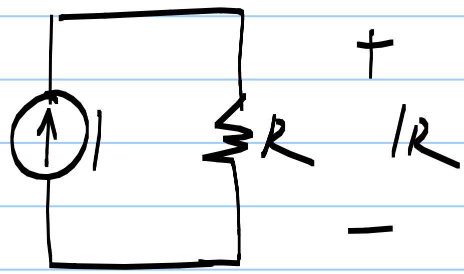
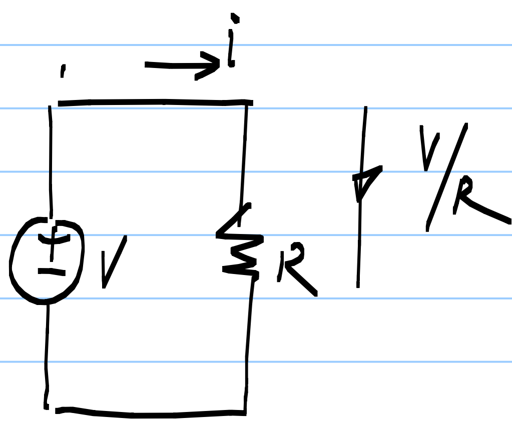
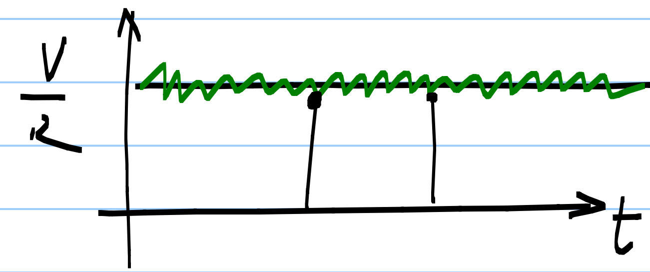
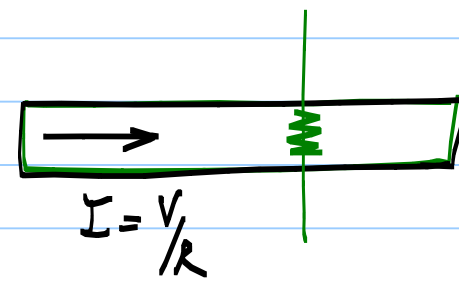
Lecture 24 Resistor noise





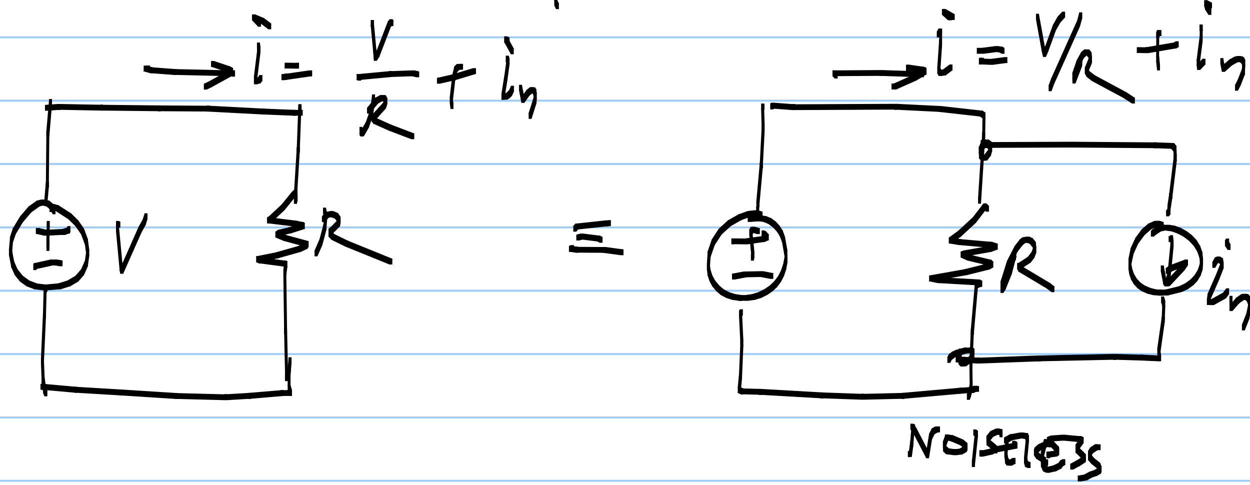




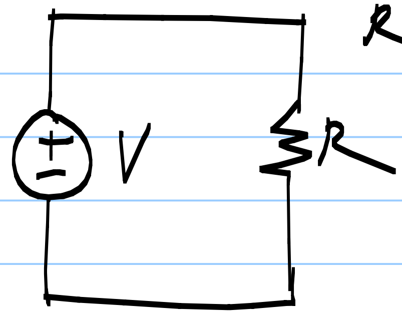
* Uncorrelated from one resistor to another

