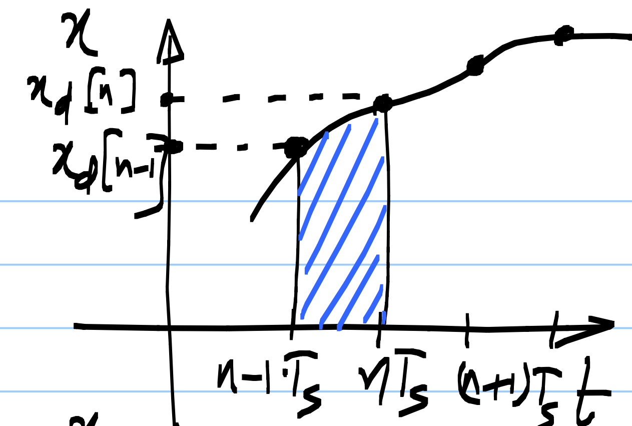
Lecture 59:

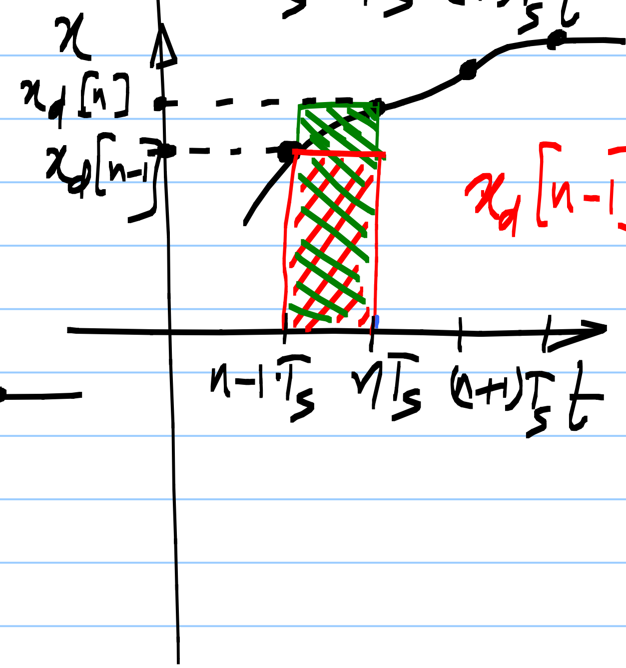
Discrete-time filters

$$y(t) = \omega_p \int x(t)\, dt$$

$$x_d[n] = x(nT_s)$$



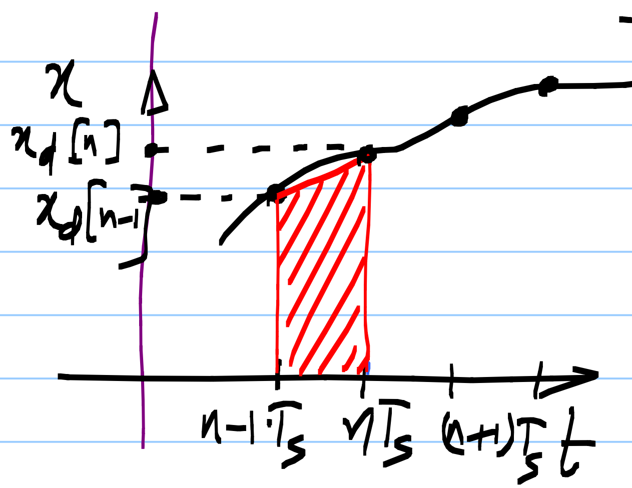
$x_d[n] \cdot T_s$



$x_d[n-1] \cdot T_s$

$$y_d[n] - y_d[n-1]$$

$$\frac{T_s}{2}\left(x_d[n] + x_d[n-1]\right)$$



$$y(t) = \omega_p \int x(t)\, dt$$

$$y_d[n] - y_d[n-1] = \omega_p T_s \cdot x_d[n-1]$$

Forward Euler Approximation

$$y_d[n] - y_d[n-1] = \omega_p T_s \, x_d[n]$$

Backward Euler

$$y_d[n] - y_d[n-1] = \frac{\omega_p T_s}{2}\left(x_d[n] + x_d[n-1]\right)$$

Bilinear

$$\boxed{\text{Bilinear}}$$

$$y_d[n] - y_d[n-1] = \frac{\omega_p T_s}{2}\left(x_d[n] + x_d[n-1]\right)$$

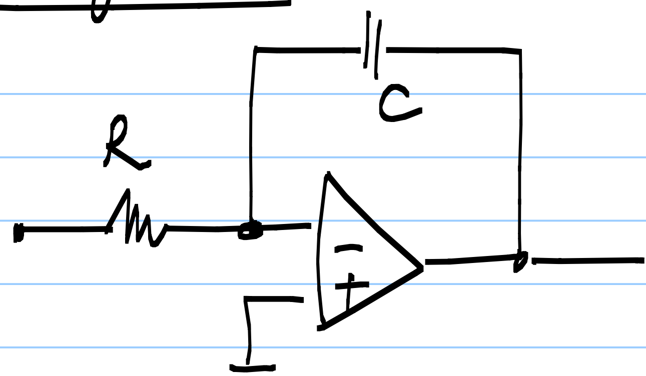
$$y(t) = \omega_p \int x(t)\,dt \longrightarrow Y(s) = \frac{\omega_p}{s} X(s)$$

$$Y_d(z)\left(1 - z^{-1}\right) = \frac{\omega_p T_s}{2} X_d(z)\left(1 + z^{-1}\right)$$

