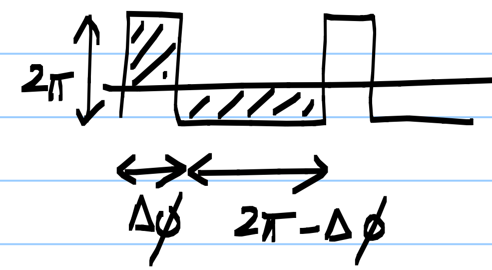
# Lecture 47

## 3 state phase detector:

* Average output $= \underbrace{\frac{V_{pd}}{2\pi}}_{K_{pd}} \Delta\phi$

* Periodic error (input referred) :

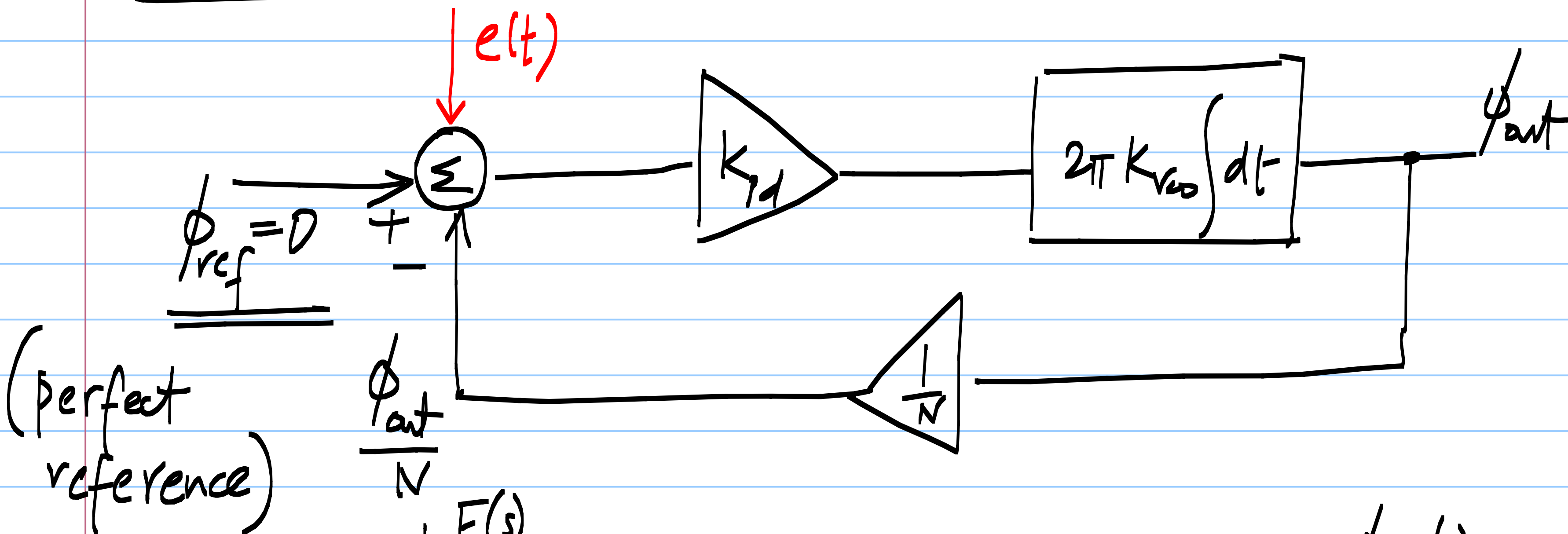


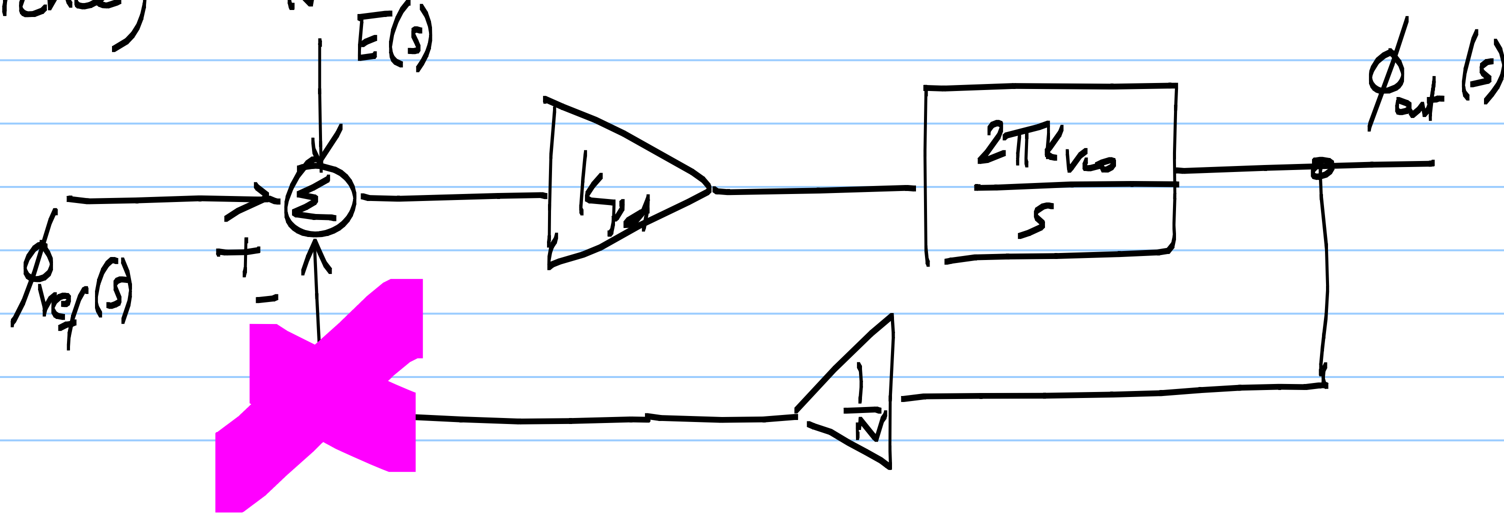

peak-peak $= 2\pi$

duty cycle $= \frac{\Delta\phi}{2\pi}$

## Type I PLL model :

$$\Delta\Phi = \Phi_{ref} - \Phi_{fb} = \frac{N f_{ref} - f_o}{K_{vco} \cdot K_{pd}}$$



$$\frac{\phi_{out}(s)}{\phi_{ref}(s)} = \frac{\phi_{out}(s)}{E(s)} = \frac{N}{1 + \frac{s \cdot N}{2\pi k_{pd} k_{vco}}}$$