* Uncorrelated from one time instant to another

* Power spectral density — distribution over frequency
* Mean squared value] variance
  (root mean squared)] standard deviation

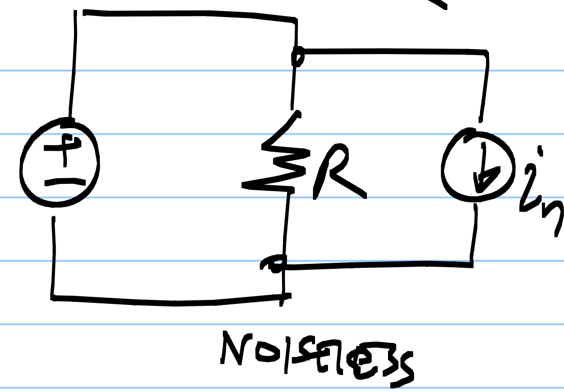


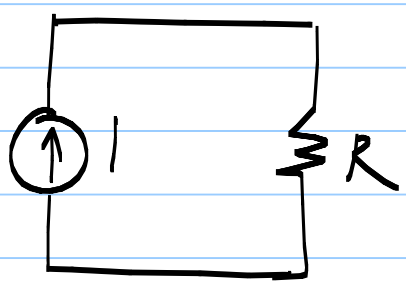
$\rightarrow i = \frac{V}{R} + i_n$



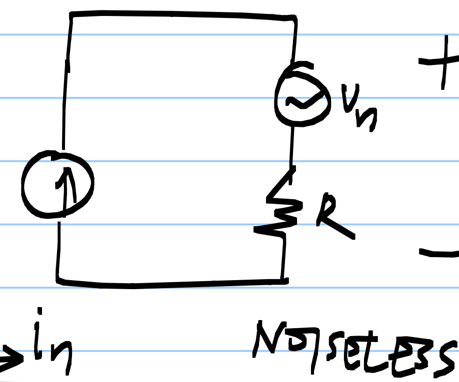
$\equiv$

$\rightarrow i = V/R + i_n$





$\equiv$



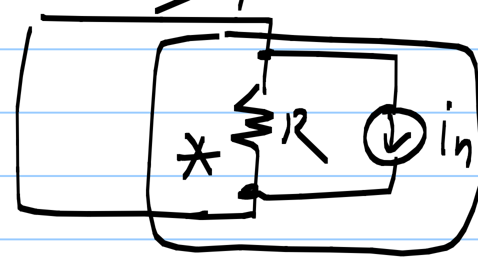