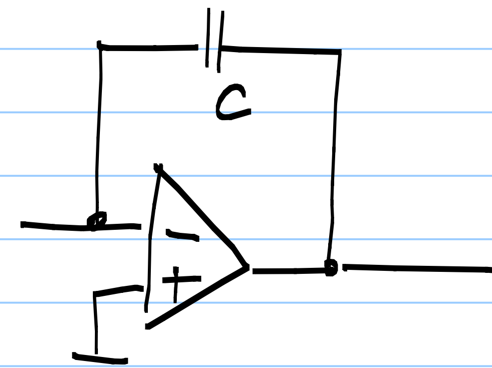
$$Y_d(z) = \frac{\omega_p T_s}{2} \cdot \frac{1+z^{-1}}{1-z^{-1}} \cdot X_d(z)$$

$$\frac{\omega_p T_s}{2} \cdot \frac{1+z^{-1}}{1-z^{-1}} \longleftrightarrow \frac{\omega_p}{s} \quad ; \quad \underbrace{s \longleftrightarrow \frac{2}{T_s}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

Integrator:



DT Integrator:



$$\frac{Y_d(z)}{X_d(z)} = \frac{2}{T_s} \cdot \frac{1+z^{-1}}{1-z^{-1}}$$

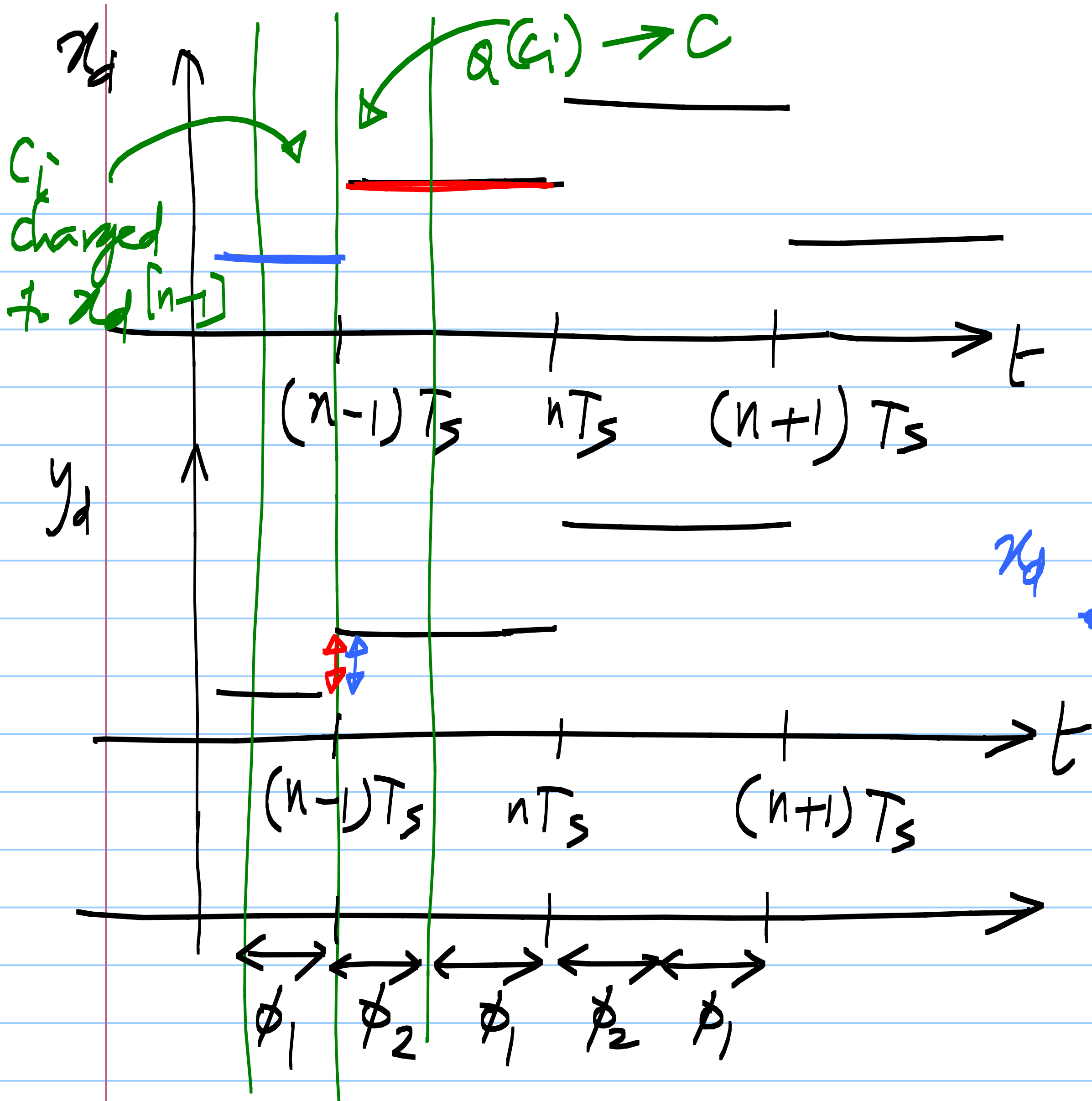
$$= \frac{2}{T_s}\left(\frac{1}{1-z^{-1}} + \frac{z^{-1}}{1-z^{-1}}\right)$$