dc gain : $N$

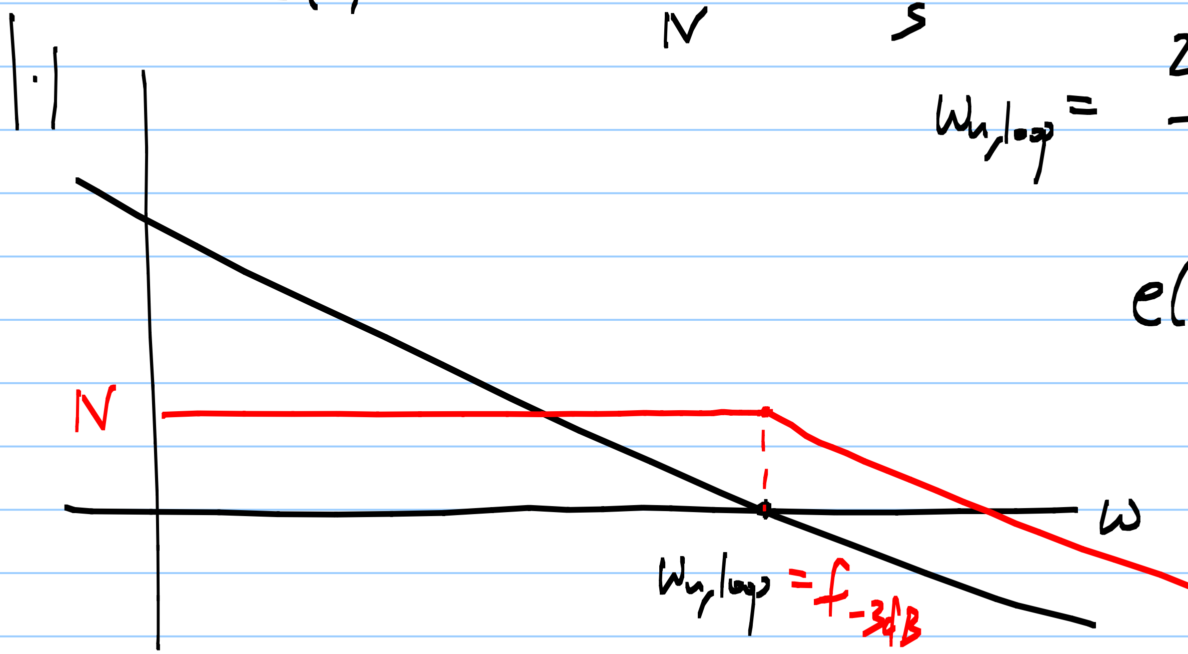
$f_{-3dB}$ : $\frac{K_{pd} K_{vco}}{N}$

Pole : $\frac{2\pi k_{pd} k_{vco}}{N}$

1st order lowpass

$$L(s) = \frac{2\pi k_{pd} k_{vco}}{N} \cdot \frac{1}{s}$$

$$\omega_{u,loop} = \frac{2\pi k_{pd} k_{vco}}{N}$$




e(t) : periodic at $f_{ref}$ → $f_{ref}$ & its harmonics

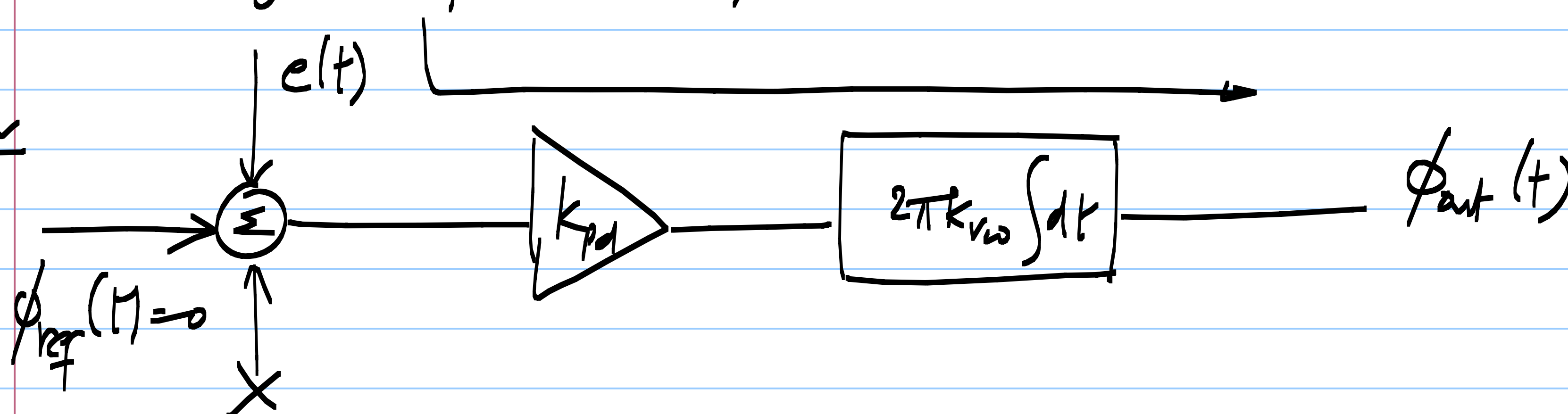
$$\frac{\phi_{out}(s)}{E(s)} = N \cdot \frac{L}{1+L} = N \cdot \frac{1}{1+1/L}$$

