Exercise 1

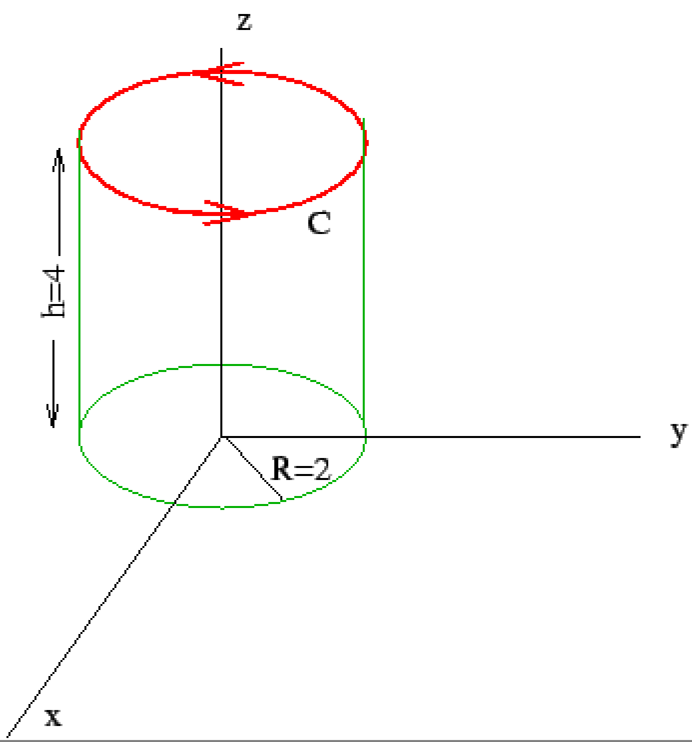
A vector field is given by $\vec{F} = -y\hat{\imath} + x\hat{\jmath} + x^2\hat{k}$. Calculate the line integral of the field along the triangular path shown above. Verify your result by Stoke's theorem.

(Ans. 1)
(Hint : To calculate the line integral along a straightline, you need the equation to the line. For instance, the equation to the line BO is $y = 2x$. Check that $\int_{AB} \vec{F} \cdot \vec{dl} = -\int y dx + \int x dy = 1$.)

Exercise 2

A vector field is given by $\vec{F} = k\rho^3 z\hat{\theta}$. Check the validity of the Stoke's theorm

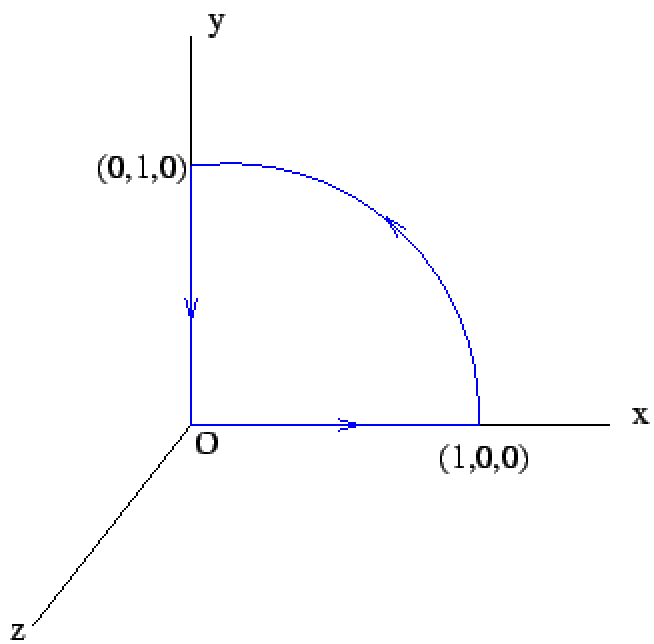
by calculating the line integral about the closed contour in the form of a circle at $z = 4$and also calculating the surface integral of the open surface of the cylinder below it, as shown. (Hint : Express the curl in cylindrical coordinates and take care of the signs of the surface elements from the curved surface and the bottom cap. Ans. $64k\pi$)



Exercise 3

Let C be a closed curve in the x-y plane in the shape of a quadrant of a circle of radius R.

If $\vec{F} = \hat{\imath}y + \hat{\jmath}z + \hat{k}x$. calculate the line integral of the field along the contour shown in a direction which is anticlockwise when looked from above the plane ($z > 0$). Take the surface of the quadrant enclosed by the curve as the open surface bounded by the curve and verify Stoke's theorem.



(Ans. $-\pi R^2/4$)

Exercise 4

Verify Stoke's theorem for a vector field $2z\hat{\imath} + 3x\hat{\jmath} + 5y\hat{k}$ where the contour is an equatorial circle of radius R and is anticlockwise when viewed from above and the surface is the hemisphere shown in the preceding example.

(Ans. $3R^2\pi$)

Exercise 5

Show that

$$\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla \cdot \vec{F}) - \nabla^2 \vec{F}$$