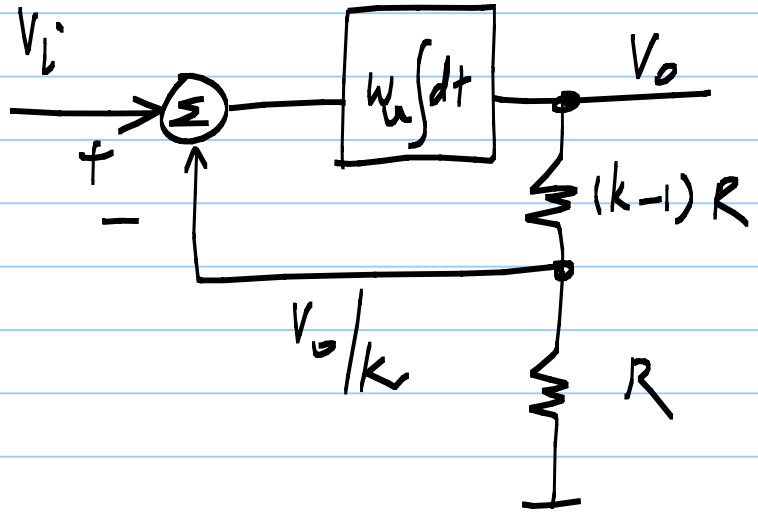
Lecture 45

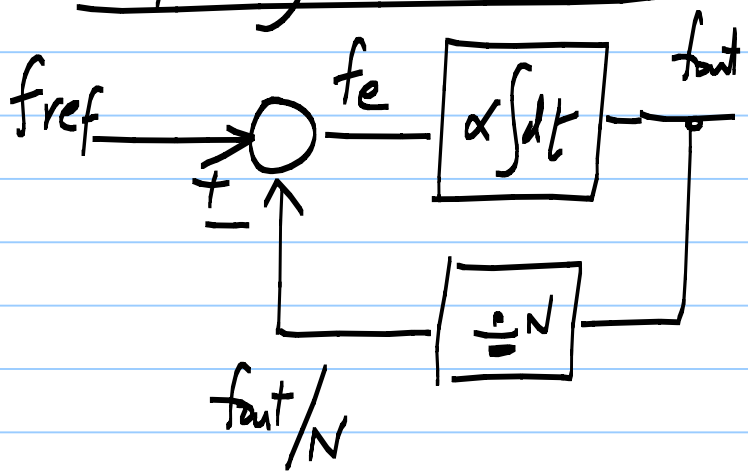
Voltage amplifier



$$V_o = kV_i$$

(for constant V_i)

Frequency multiplier



$$f_{out} = N \cdot f_{ref}$$

(Constant f_{ref})

Frequency and phase:

$\cdot \cos(2\pi f_0 t)$

$\cos(\theta)$: periodic in θ with a period 2π

$\cos(\theta + n \cdot 2\pi) = \cos(\theta)$

$\theta = 2\pi f_0 t$

$\cos(2\pi f_0 t)$: periodic in t with a period $\frac{1}{f_0}$

$\cos(\theta)$

$\theta = 2\pi f_0 t$

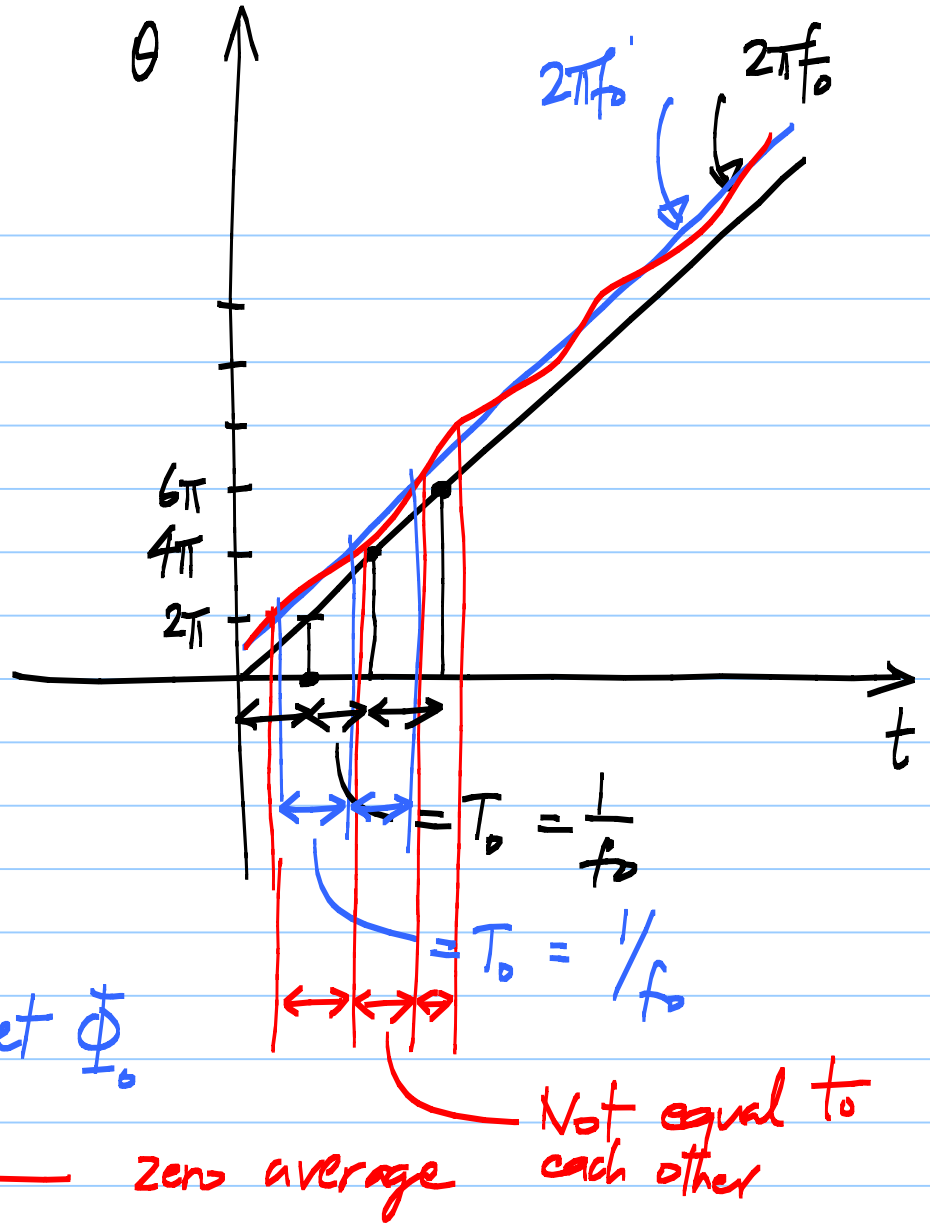
Periodic signal in time:

phase: ideal ramp {linearly with time}

$\theta = 2\pi f_0 t + \Phi_0$

Constant offset Φ_0

$\theta = 2\pi f_0 t + \Phi_0 + \phi(t)$ ← zero average



$p(\theta)$: periodic in θ, with a period 2π

$\cos(\theta)$: periodic in θ, with a period 2π

$$\theta = 2\pi f_0 t + \Phi_0 + \phi(t)$$

$\phi(t)$: time varying part (excluding the ramp)

Φ_0: average (dc) phase shift

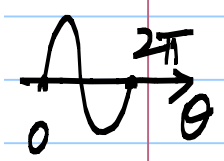
$2\pi f_0 t$: ramp, represents the average slope