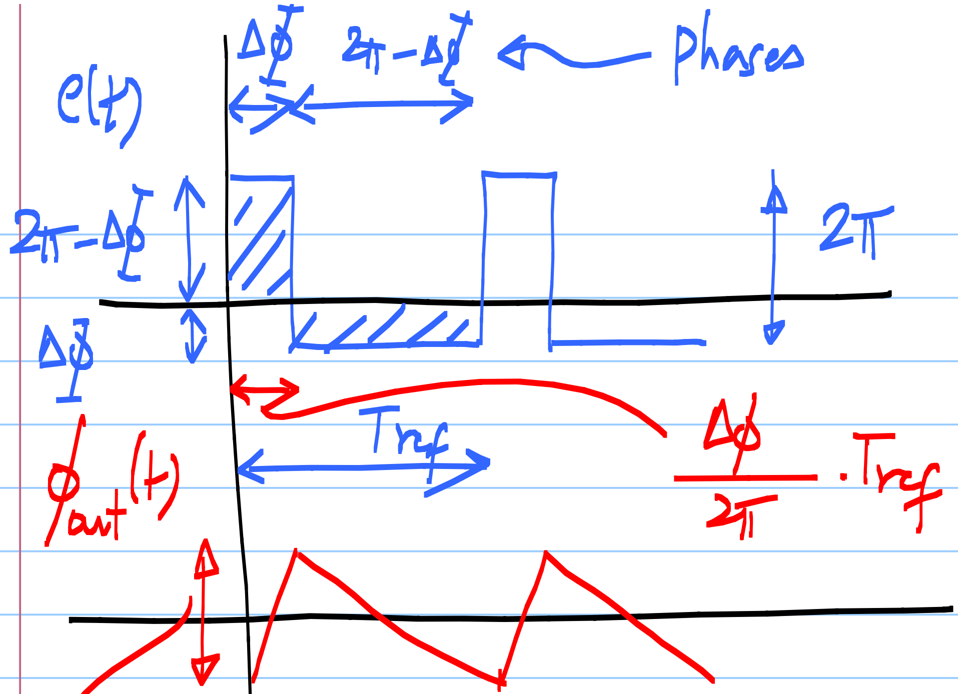
$$\simeq N \qquad |L| \gg 1$$

$$\simeq N \cdot L \qquad |L| \ll 1$$

Assuming $f_{ref} > f_{-3dB}$, the feedback is negligible

$\simeq$






$$\frac{\Delta\phi}{2\pi}\cdot T_{ref}$$

$$2\pi K_{vco} K_{pd} \cdot (2\pi - \Delta\phi) \cdot \frac{\Delta\phi}{2\pi} \cdot T_{ref}$$

$$= \frac{K_{vco} K_{pd}}{f_{ref}} (2\pi - \Delta\Phi) \cdot \Delta\Phi$$

maximum when $\Delta\Phi = \pi$

$$= \pi^2 \cdot \frac{K_{vco} K_{pd}}{f_{ref}}$$


$$\pi^2 \cdot \frac{K_{pd} K_{vco}}{f_{ref}}$$

## Output signal:

Periodic in $t$ : period of $\frac{1}{N f_{ref}}$

ideal case: $\cos\left(2\pi \cdot N f_{ref} t + \Phi_{out}\right)$

with phase error $\phi(t)$: $\cos\left(2\pi N f_{ref} t + \Phi_{out} + \phi(t)\right)$

Not periodic in $t$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos\left(2\pi N f_{ref} t + \Phi_{out}\right) \cdot \underbrace{\cos(\phi(t))}_{\simeq 1} - \sin\left(2\pi N f_{ref} t\right) \cdot \underbrace{\sin(\phi(t))}_{\simeq \phi(t)}$$

