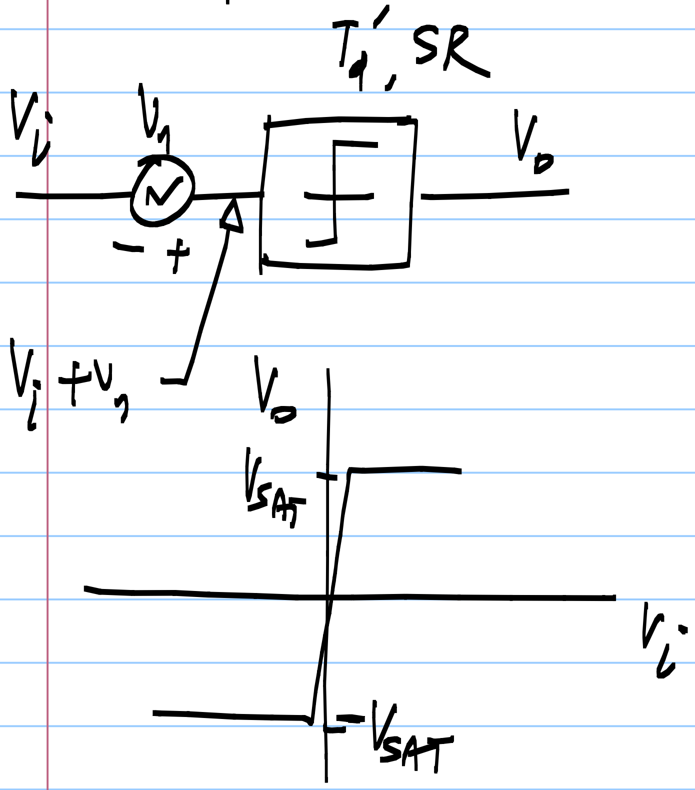
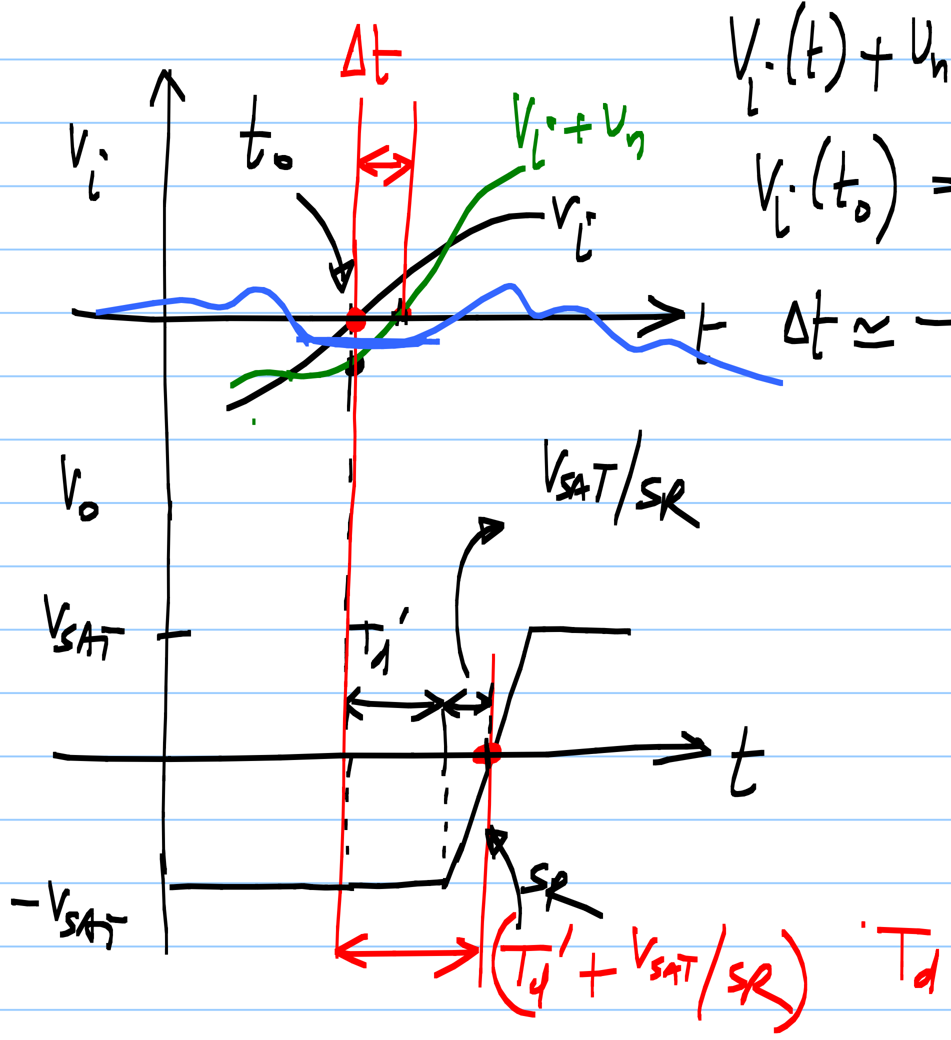
Lecture 51 : Oscillator phase noise