Bkwd Euler

Fwd Euler

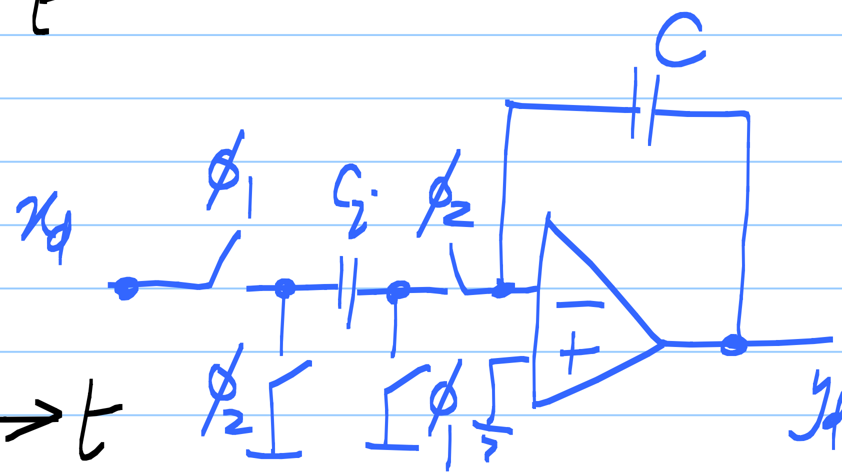
$$y_d[n] - y_d[n-1] = \frac{2}{T_s} \cdot x_d[n]$$