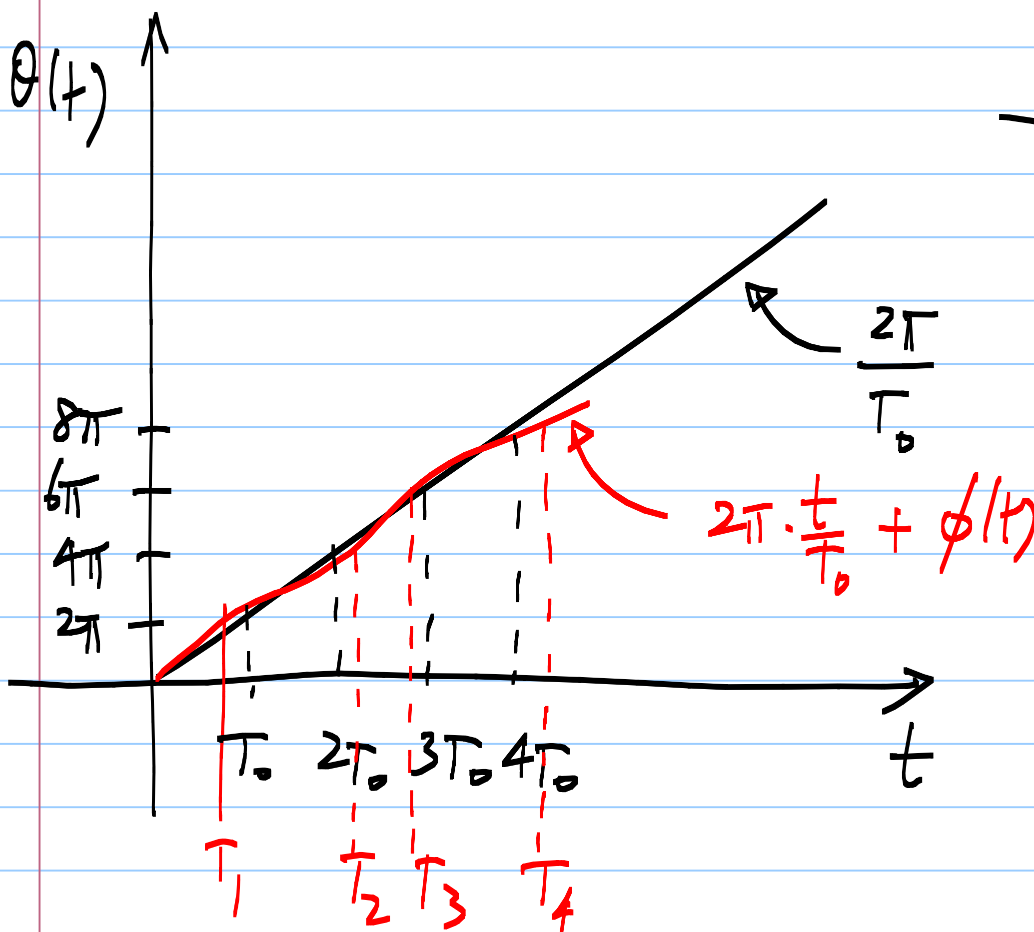
If $|\phi(t)| \ll 1$ rad

$$\cos(2\pi N f_{ref} t + \Phi_{out}) \cdot \underbrace{\cos(\phi(t))}_{\simeq 1} - \sin(2\pi N f_{ref} t) \cdot \underbrace{\sin(\phi(t))}_{\simeq \phi(t)}$$

If $|\phi(t)| \ll 1$ rad

$$\simeq \underbrace{\cos(2\pi N f_{ref} t + \Phi_{out})}_{\text{Ideal periodic output}} - \underbrace{\phi(t)\sin(2\pi N f_{ref} t)}_{\phi(t) \text{ modulating a carrier at } N f_{ref}}$$

Jitter: Description of aperiodicity in the time domain




absolute jitter:

$T_1 - T_0$

$T_2 - 2T_0$

$\vdots$

$T_k - k \cdot T_0$

average → 0

$\phi(t)$:

$T_k - kT_0$

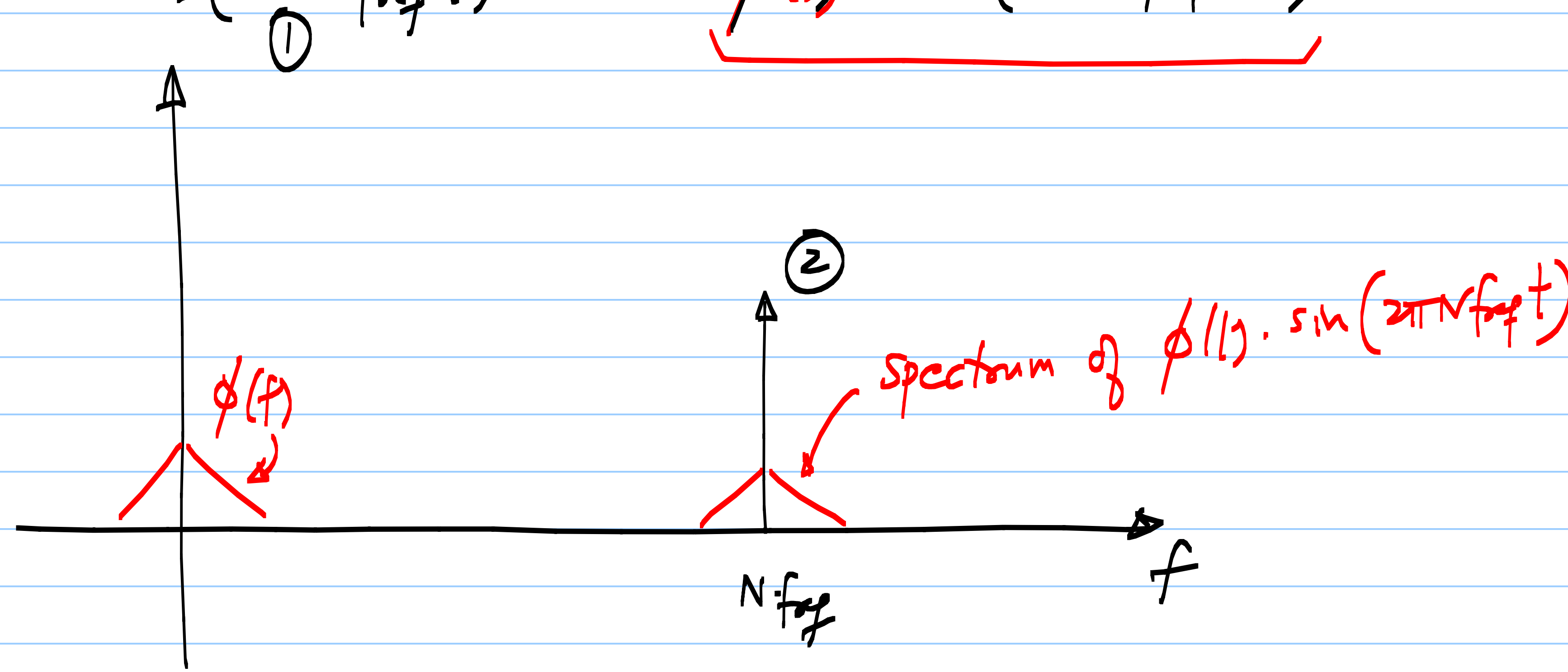
$\simeq -\phi(kT_0) \cdot \frac{T_0}{2\pi}$