only specify the frequency f_0

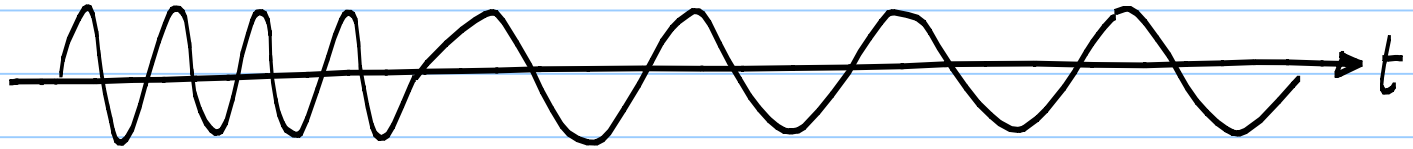
f_0: average frequency

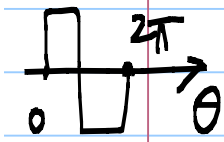
$\frac{1}{f_0} = T_0$: average period

$p(\cdot)$

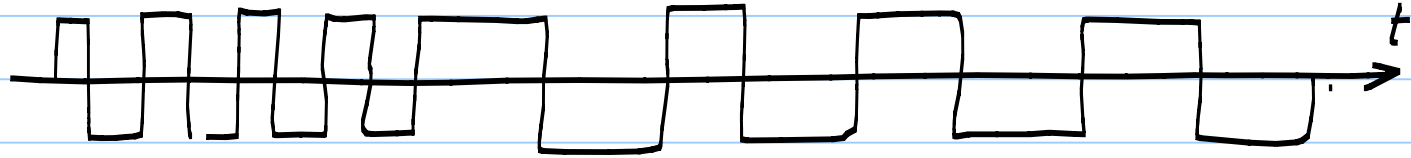


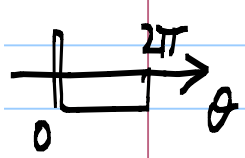
sine :



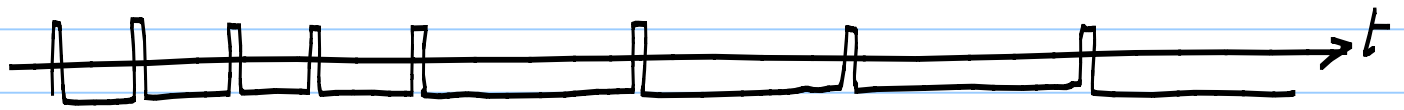


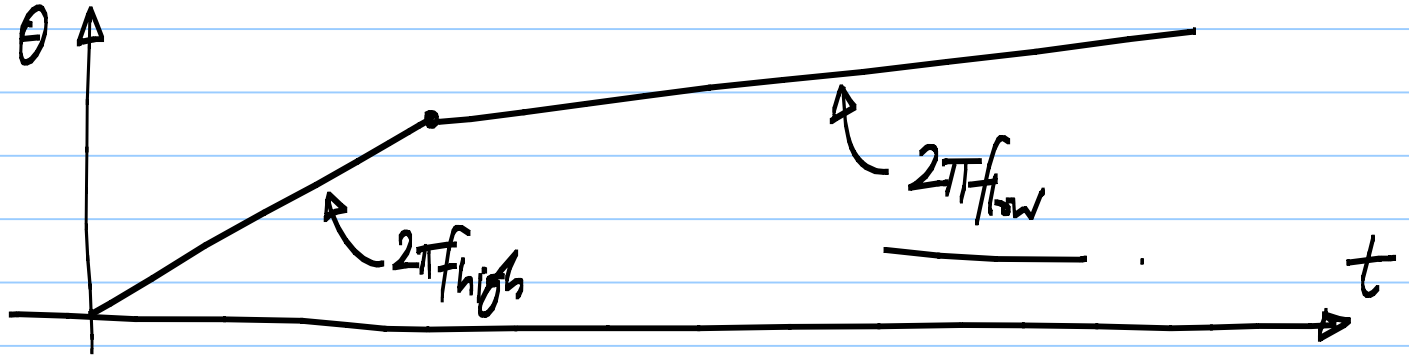
square :





pulse :





Phase $\theta(t) = 2\pi f_0 t + \Phi_0 + \phi(t)$ → average frequency

Instantaneous Frequency $f = \frac{1}{2\pi} \cdot \frac{d\theta}{dt} = f_0 + \frac{1}{2\pi}\frac{d}{dt}\phi(t)$