+

$I \cdot R + v_n$

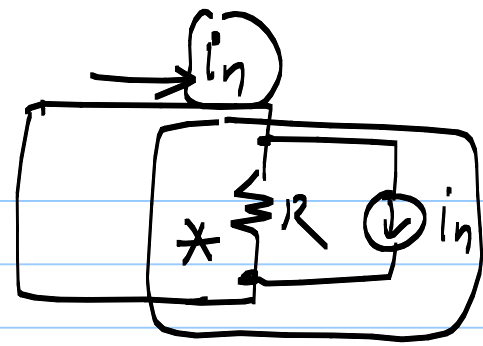
−

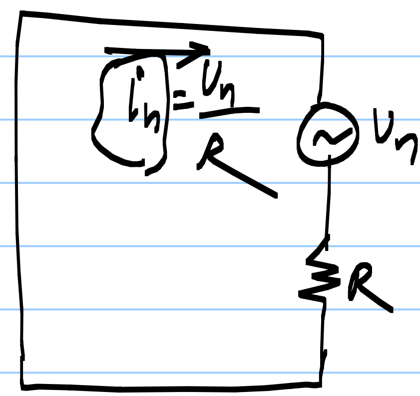
$i_n \cdot R = v_n$

$\rightarrow i_n$

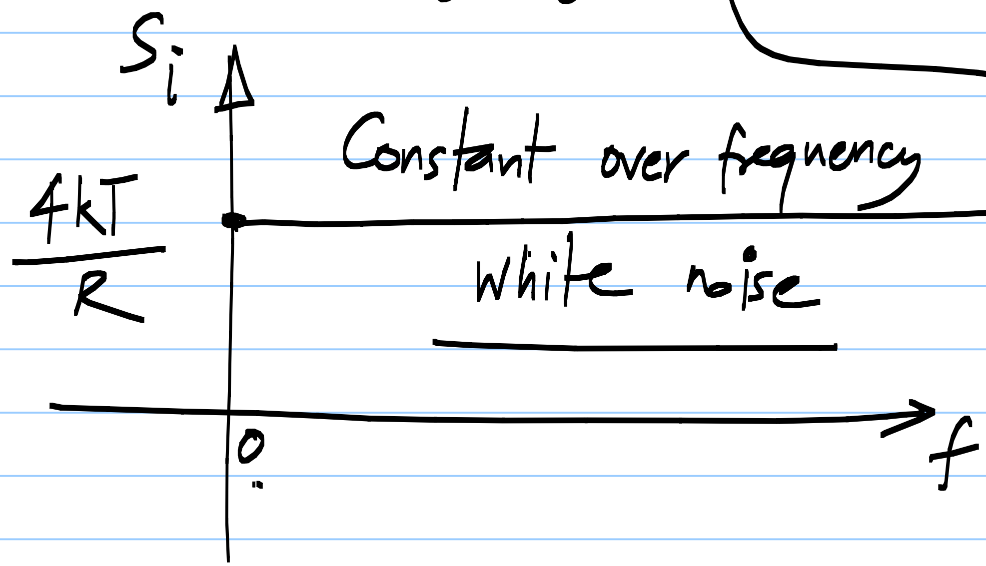


$$\boxed{i_n \cdot R = v_n}$$





Spectral density of i_n, v_n {noise in a resistor R}



$S_i(f)$

Fourier transform

Autocorrelation function

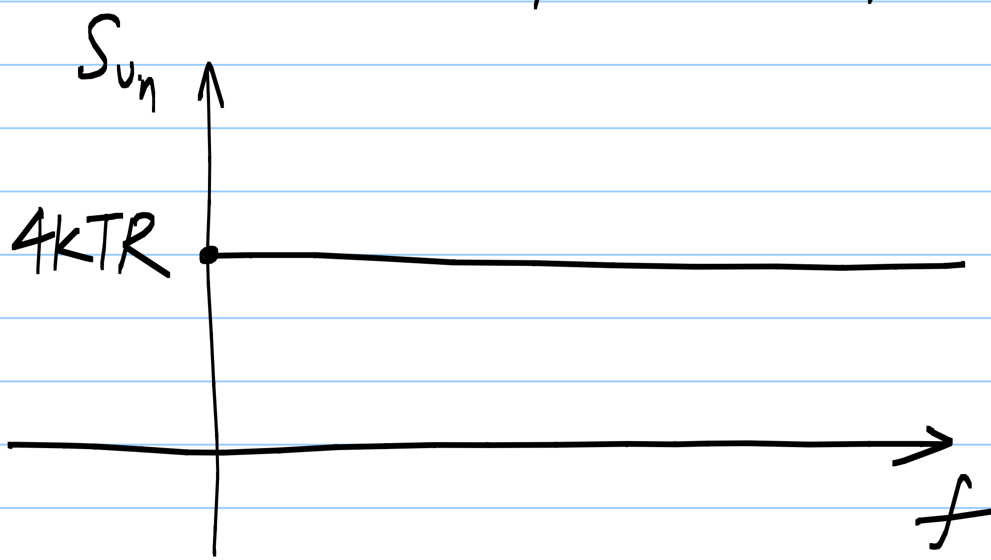
k: Boltzmann's constant 1.38×10^{-23} J/K