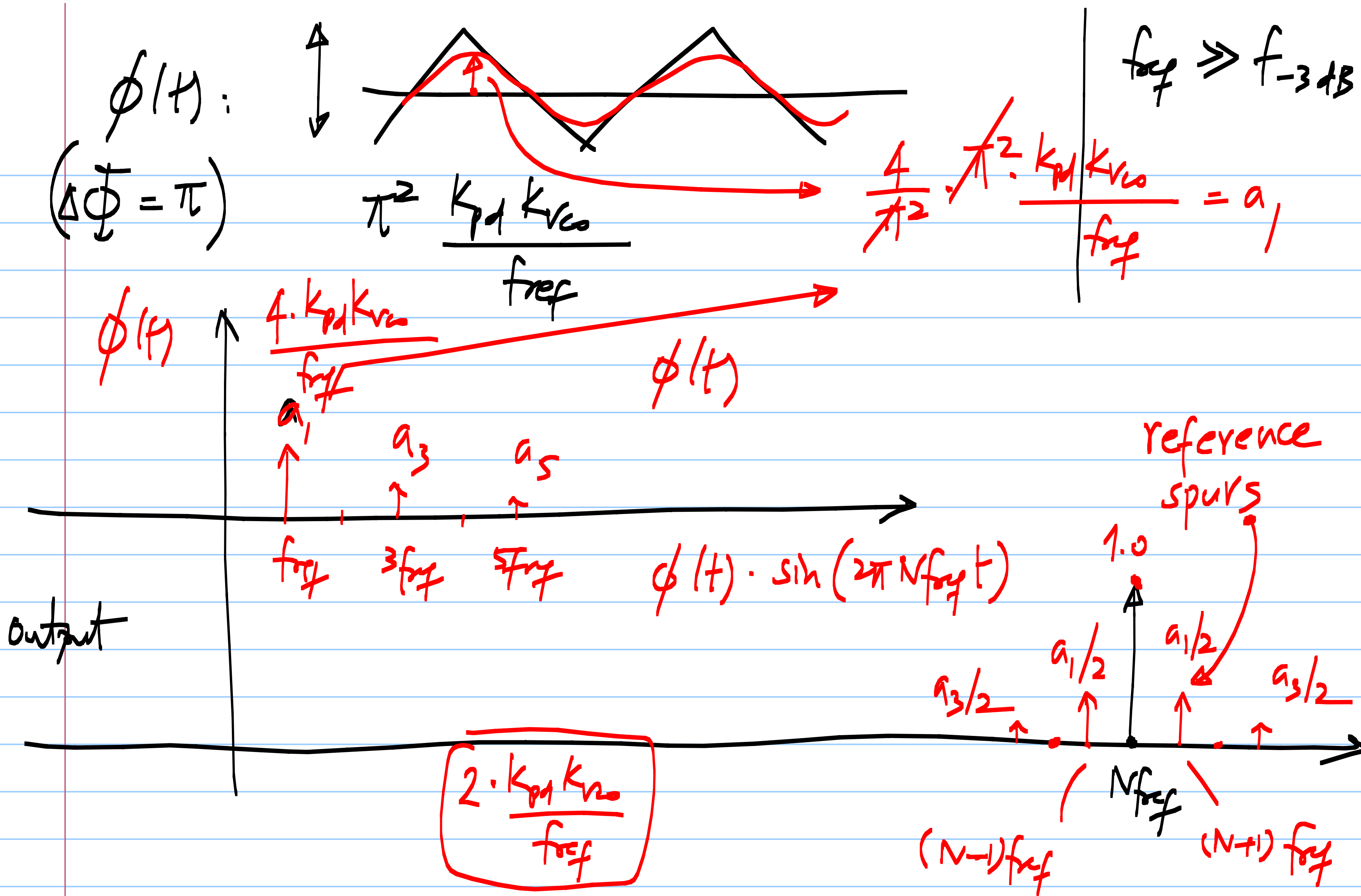
absolute jitter (phase) $\phi(kT_0)$ → **phase error**

