Exercise 1

A force $\vec{F} = xy\hat{\imath} + (x^2 - z^3)\hat{\jmath} - xz^2\hat{k}$ acts on a particle. Calculate the work done if the particle is taken from the point $(0,0,0)$ to the point $(2,1,3)$ along straight line segment connecting $(0,0,0) \rightarrow (0,1,0) \rightarrow (2,1,0) \rightarrow (2,1,3)$. What would be the work done if the particle directly moved to the final point along the straightline connecting to origin.

(Ans. -16, -13.8.)

Exercise 2

A vector field is given by $\vec{F} = (2x + 3y)\hat{\imath} + (3x + 2y)\hat{\jmath}$

Evaluate the line integral of the field around a circle of unit radius traversed in clockwise fashion.

(Ans. 6π)

Exercise 3

Evaluate the line integral of a *scalar function* xy along a parabolic path $y = x^2$ connecting the origin to the point $(1,1)$.

[Hint : Remember that the arc length along a curve is given by $\sqrt{(dx)^2 + (dy)^2}$. The curve can be parametrized by $x = t$ and $y = t^2$.] [Ans. $(25\sqrt{5} + 1)/120$]

Exercise 4

Find the flux of the vector field $\vec{V} = Ax\hat{\imath} + By^2\hat{\jmath}$ through a rectangular surface in the x-y plane having dimensions $a \times b$. The origin of the coordinate system is at one of the corners of the rectangle and the x-axis along its length.

(Ans. $Bab^3/3$)

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Exercise 5

Find the flux through a hemispherical bowl with its base on the x-y plane and the origin at the centre of the base. The vector field, in spherical polar coordinates is $\vec{V} = r\sin\theta\hat{r} + \hat{\theta} + \hat{\phi}$.

(Ans. $\pi(1 + \pi/2)$)

Exercise 6

Find the flux of the vector field $\vec{V} = 2\hat{\rho} - 3r\hat{\theta} + z\rho\hat{k}$ through surfaces of a right cylinder of radius 1 and height 2. The base of the cylinder is in the $z = 0$ plane with the origin at the centre of the base.

(Ans. $28\pi/3$)