of 20 links of steel, each 10 cm in length. The equilibrium shape of the chain is found by formulating the problem as

$$\text{minimize} \sum_{i=1}^{n} c_i y_i \quad \text{subject to} \quad \sum_{i=1}^{n} y_i = 0 \quad \text{and} \quad L - \sum_{i=1}^{n} \sqrt{l^2 - y_i^2} = 0,$$

where $c_i = n - i + 1/2$, $n = 20$, $l = 10$, $L = 160$. Derive the dual function for this problem and work out a complete *steepest ascent* formulation for maximizing the dual function, and hence solving the original problem.
Implement this formulation in a steepest ascent loop and obtain optimal values of Lagrange multipliers, equilibrium configuration and the corresponding (minimum) potential energy, i.e. $(\sum_{i=1}^{n} c_i y_i)$.