Comparator:





$$V_i(t) + v_n(t) = 0$$

$$V_i(t_0) = 0$$

$$\Delta t \simeq -\frac{v_n(t_0)}{\left.\frac{dV_i}{dt}\right|_{t_0}}$$

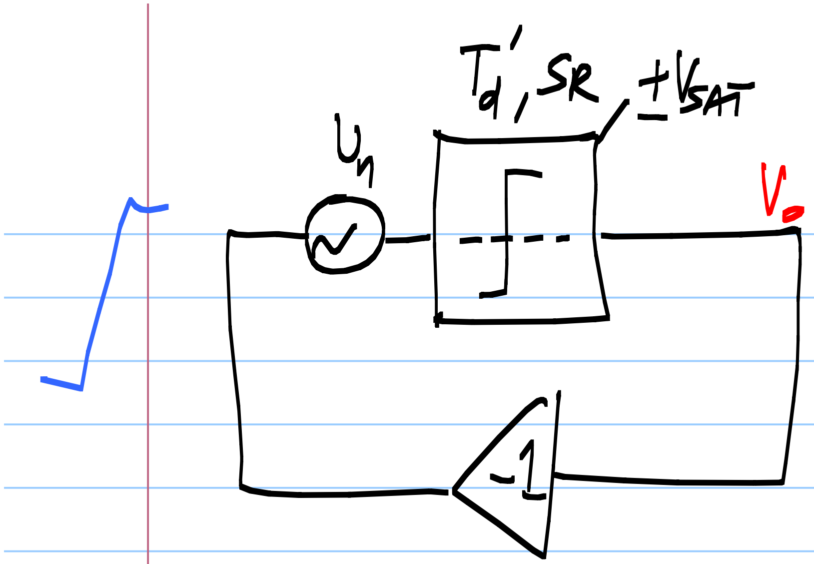
$$V_i(t) + v_n(t) = 0$$

$$\underbrace{V_i(t_0)}_{=0} + \frac{dV_i}{dt}\Big|_{t_0}(t-t_0) + v_n(t_0) + \underbrace{\cancel{\frac{dv_n}{dt}\Big|_{t_0}(t-t_0)}}_{=0.} = 0$$

{not changing much}

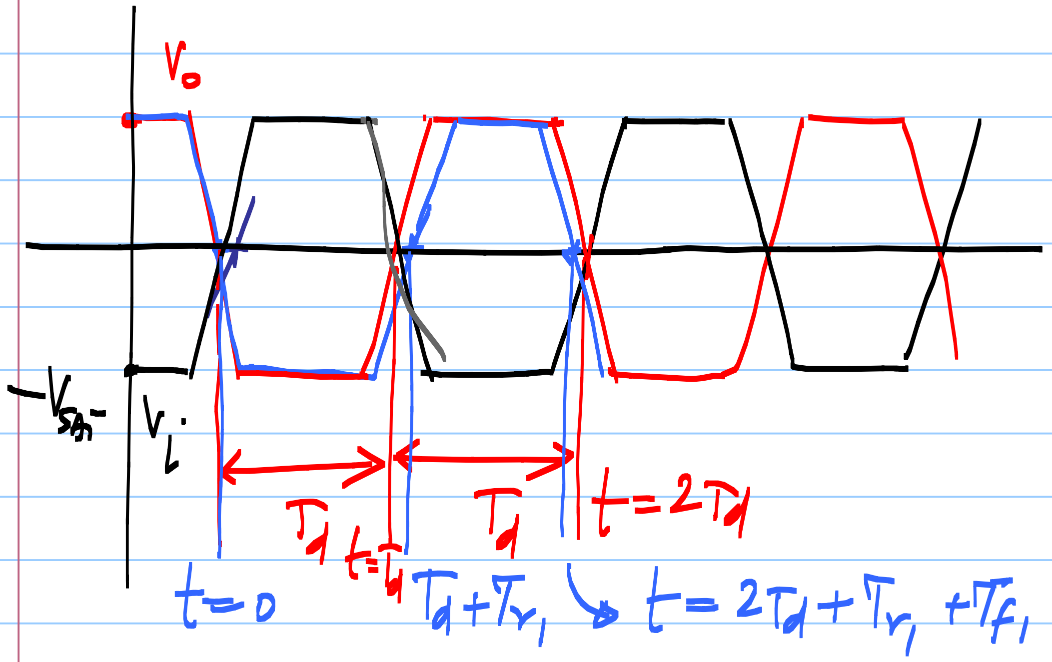
$$(t-t_0) = \frac{-v_n(t_0)}{\frac{dV_i}{dt}\Big|_{t_0}}$$