$$y_d[n] - y_d[n-1] = \frac{2}{T_s} \cdot x_d[n-1]$$

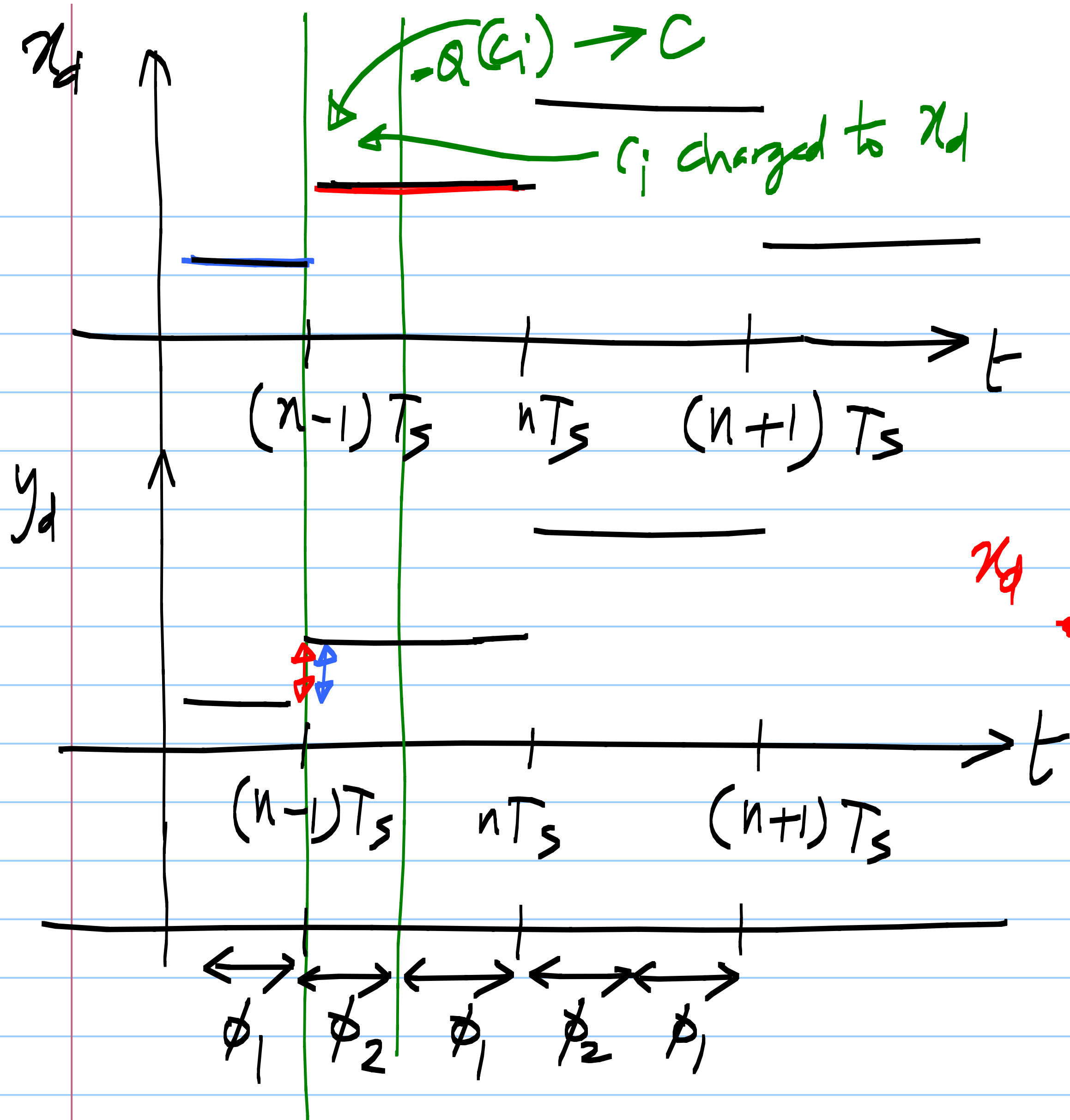


$$y_d[n] - y_d[n-1] = \frac{2}{T_s} \cdot x_d[n]$$

$$y_d[n] - y_d[n-1] = \frac{2}{T_s} \cdot x_d[n-1]$$



$$y_d[n] - y_d[n-1] = \frac{C_i}{C} \cdot x_d[n-1]$$



$$y_d[n] - y_d[n-1] = \frac{2}{T_s} \cdot x_d[n]$$

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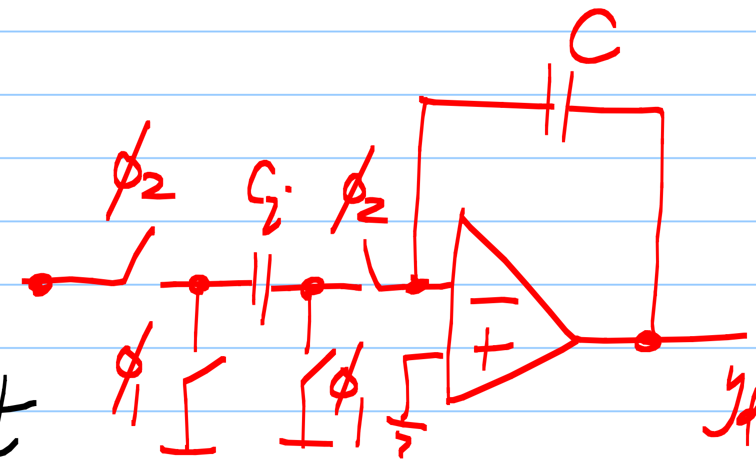
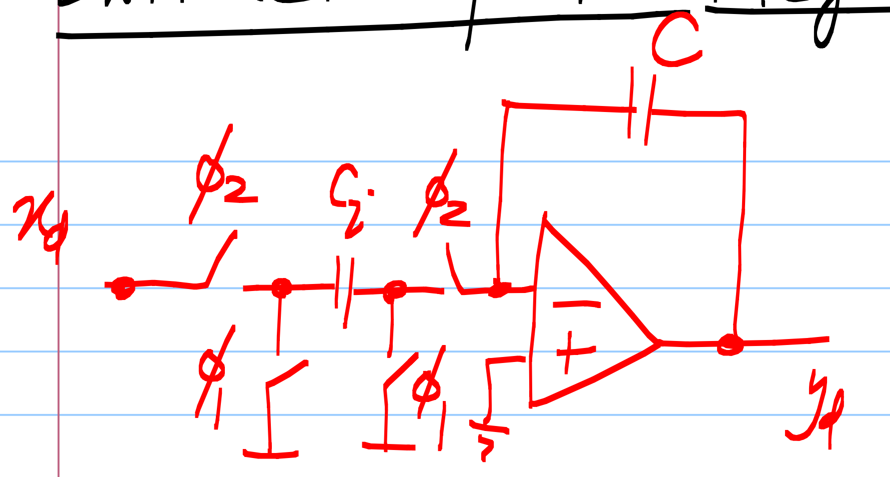
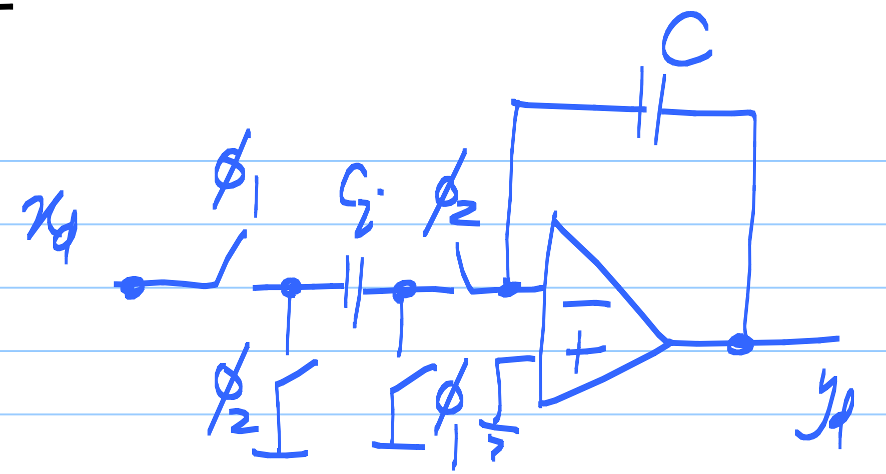