T: Absolute temp. 300 K

$$\int_0^\infty S_i(f)\,df = \text{Variance} \quad [\sigma^2]$$

$$v_n = i_n \cdot R$$

$$S_{v_n} = S_{i_n} \cdot R^2 = \frac{4kT}{R} \cdot R^2 = 4kT \cdot R$$



$kT = 4 \times 10^{-21}\,J$

@ 300K

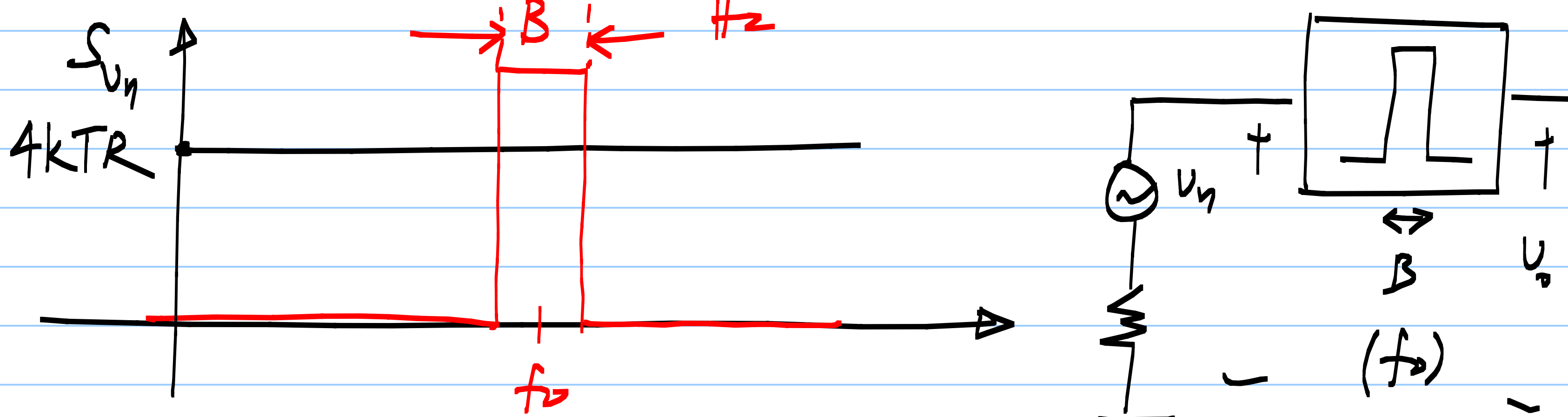
$R = 1k\Omega$

$4 \times 4 \times 10^{-21} J \times 10^{3} \Omega$

$= 16 \times 10^{-18}\ V^2/Hz$

$$S_i = \frac{4kT}{R} \quad A^2/Hz \; ; \quad S_v = 4kT \cdot R \quad \frac{V^2}{Hz}$$

For a $1k\Omega$ resistor $\quad S_i = 16 \times 10^{-24} \; A^2/Hz \quad S_v = 16 \times 10^{-18} \; \frac{V^2}{Hz}$



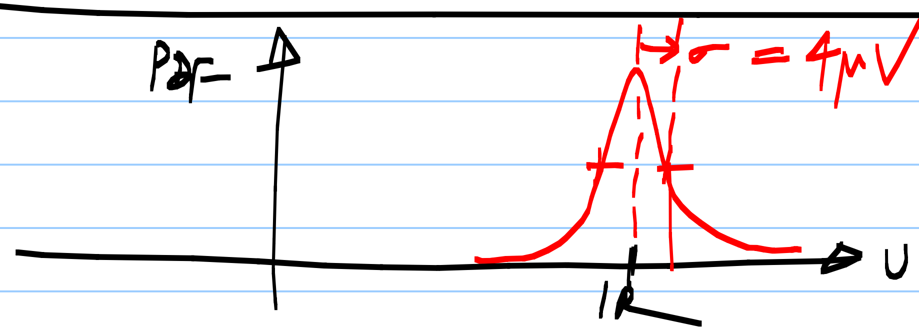
$$(4kT \cdot R) \cdot B = \text{Variance of } v_0$$

$1k\Omega$ resistor: $S_v = 16 \times 10^{-18}\ V^2/Hz$

$1MHz$ BW: $B = 10^6\ Hz$

$S_v \cdot B = \underbrace{16 \times 10^{-12}\ V^2}$ $= \sigma_{v_o}^2$

$$\boxed{\sigma_{v_o} = \sqrt{S_v \cdot B} = 4 \times 10^{-6}\ V = 4\mu V}$$



$I \cdot R$

$$S_v = 4kTR \ \frac{V^2}{Hz} \equiv \sqrt{4kTR} \ \frac{V}{\sqrt{Hz}}$$

$1k\Omega$: $$16\times10^{-18} \ \frac{V^2}{Hz} \equiv 4\times10^{-9} \ \frac{V}{\sqrt{Hz}}$$

