Mathematical Methods in Engineering and Science

IIT Kanpur

Assignment Policy of the Course

- Assignments of this course are of the traditional problem-solving type. Not only the result, but also the approach will be considered for evaluation.

- Submissions are invariably to be in the PDF format. PDF version of scanned copy of neat handwritten work is acceptable. Brevity and to the point solutions will be rewarded. Illegible writing, sloppy work and beating around the bush will be penalized.

- For many segments of the course, a programming background will be useful, though it is not essential.

- For some problems in the assignments, programming will be needed to get the complete solution. But, the programme does not need to be submitted.
  Those students who do not have a programming background may conduct the intial few (2 to 5) steps or iterations manually and indicate through what repititive and automated procedure a programme would need to proceed in order to get the final complete solution. Such solutions will be considered for partial credit.

- There may also be some problems, for which programming may not be needed as such, but may be useful in solving the problem systematically. Students having a programming background are encouraged to take advantage of this.

MMES (2017) Assignment 3

(Full marks = 100)

1. Find out the canonical form of each of the following matrices:

(a) $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$, (b) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, (c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$, and (d) $\begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$.

2. For the matrix
$$\mathbf{A} = \begin{bmatrix} 6.4 & -7.2 & 7.2 \\ -7.2 & 10.6 & -9.6 \\ 7.2 & -9.6 & 9 \end{bmatrix},$$

it is already known that $[\ 12\ \ -16\ \ 15\ ]^T$ is an eigenvector corresponding to eigenvalue 25. Find out a matrix that has all its eigenvalues and eigenvectors identical to $\mathbf{A}$ except that the eigenvalue corresponding to this eigenvector is 0. Hence, determine the *other* eigenvalues and eigenvectors of $\mathbf{A}$.

3. Find the eigenvalues, eigenvectors and the determinant of the matrix

$$\mathbf{Q} = \begin{bmatrix} 0.80 & 0 & 0.6 \\ 0.36 & 0.8 & -0.48 \\ -0.48 & 0.6 & 0.64 \end{bmatrix}.$$

Does it represent a rotation? If yes, then in which plane?

4. Tridiagonalize the matrix
$$\begin{bmatrix} 9 & 3 & 2 & & \\ 3 & 8 & 2 & & \\ 2 & 2 & 7 & 3 & \\ & & 3 & 6 & 1 \\ & & & 1 & 8 \end{bmatrix}$$

by an economical use of Givens rotations.

5. Find a matrix that will map the vector $[1\ 1\ 1\ 1]^T$ in the direction of the vector $[1\ 0\ 0\ 0]^T$.

6. (a) Find the eigenvalues, eigenvectors and determinant of a Householder matrix.

   (b) Randomly construct five or six orthogonal matrices of 5×5 size. [*Hint:* Use the method of problem 4 in assignment 1.] Find out their eigenvalues, eigenvectors and determinants. Based on the eigenstructure, work out a characterization and classification of orthogonal matrices.

7. Find the number of positive eigenvalues of the symmetric tridiagonal matrix for which the diagonal is $\{3, 1, 2, 4, 1, 3, 2\}$ and the super-diagonal is $\{2, 4, 6, 8, 0, 2\}$.

8. For the real symmetric tridiagonal matrix

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & \\ 1 & 5 & 1 \\ & 1 & 2 \end{bmatrix},$$

use an ordinary QR decomposition algorithm till the largest off-diagonal element is reduced below 0.1 in magnitude. Hence, determine approximate eigenvalues of $\mathbf{A}$. Note your experience about the progress of the algorithm. What additional effort is needed to find corresponding eigenvectors as well?

9. Apply a few QR iterations on the symmetric tridiagonal matrix with the leading diagonal as $\{8, 6, 8, 2, 9, 9\}$ and the super-diagonal as $\{0, 2, 9, 0, 1\}$, *without splitting* and study the progress of the algorithm.

10. An approximate eigenvalue of the matrix

$$\begin{bmatrix} 5 & 7 & 6 & 5 \\ 7 & 10 & 8 & 7 \\ 6 & 8 & 10 & 9 \\ 5 & 7 & 9 & 10 \end{bmatrix}$$

is 30. Find the corresponding eigenvector and improve the eigenvalue. If, in addition, you know that the given matrix is ill-conditioned, how does this much information help in solving the eigenvalue problem completely?