$$y_d[n] - y_d[n-1] = -\frac{C_i}{C} \cdot x_d[n]$$

Switched capacitor integrators





$$y_d[n] - y_d[n-1] = -\frac{C_i}{C} \cdot x_d[n]$$

$$\frac{Y_d(z)}{X_d(z)} = -\frac{C_i}{C}\,\frac{1}{1-z^{-1}}$$

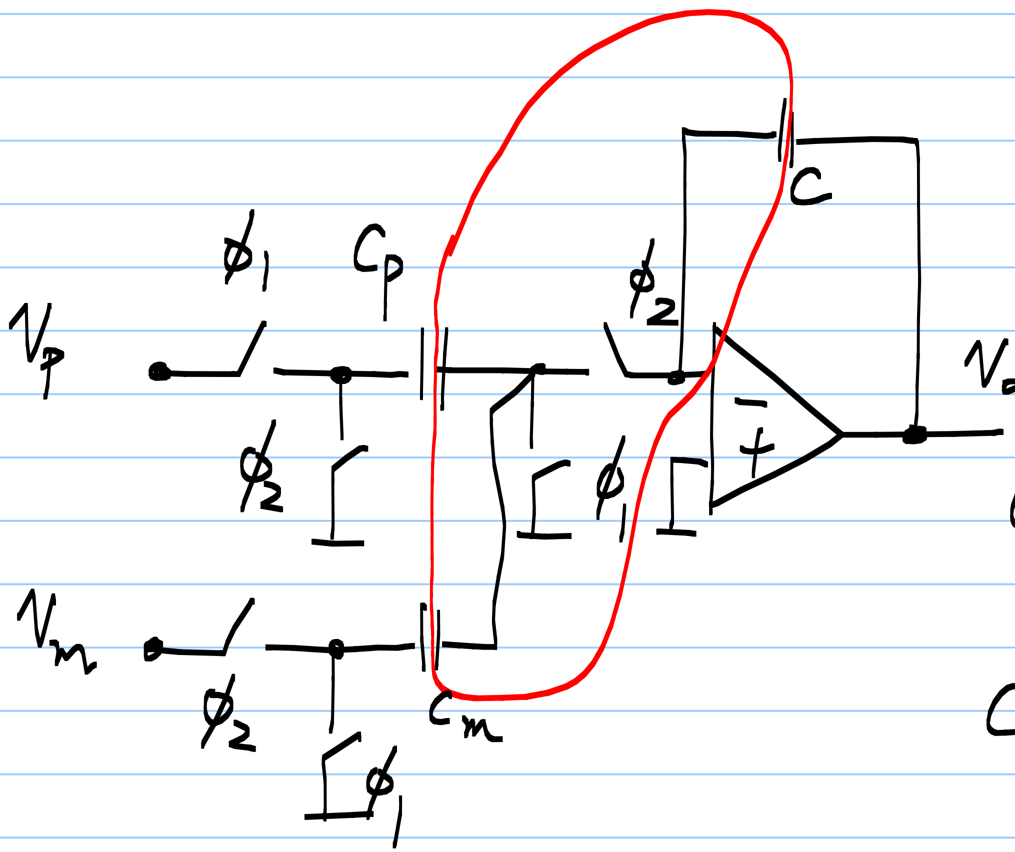
Delay-free, inverting integrator

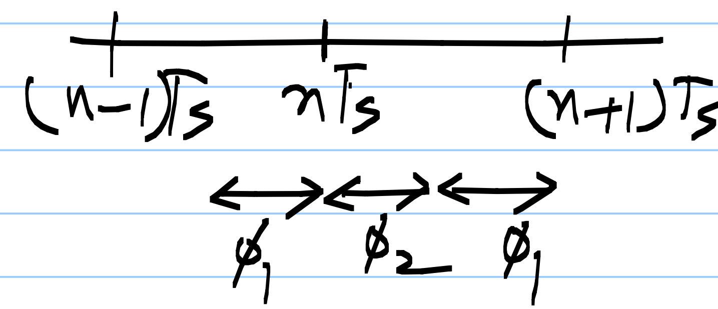
$$y_d[n] - y_d[n-1] = \frac{C_i}{C} \cdot x_d[n-1]$$

$$\frac{Y_d(z)}{X_d(z)} = +\frac{C_i}{C} \cdot \frac{z^{-1}}{1-z^{-1}}$$

Delayed, non-inverting integrator

Bilinear transformed integrator:





	ϕ_1	ϕ_2
C_p	$-C_p V_p[n-1]$	0
C_m	0	$-C_m V_m[n]$
C	$-C\cdot V_o[n-1]$	$-C\cdot V_o[n]$