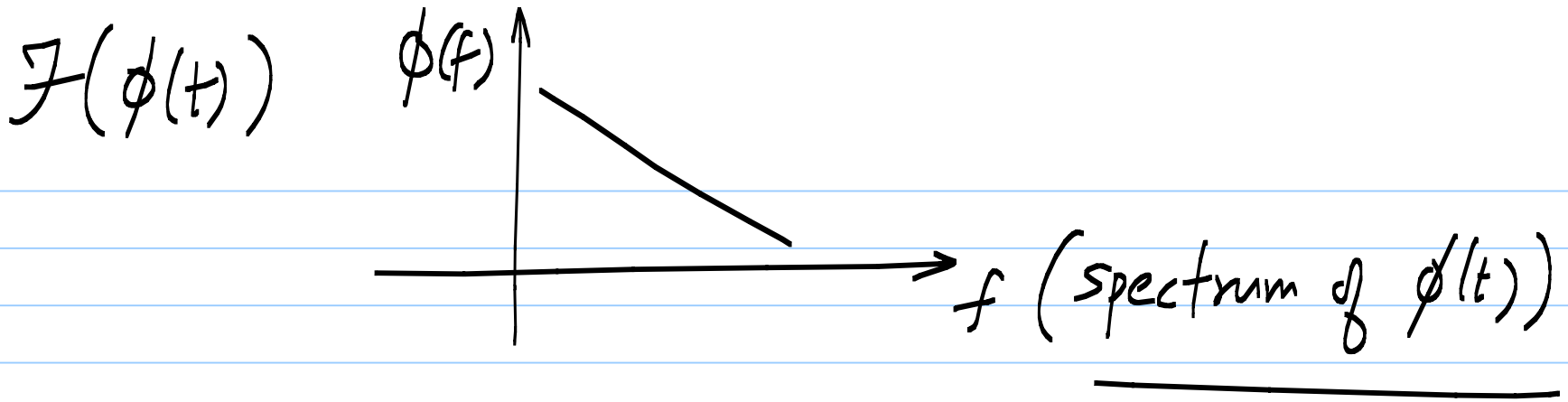
$= f_0$ if $\phi(t) = 0$ → average $= 0$

Instantaneous phase → $\theta(t) = 2\pi f_0 t + \Phi_0 + \phi(t)$ → (avg.) frequency

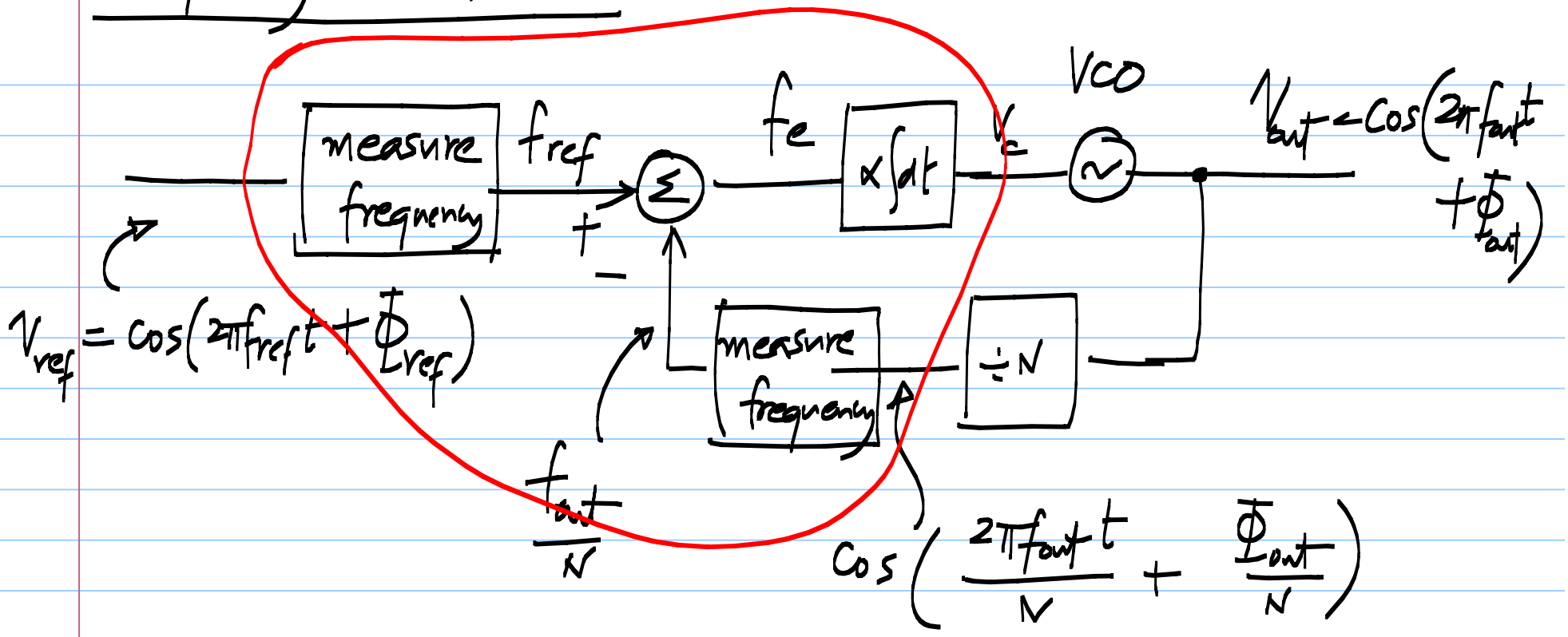
Instantaneous frequency $f(t) = f_0 + \frac{1}{2\pi} \cdot \frac{d\phi(t)}{dt}$