(time) : $-\phi(kT_0) \cdot \frac{T_0}{2\pi}$

## Description of jitter in the frequency domain:

$$\simeq \cos(2\pi N f_{ref} t) - \phi(t) \sin(2\pi N f_{ref} \cdot t)$$




φ(t) :
(ΔΦ = π)
π² Kpd Kvco / fref
fref ≫ f-3dB
4/π² · π² · Kpd Kvco / fref = a1
φ(t)
4·Kpd Kvco / fref
φ(t)
a1
a3
a5
fref
3fref
5fref
φ(t) · sin(2π N fref t)
reference spurs
1.0
a1/2
a1/2
a3/2
a3/2
output
2·Kpd Kvco / fref
Nfref
(N−1)fref
(N+1) fref

$$\frac{a_1}{2} = 10^{-2}$$ [reference spur = 40dB below the component at $Nf_{ref}$]

$$2 \cdot \frac{K_{pd} K_{vco}}{f_{ref}} = 10^{-2}$$

$$K_{pd} \cdot K_{vco} = \frac{10^{-2}}{2} \cdot f_{ref} = 5 \cdot 10^{-3} \cdot f_{ref}$$

Lock range: $|Nf_{ref} - f_o| < 2\pi \cdot K_{pd} \cdot K_{vco}$

$$\pi \cdot 10^{-2} \cdot f_{ref} \ll f_{ref}$$

| Division modulus | |
|---|---|
| $N-1$ | $(N-1)f_{ref}$ |
| $N$ | $N f_{ref}$ |
| $N+1$ | $(N+1)f_{ref}$ |

$\updownarrow$ $2f_{ref}$

* Periodicity of phase detection limits the lock range to $2\pi \cdot K_{pd} \cdot K_{vco}$ $\rightarrow$ Make $K_{pd} \cdot K_{vco}$ large

* Periodic errors in the phase detector limit the value of $K_{pd} \cdot K_{vco}$.

  $-40$ dBc reference spur $\Rightarrow$ $K_{pd} \cdot K_{vco} = 5 \cdot 10^{-3} f_{ref}$

  $\Rightarrow$ Lock range $= \pi \cdot 10^{-2} f_{ref} \ll f_{ref}$

Cannot change N at all!