$$= \underbrace{4\,nV/\sqrt{Hz}}$$

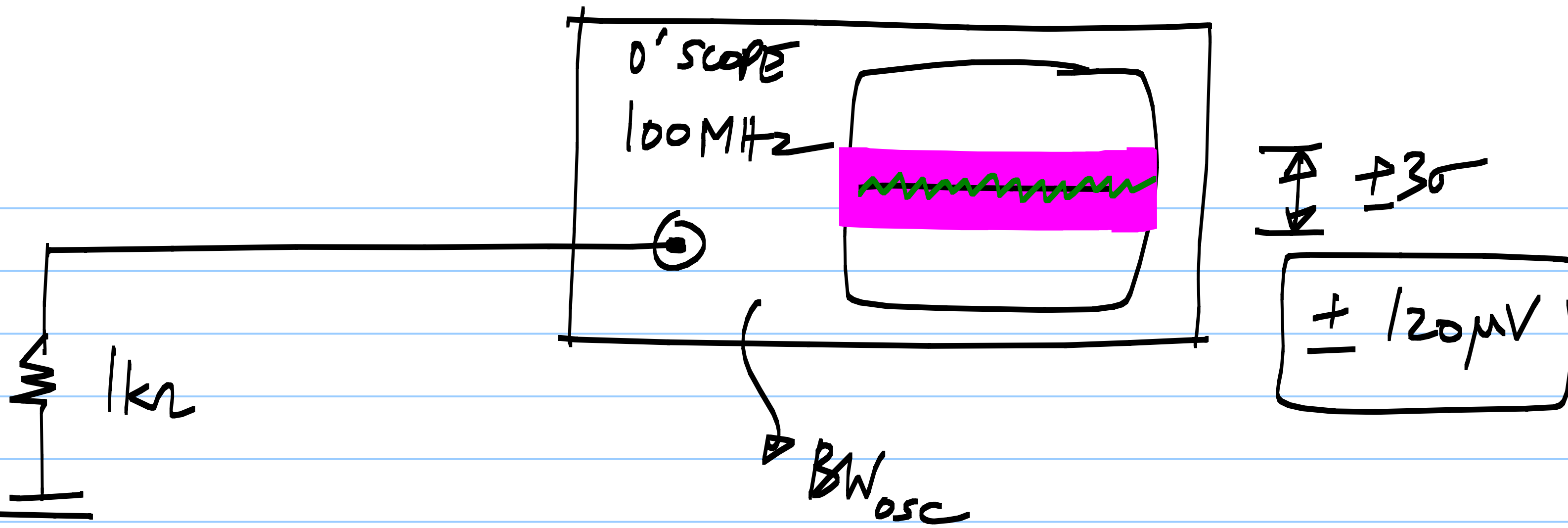
1MHz BW $$\underbrace{4\,nV/\sqrt{Hz} \cdot \sqrt{1MHz} = 4\mu V}$$

$$S_i = \frac{4kT}{R} \ \frac{A^2}{Hz} \equiv \sqrt{\frac{4kT}{R}} \ \frac{A}{\sqrt{Hz}}$$

$1k\Omega$

$$4\times10^{-12} \frac{A}{\sqrt{Hz}}$$

$$= 4pA/\sqrt{Hz}$$



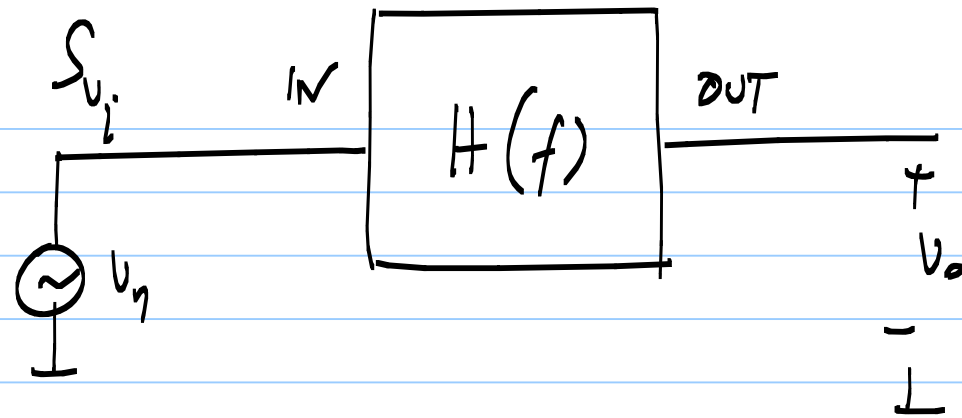
$$\sqrt{\overline{v_n^2}} = \sqrt{4kTR \cdot BW_{osc}}$$