$$-C_p V_p[n-1] - 0 - C V_o[n-1]$$

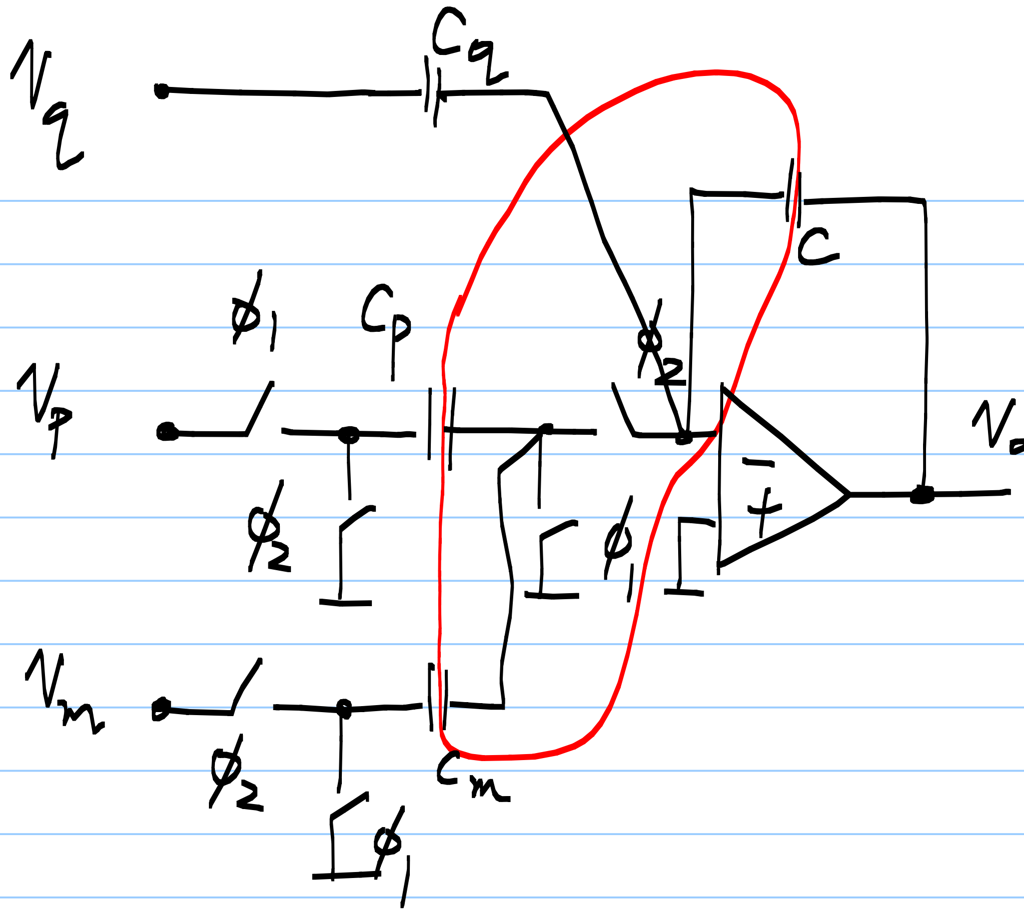
$$= 0 - C_m V_m[n-1] - C V_o[n]$$

$$V_o[n] - V_o[n-1] = \frac{C_p}{C} V_p[n-1] - \frac{C_m}{C} V_m[n]$$

For bilinear transformed integrator,

$$V_p = V_i \; ; \; V_m = -V_i \; ; \; C_p = C_m$$

$$V_o[n] - V_o[n-1] = \frac{C_p}{C} \cdot \left(V_i[n-1] + V_i[n] \right)$$



$$V_o[n] - V_o[n-1] = \frac{C_p}{C} V_p[n-1] - \frac{C_m}{C} V_m[n] + \frac{C_q}{C}\left(V_q[n-1] - V_q[n]\right)$$