→ frequency of $\phi(t)$

→ frequency of $\frac{1}{2\pi}\frac{d\phi(t)}{dt}$


$\mathcal{F}(\phi(t))$
$\phi(f)$
f (spectrum of $\phi(t)$)

Frequency multiplier:



Steady state when $f_e = 0$

$f_{out}/N = f_{ref}$ or $\boxed{f_{out} = N \cdot f_{ref}}$

To control the o/p frequency f_{out}:

Voltage controlled oscillator (VCO)

$$f_{out} = K_{VCO} \cdot V_c + f_0$$

K_{VCO}: VCO gain, Hz/V

f_0: free running frequency (freq. when $V_c = 0$)

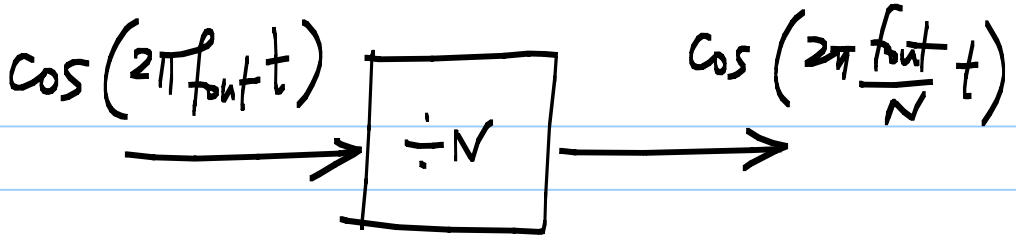
VCO output: $\cos(\theta_{out}(t))$

$$\theta_{out} = 2\pi f_0 t + 2\pi K_{VCO} \int V_c(t)\, dt$$

$$= 2\pi(f_0 + K_{VCO} \cdot V_c) \cdot t$$

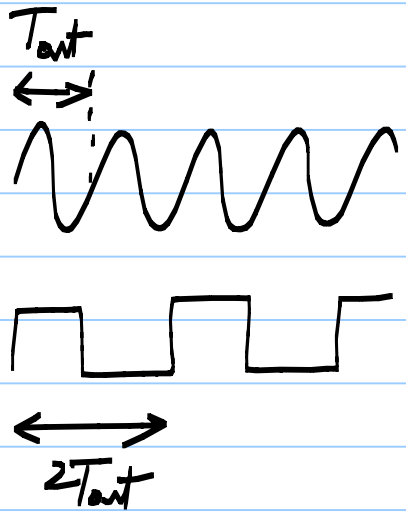
(constant V_c)

