Exercise 1

A long wire of cross sectional radius R carries a current I. The current density varies as the square of the distance from the axis of the wire. Find the magnetic field for $r < R$ and for $r > R$.

(Hint : First show that the current density $J = 2Ir^2/\pi R^4$ and obtain an expression for current enclosed for $r < R$.

Answer : $B = \mu_0 I/2\pi r$ for $r > R$ and $B = \mu_0 I r^3/2\pi R^4$ for $r < R$.)

Exercise 2

A hollow cylindrical conductor of infinite length carries uniformly distributed current I from $a < r < b$. Determine magnetic field for all r.
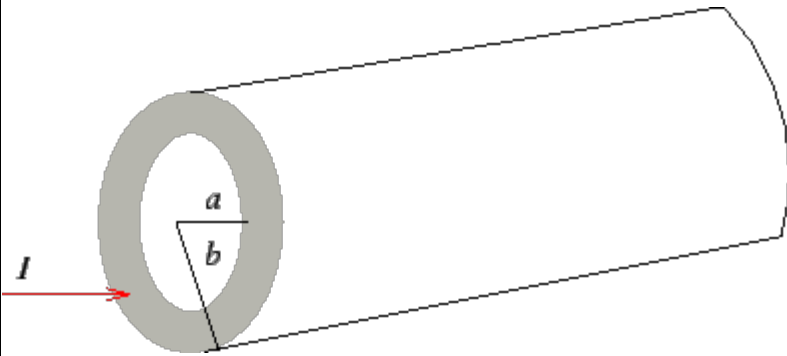
(Answer : Field is zero for $r < a$, $B = \dfrac{\mu_0 I}{2\pi r}\dfrac{r^2 - a^2}{b^2 - a^2}$ for $a < r < b$ and $B = \mu_0 I/2\pi r$ for $r > b$.)

Exercise 3

A coaxial cable consists of a solid conductor of radius a with a concentric shell of inner radius b and outer radius c. The space between the solid conductor and the shell is supported by an insulating material.
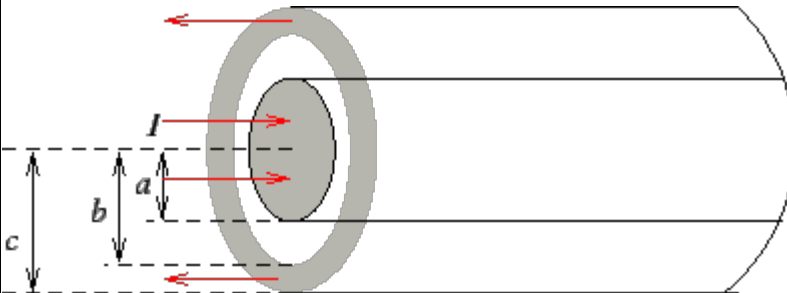
A current I goes into the inner conductor and is returned by the outer shell. Assume the cuurent densities to be uniform both in the shell and in the inner conductor. Calculate magnetic field everywhere.

(Ans. $B = \mu_0 I r / 2\pi b^2$ inside the inner conductor, $B = \mu_0 I / 2\pi r$ between the shell and the inner conductor,

$$B = \frac{\mu_0 I}{2\pi r} \frac{c^2 - r^2}{c^2 - b^2})$$

Exercise 4

Determine the magnetic field in a cylindrical hole of radius a inside a cylindrical conductor of radius b. The cylinders are of infinite length and their axes are parallel, being separated by a distance d. The conductor carries a current I of uniform density.

(Hint : The problem is conveniently solved by imagining currents of equal and opposite densities flowing in the hole and using superposition principle to calculate the field. Answer : The field inside the hole is constant $B = \mu_0 I d / 2\pi (b^2 - a^2)$)

Exercise 5

A toroid is essentially a hollow tube bent in the form of a circle. Current carrying coils are wound over it. Use an amperian loop shown in the figure to show that the field within the toroid is $\mu_0 N I / L$, where N is the number of turns and L the circumference of the circular path.
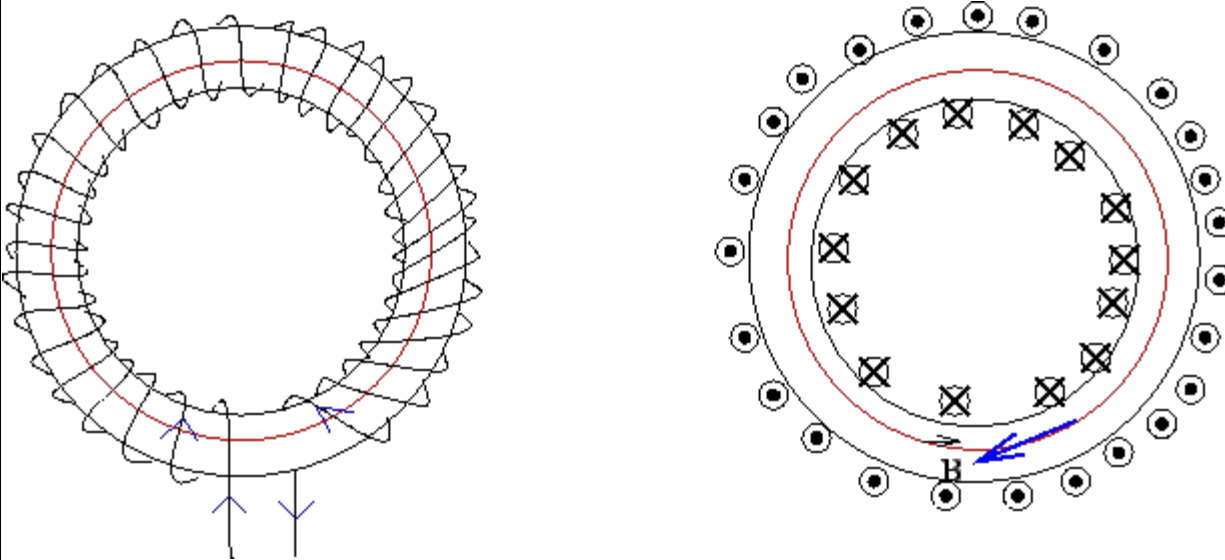
Note that as the circumference of the circular path varies with the distance of the amperian loop from the toroid axis, the magnetic field in the toroid varies over its cross section. Take the inner radius of the toroid to be 20cm and the outer radius as 21cm. Find the percentage variation of the field over the cross section of the toroid.

(Ans. 2.9%)

Exercise 6

Calculate the force per unit area between two parallel infinite current sheets with current densities λ_1 and λ_2 in the same direction.

(Ans. $\mu_0 \lambda_1 \lambda_2 / 2$)