shift in zero crossing due to noise



* Oscillator with a period $T_o = 2T_d$

frequency $f_o = \frac{1}{T_o} = \frac{1}{2T_d}$



* output changes sign at $\underline{t=0}$

(noise)

Input changes sign at $t = T_{r_1}$

* output changes sign at $t = T_d + T_{r_1}$

(noise)

Input " " " $t = T_d + T_{r_1} + T_{f_1}$

* output " " " $\underline{t = 2T_d + T_{r_1} + T_{f_1}}$

output " " " $t = 3T_d + T_{r_1} + T_{f_1} + T_{r_2}$

output " " " $\underline{t = 4T_d + T_{r_1} + T_{f_1} + T_{r_2} + T_{f_2}}$

$2T_d$: T_0

$$T_k = T_{r_k} + T_{f_k}$$

(jitter){phase noise}

Accumulated jitter

Each period	ideal zero crossings	with noise
	0	0
T_1	T_0	$T_0 + T_1$
T_2	$2T_0$	$2T_0 + T_1 + T_2$
	$\vdots$	$\vdots$
T_n	nT_0	$nT_0 + T_1 + T_2 + .. T_n$
	$\vdots$	$\vdots$

Period jitter

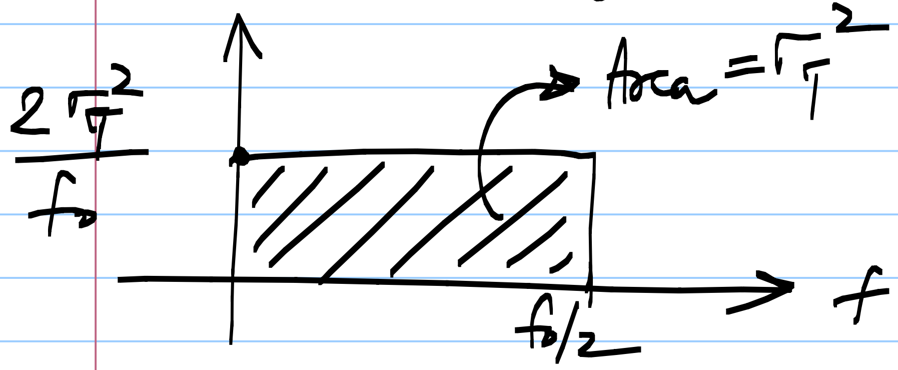
$$T_k = T_{rk} + T_{fk} = \frac{v_{nr,k}}{SR} + \frac{v_{nf,k}}{SR}$$

$$T_{k+1} = \frac{v_{nrk+1}}{SR} + \frac{v_{nfk}}{SR}$$

v_n : white noise $\Rightarrow \{T_k\}$ is $\underline{\underline{\text{white}}}$