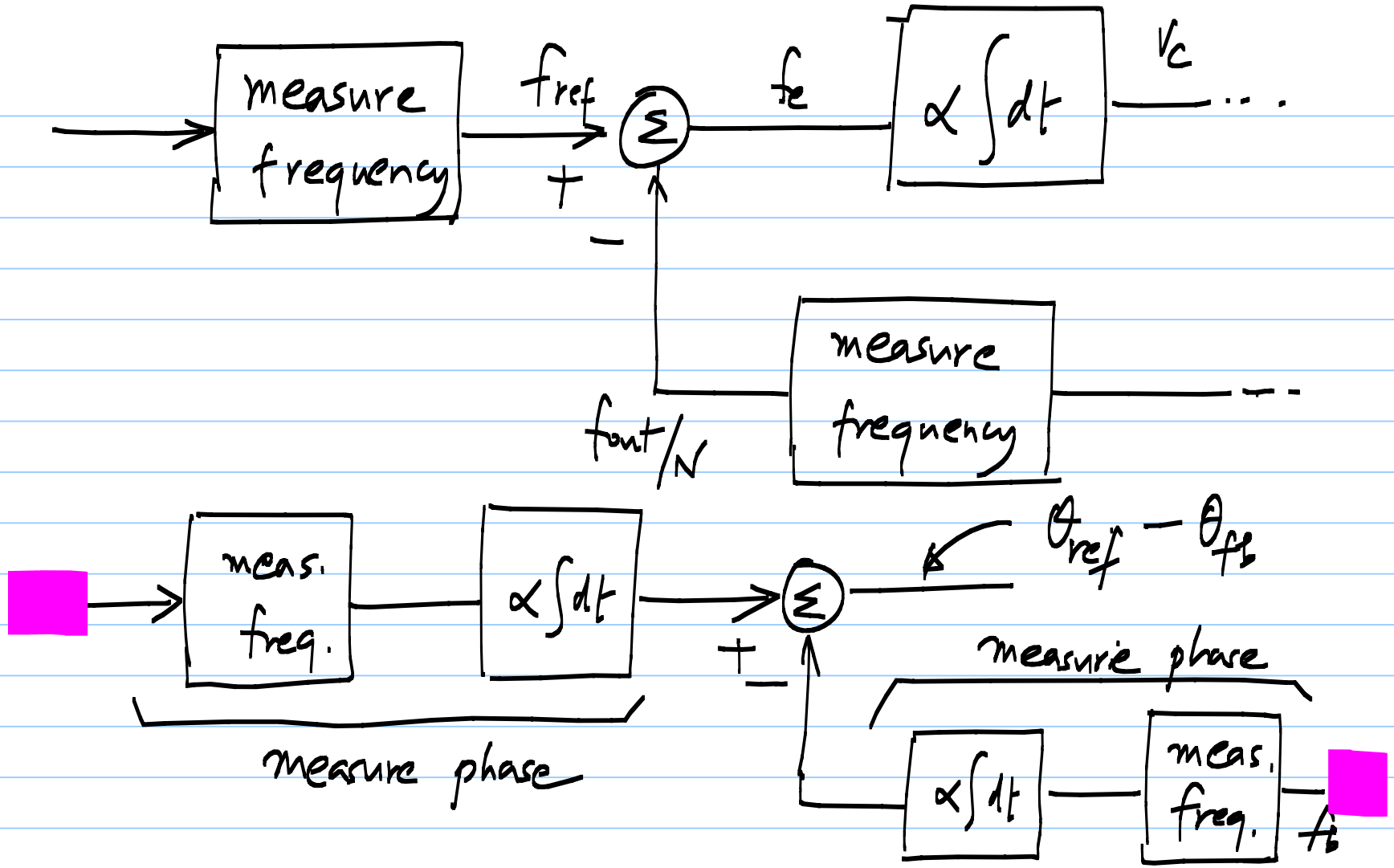
Frequency divider ?



When N is an integer:

