Problem Sheet 09

Problem 1: Method of Lines

Consider the following transient heat conduction (dimensionless form) equation:

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}, \qquad T'(t,0) = 0, \qquad T(t,1) = 1$$

The corresponding initial condition is $T(0,x) = x^4$. Note that one boundary is insulated and the other is kept at a constant temperature (dimensionless value of 1).

(1) Split the spatial domain into five equal intervals. Convert the PDE into ODE-IVP.

(2) Solve the resulting ODE-IVP using RK-2 method (with appropriate step size) to obtain temperature at $t = 0.1$ and $t = 1$.

Problem 2: Finite Difference Method

We will solve the above problem using Finite Difference type methods

(1) Discretize the time derivative using Euler's explicit method. Express temperature at any time m and location i as $T_{i,m}$. Use the central difference in space with $\Delta x = 0.2$.

(2) Use the above expression to find the temperature at various times using $\Delta t = 0.05$. Use five steps to reach $t = 0.25$.

(3) Comment on the stability of the solution technique

(4) Repeat using Euler's implicit method with the same time step.

Problem 3: Transient Reactor

Convert Problem 4 of the previous module (Problem Sheet 8) to a transient model and solve it using method of lines. As initial condition, assume the entire reactor to have uniform concentration of $c_{(t=0)} = 1$.