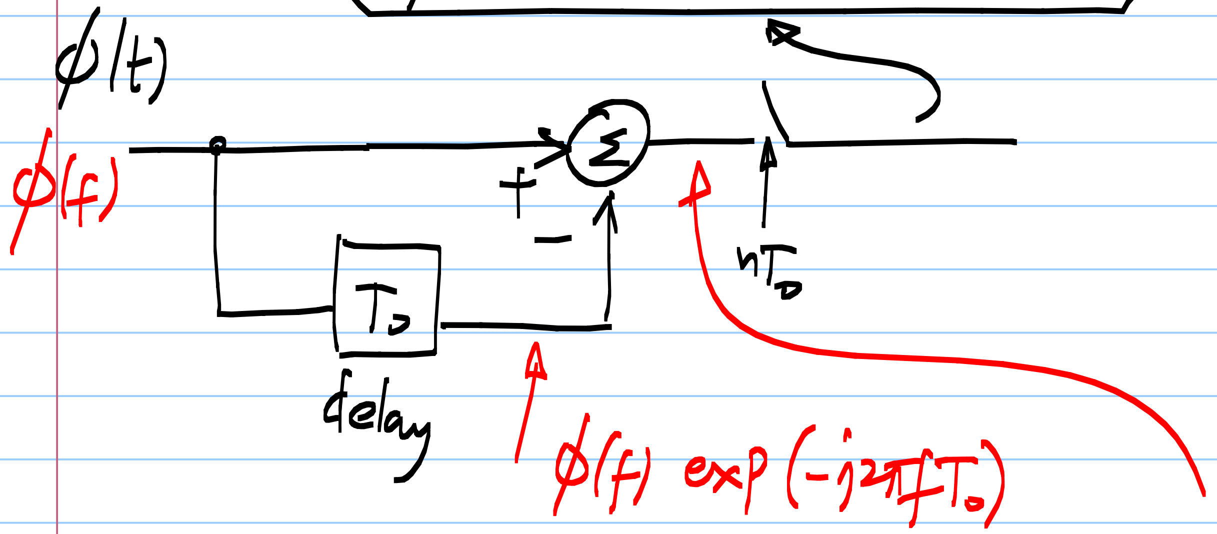
$\hookrightarrow$ mean-squared value σ_T^2

Period jitter
<u>spectral density</u>:



Phase noise: $\underline{\underline{\phi(nT_o)}} = \sum_{k=1}^{n} T_k \cdot \frac{2\pi}{T_o}$

$$\underbrace{\phi(nT_o) - \phi((n-1)\,T_o)} = \underbrace{T_n \cdot \left(\frac{2\pi}{T_o}\right)}$$



$$\underline{\phi(f)\left(1 - \exp(-j2\pi fT_o)\right)}$$

$$S_\phi(f)\left|1-\exp(-j2\pi f T_0)\right|^2$$

$$=\underline{S_\phi(f)\ 4\cdot\sin^2(\pi f T_0)} = S_T\cdot\frac{4\pi^2}{T_0^2}$$

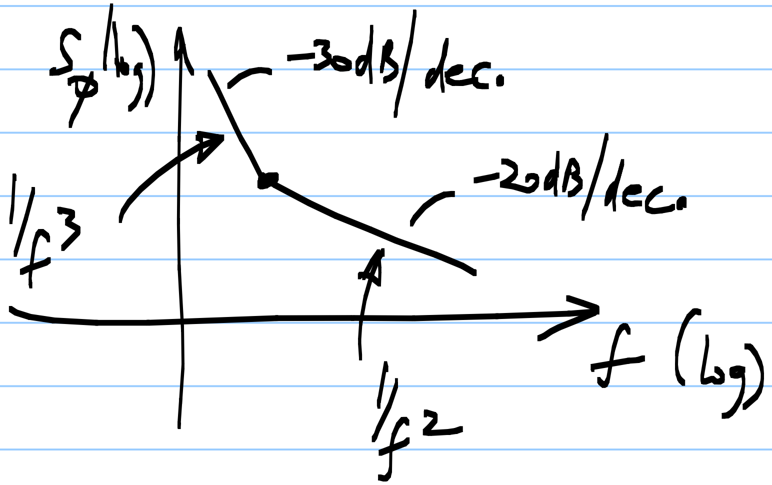
$$S_\phi(f) = \frac{2\sigma_T^2}{f_0}\cdot\frac{4\pi^2}{T_0^2}\cdot\frac{1}{4\sin^2(\pi f T_0)}$$