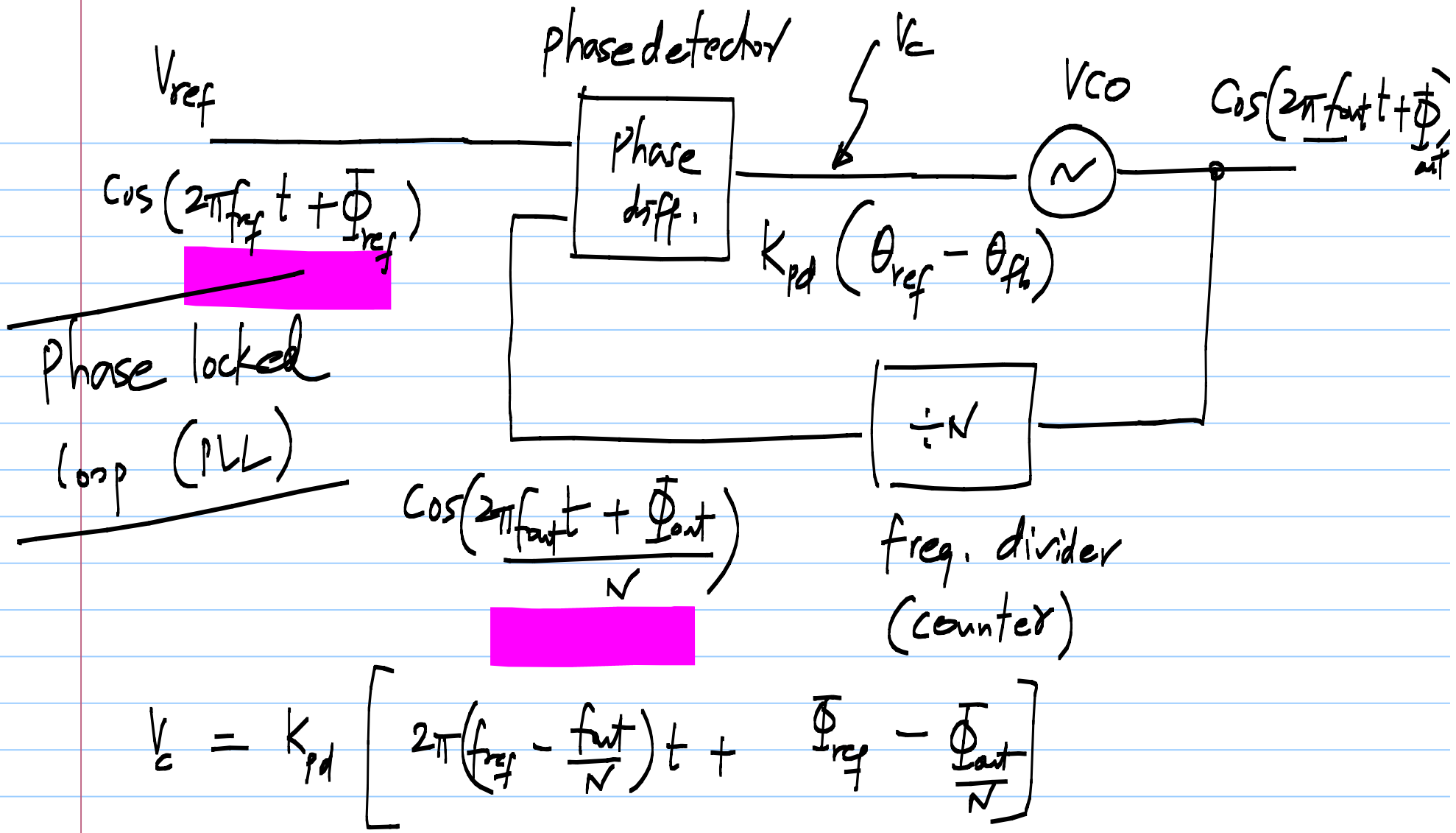
Digital counter.




measure frequency
f_{ref}
f_e
$\alpha \int dt$
V_c
measure frequency
f_{out}/N
meas. freq.
$\alpha \int dt$
$\theta_{ref} - \theta_{fb}$
measure phase
measure phase
$\alpha \int dt$
meas. freq.
f_1

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$$

$$\theta(t) = 2\pi \int f_i(t)\, dt$$


Phase detector
V_c
VCO
V_{ref}
$\cos(2\pi f_{out} t + \Phi_{out})$
Phase det.
$\cos(2\pi f_{ref} t + \Phi_{ref})$
$K_{pd}(\theta_{ref} - \theta_{fb})$
Phase locked loop (PLL)
$\div N$
$\cos\left(2\pi f_{out} t + \frac{\Phi_{out}}{N}\right)$
freq. divider (counter)
$V_c = K_{pd}\left[2\pi\left(f_{ref} - \frac{f_{out}}{N}\right)t + \Phi_{ref} - \frac{\Phi_{out}}{N}\right]$