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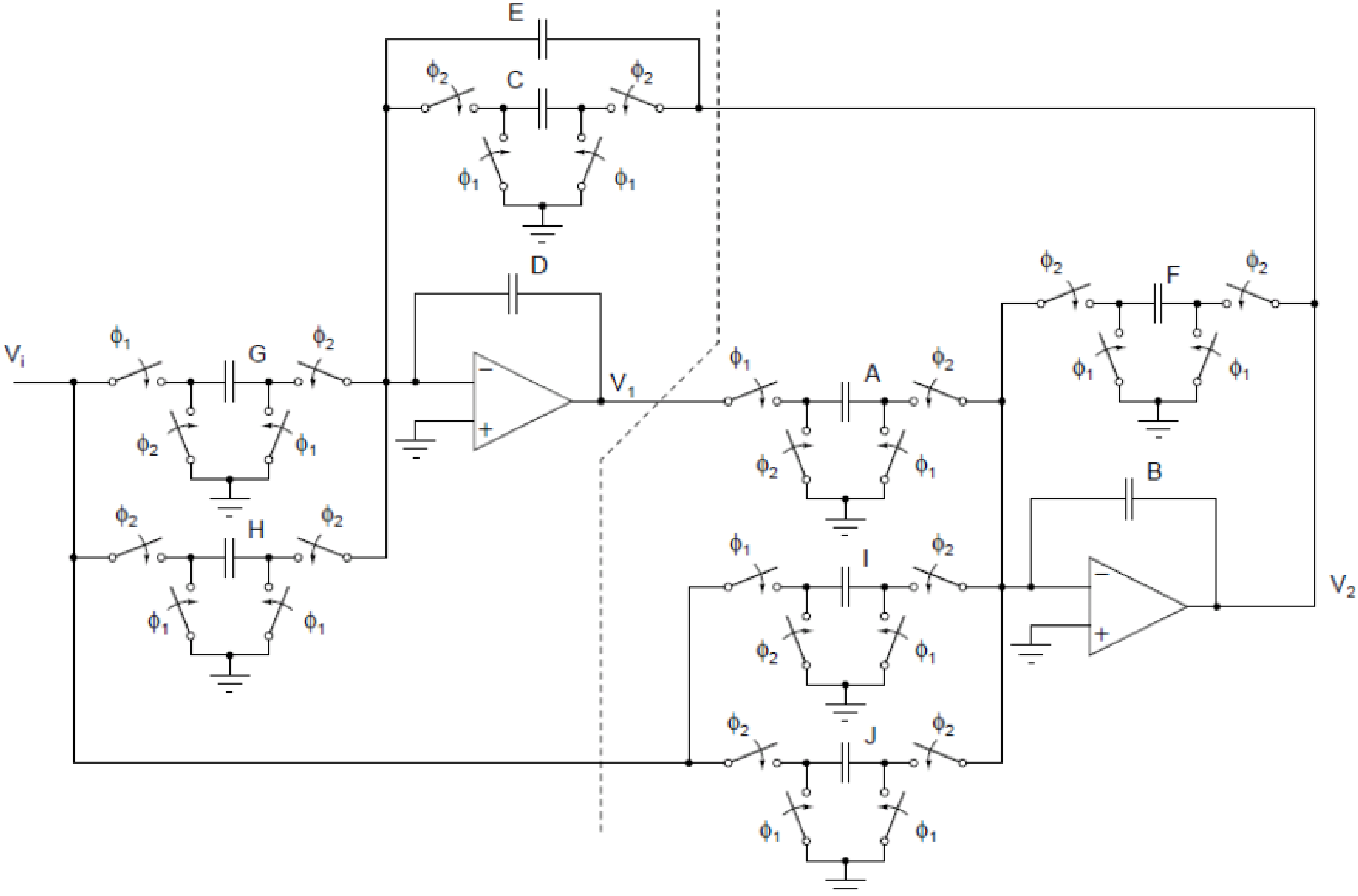
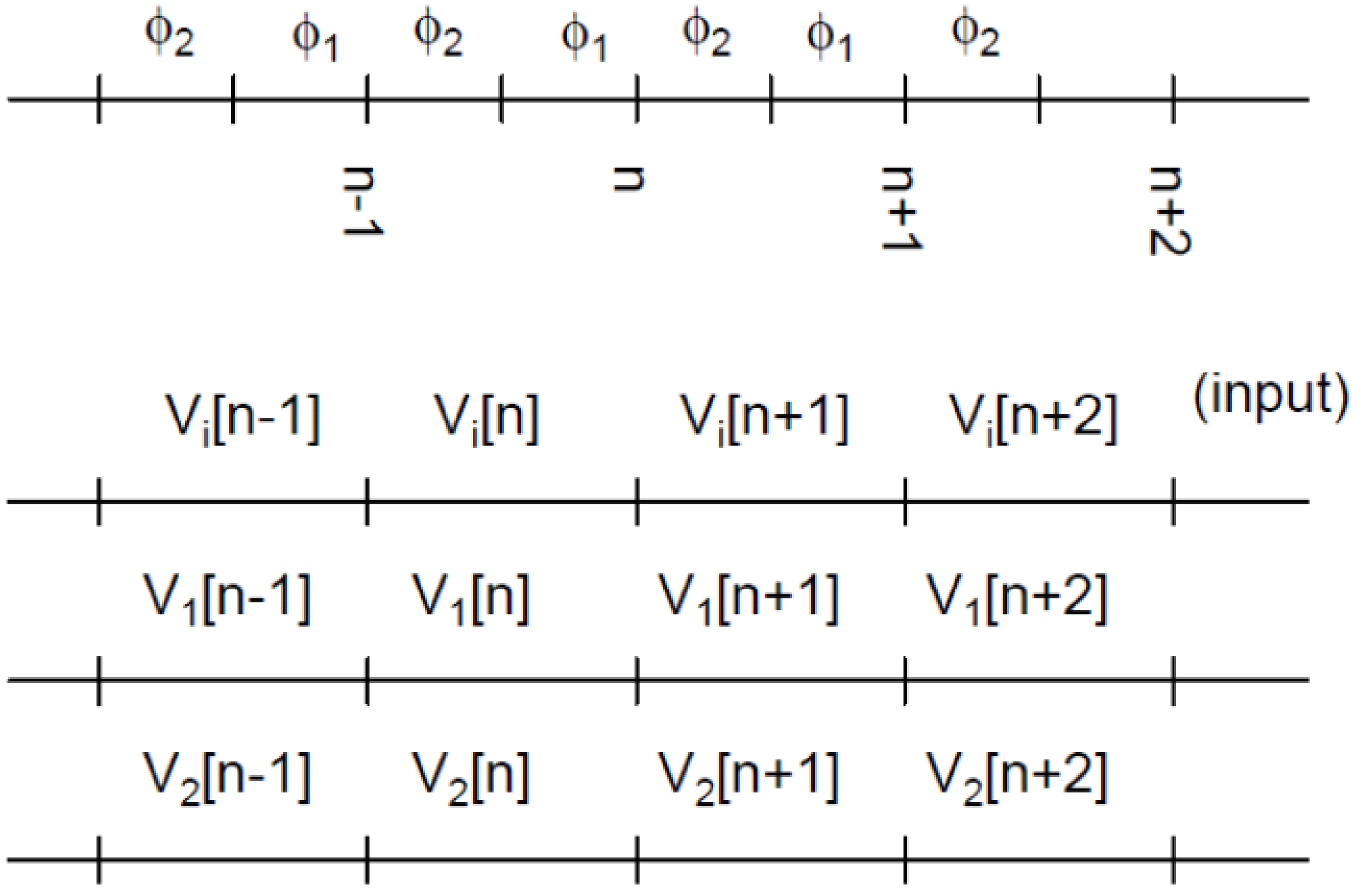
Indian Institute of Technology, Madras