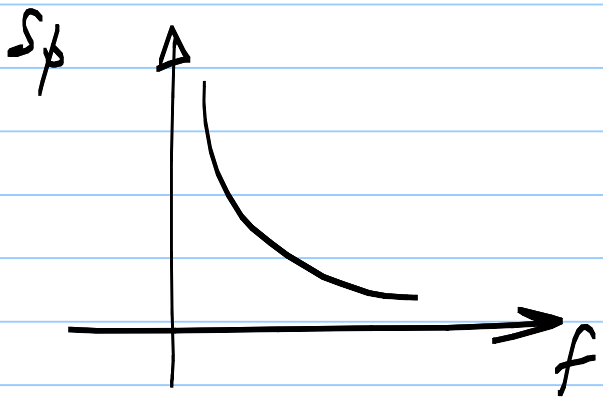
$$\simeq \frac{2\sigma_T^2}{f_0}\cdot\frac{4\pi^2}{T_0^2}\cdot\frac{1}{4\pi^2 f^2 T_0^2} = \frac{2\sigma_T^2}{T_0^3}\cdot\frac{1}{f^2}$$

Oscillator phase noise: * Accumulated jitter

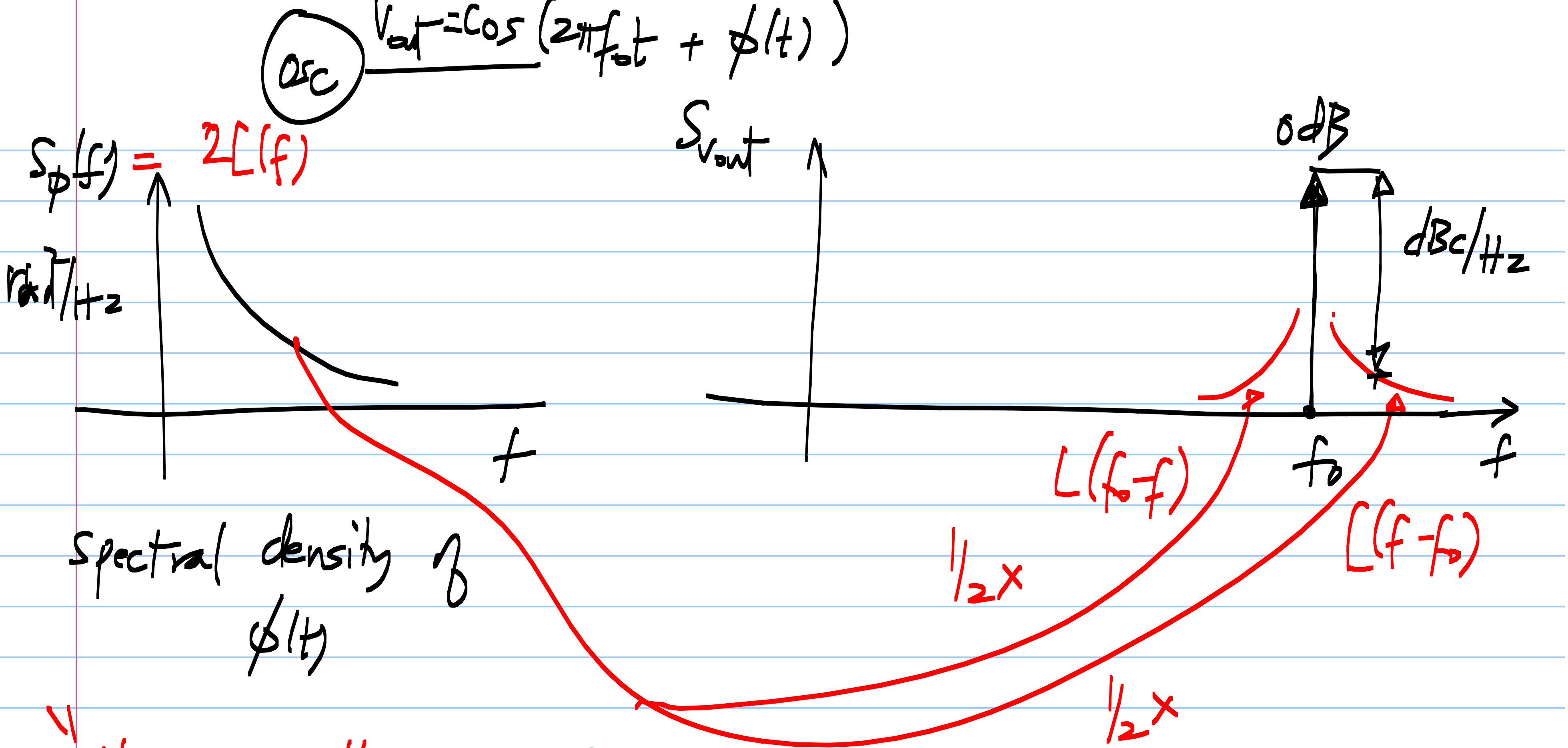
* Phase noise : $\frac{1}{f^2}$ dependence

