# Mathematical Methods in Engineering and Science

# IIT Kanpur

## Assignment Policy of the Course

- Assignments of this course are of the traditional problem-solving type. Not only the result, but also the approach will be considered for evaluation.
- Submissions are invariably to be in the PDF format. PDF version of scanned copy of neat handwritten work is acceptable. Brevity and to the point solutions will be rewarded. Illegible writing, sloppy work and beating around the bush will be penalized.
- For many segments of the course, a programming background will be useful, though it is not essential.
- For some problems in the assignments, programming will be needed to get the complete solution. But, the programme does not need to be submitted.
  Those students who do not have a programming background may conduct the intial few (2 to 5) steps or iterations manually and indicate through what repititive and automated procedure a programme would need to proceed in order to get the final complete solution. Such solutions will be considered for partial credit.
- There may also be some problems, for which programming may not be needed as such, but may be useful in solving the problem systematically. Students having a programming background are encouraged to take advantage of this.

# MMES (2017) Assignment 12

## (Full marks = 100)

1. From the Taylor's series of $\frac{1}{1+z^2}$ about $z = 0$, develop a Taylor's series for $\tan^{-1} z$.

2. Develop the Laurent's series for $\frac{z^2+1}{z^3-4z}$ about the origin and determine its region of convergence.

3. Determine the locations and nature of zeros of $\sin z$ and $\sin\left(\frac{1}{z}\right)$.

4. Examine the behaviour of $e^{1/z}$ near the origin and determine $z$ to satisfy $e^{1/z} = Ae^{i\phi}$ for any complex number $Ae^{i\phi}$.

5. Evaluate the contour integral $\oint_C \frac{1+z^2}{(z-1)^2(z+2i)} dz$, where $C$ is an ellipse with major radius 10, minor radius 4 and centre at the origin.

6. For a particle of mass $m$, define the (*Lagrangian*) function

$$\mathcal{L}(t, x, y, \dot{x}, \dot{y}) = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - mgy$$

and develop the integral (action)

$$s = \int \mathcal{L} dt.$$

The problem is to determine the trajectory $x(t), y(t)$ of the particle from $(0, 0)$ at time $t = 0$ to $(a, b)$ at time $t = T$, along which $s$ is minimum, or at least stationary.

   (a) Verify that $x(t) = \alpha t(t - T) + at/T$, $y(t) = \beta t(t - T) + bt/T$ is a feasible trajectory, and develop the function $s(\alpha, \beta)$.

   (b) Formally find out values of $\alpha$ and $\beta$ to minimize $s$, and hence determine the required trajectory $x(t), y(t)$. Find out $\dot{x}, \dot{y}, \ddot{x}, \ddot{y}$. Which *law* of physics did you *derive* just now, in a way?

   (c) Now, bypass the work of the two previous steps and work on a more direct theme. Work out the variation $\delta s$ as a result of *arbitrary* variations $\delta x(t)$, $\delta y(t)$, that respect the given boundary conditions, and consistent variations in their rates as well. Insist on $\delta s = 0$ to derive the same result as above. [*Hint:* To get rid of the $\delta \dot{x}$ and $\delta \dot{y}$ terms, integrate the corresponding terms by parts.]

7. We want to solve the Blasius problem (see Problem 2, Assignment 8) in the form

$$f'''(x) + f(x)f''(x) = 0, \quad f(0) = f'(0) = 0, \quad f'(5) = 1$$

by Galerkin method. Let us choose $x^2$, $x^3$, $\cdots$, $x^8$ as the basis functions, which already satisfy the first two conditions. Taking 1, $x$, $x^2$, $\cdots$, $x^5$ as trial functions and using the boundary condition at $x = 1$, determine the solution.