E
C
D
G
H
A
I
J
F
B
V_i
V_1
V_2
ϕ_1
ϕ_2


ϕ_2 ϕ_1 ϕ_2 ϕ_1 ϕ_2 ϕ_1 ϕ_2
n-1
n
n+1
n+2
$V_i[n-1]$ $V_i[n]$ $V_i[n+1]$ $V_i[n+2]$
(input)
$V_1[n-1]$ $V_1[n]$ $V_1[n+1]$ $V_1[n+2]$
$V_2[n-1]$ $V_2[n]$ $V_2[n+1]$ $V_2[n+2]$

$$
\begin{aligned}
D(V_1[n] - V_1[n-1]) + G(0 - V_i[n-1]) + & \\
H(V_i[n] - 0) + C(V_2[n] - 0) + E(V_2[n] - V_2[n-1]) &= 0 \\
B(V_2[n] - V_2[n-1]) + F(V_2[n] - 0) + A(0 - V_1[n-1]) + & \\
I(0 - V_i[n-1]) + J(V_i[n] - 0) &= 0
\end{aligned}
$$

$$
\frac{V_2}{V_{in}} = \frac{-DJ + (ID + DJ - HA)z^{-1} + (GA - ID)z^{-2}}{D(B+F) + (-2BD - DF + AC + AE)z^{-1} + (BD - AE)z^{-2}}
$$

$B = D = 1$; Any one of $G, H, I, J = 0$; $A = C$

2^{nd} order lowpass filter:

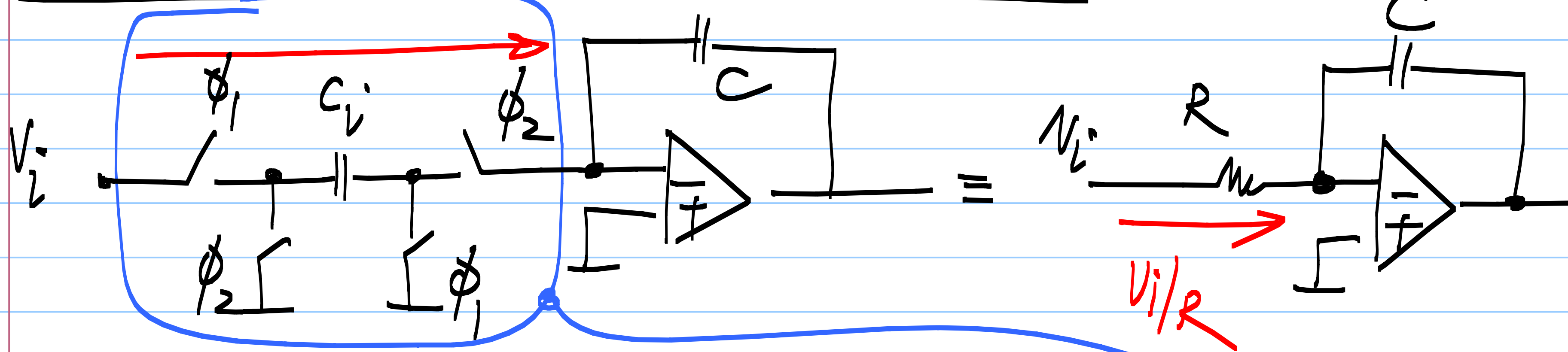
$$s \leftrightarrow \frac{2}{T_s} \cdot \frac{1-z^{-1}}{(1+z^{-1})}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + \frac{s}{Q\omega_p} + \frac{s^2}{\omega_p^2}}$$

$$\frac{V_o(z)}{V_i(z)} = \frac{1}{1 + \frac{2}{Q\omega_p T_s} \cdot \frac{1-z^{-1}}{(1+z^{-1})} + \frac{4}{(\omega_p T_s)^2} \cdot \frac{(1-z^{-1})^2}{(1+z^{-1})^2}}$$

$$= \frac{(1+z^{-1})^2}{(1+z^{-1})^2 + \frac{2}{Q\omega_p T_s}(1-z^{-1})(1+z^{-1}) + \frac{4}{(\omega_p T_s)^2}(1-z^{-1})^2}$$

Switched capacitor $\longleftrightarrow$ resistor:



In each cycle (of T_s), a charge $C_i V_i$ is transferred to C

$$\therefore \text{ Average current} = \frac{C_i V_i}{T_s} = \frac{V_i}{(T_s/C_i)}$$

$$R = \frac{T_s}{C_i} = \frac{1}{f_s C_i}$$