* $\frac{1}{f^3}$ region if $\frac{1}{f}$ noise contributes significantly





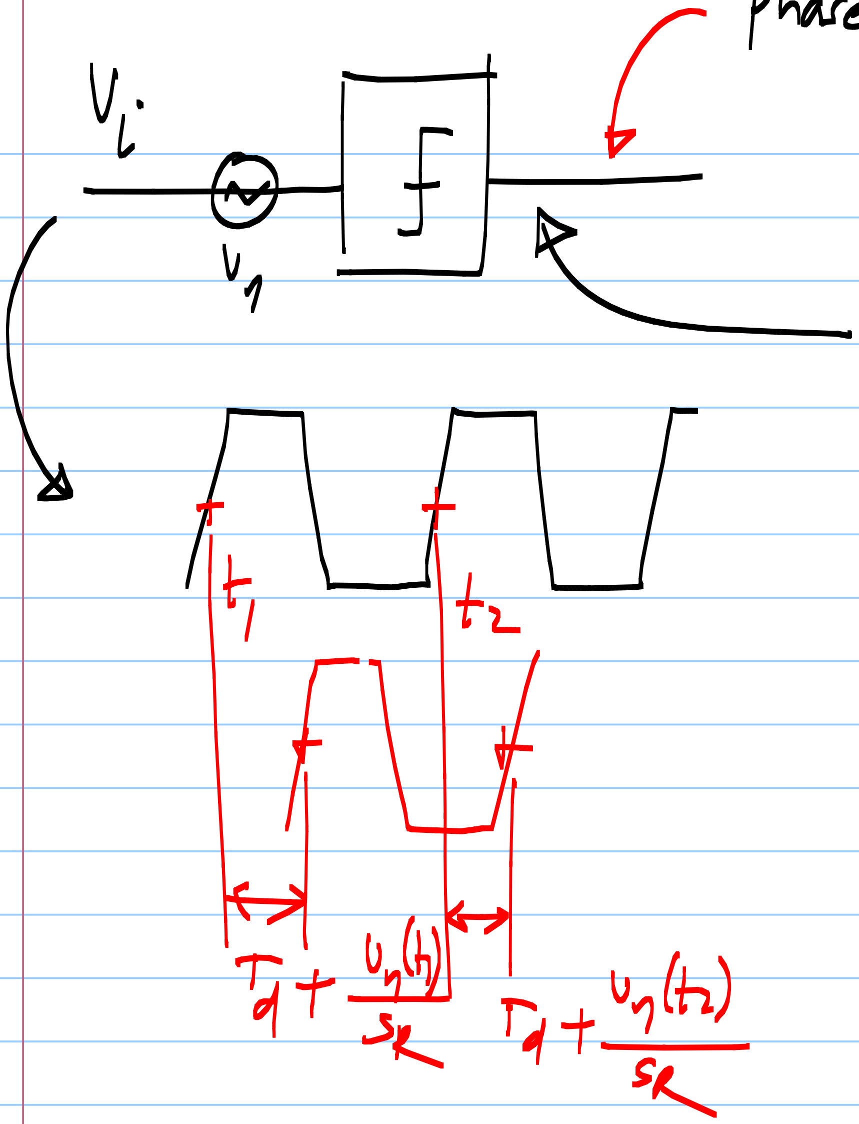
$V_{out} = \cos(2\pi f_0 t + \phi(t))$



Spectral density of $\phi(t)$

"phase noise" $\mathcal{L}(f) = \frac{1}{2} S_\phi(f)$

$$\overline{\phi^2} = \int_0^\infty 2\mathcal{L}(f)\, df$$



phase noise is white if v_n is white

Comparator driven by an ideal periodic signal

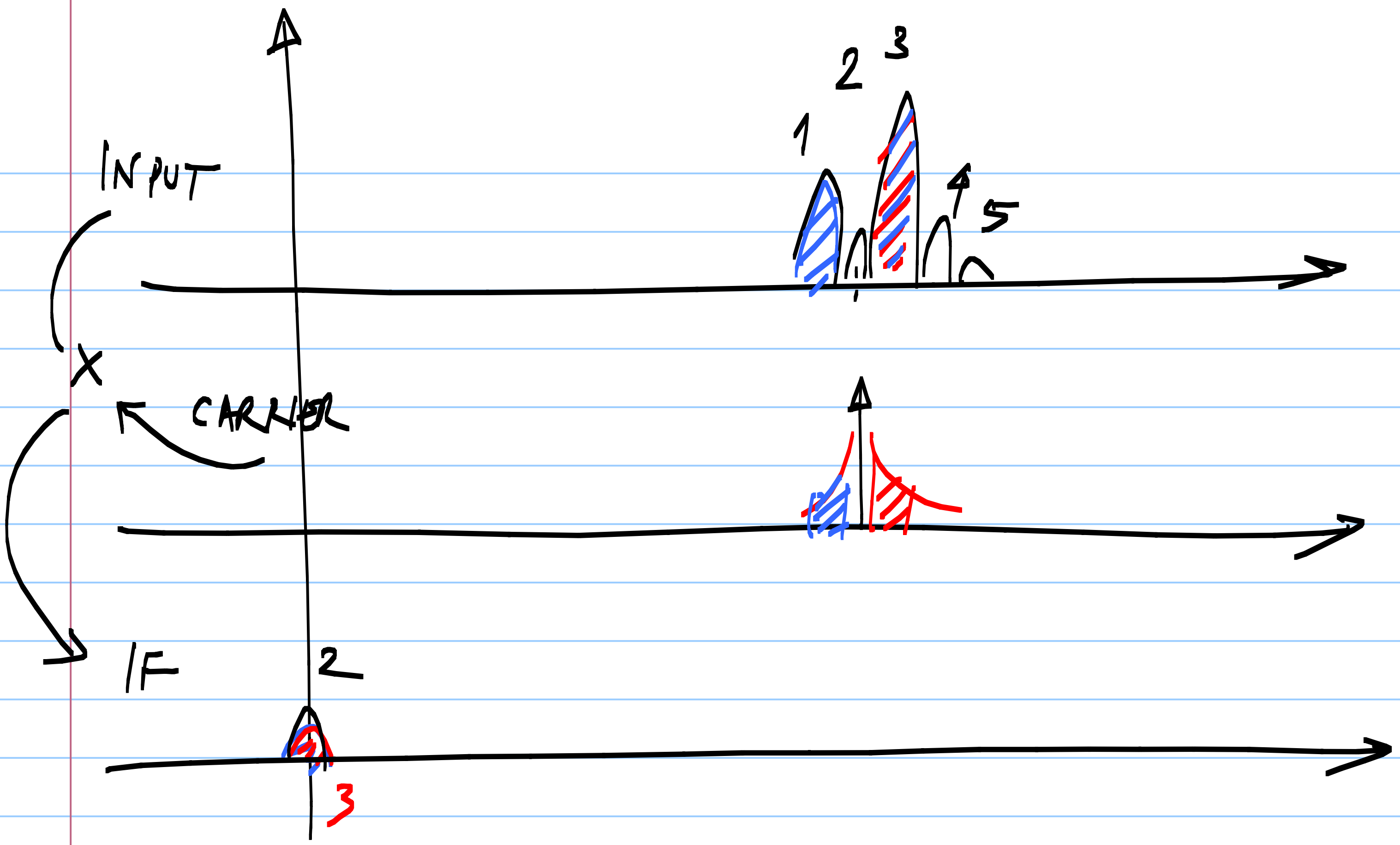
$$T_n = \frac{v_{n,r}}{SR_+} + \frac{v_{n,f}}{SR_-}$$

Reduce this to reduce phase noise

* Reduce $v_{n,r}$, $v_{n,f}$ → Impedance scaling

  → Increase the power dissipation

* Increase SR_+ → Increase the power dissipation


1
2
3
4
5
INPUT
X
CARRIER
IF
2
3