$$\overline{v_n^2} = 4kT \cdot R \cdot BW_{osc}$$

$$16\times10^{-18}\,\frac{V^2}{Hz} \times 10^{8}\,Hz$$

$$= 16\times10^{-10}\,V^2 = \overline{v_n^2}$$

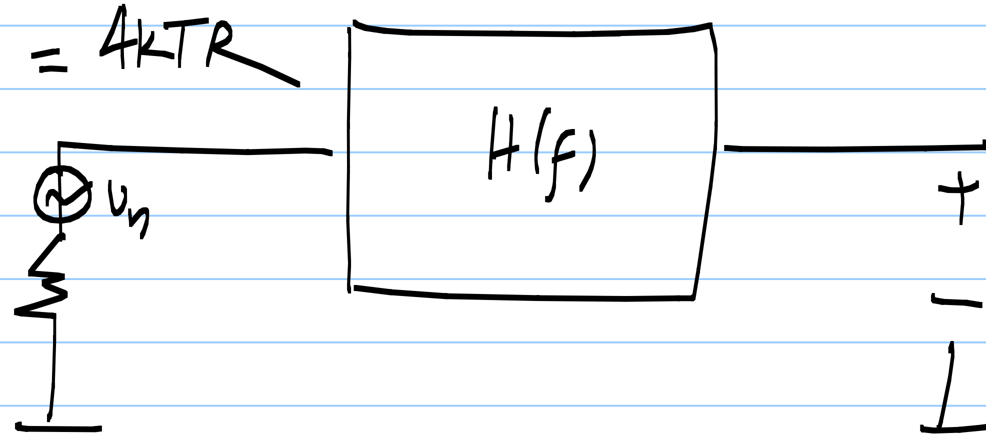
$$\boxed{40\mu V = \sqrt{\overline{v_n^2}}}$$



$$S_{v_o} = S_{v_i} \left| H(f) \right|^2$$

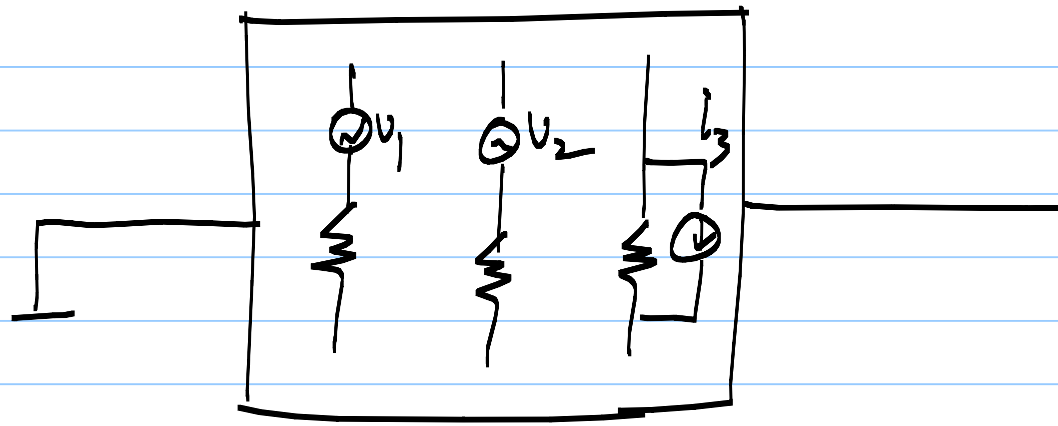
$$\boxed{\int_0^\infty S_{v_o}(f) \cdot df}$$

$$S_{v_i} = 4kTR$$



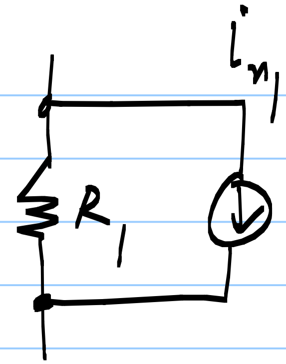
$$S_{v_o} = 4kTR \left| H(f) \right|^2$$

$$\overline{v_o^2} = 4kTR \int_0^\infty \left| H(f) \right|^2 \cdot df$$

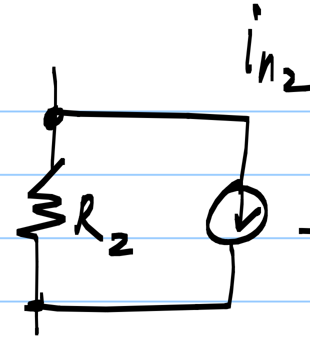


$$V_o = V_1 \cdot H_1(f) + V_2 \cdot H_2(f) + I_3 \cdot H_3(f) + \ldots$$

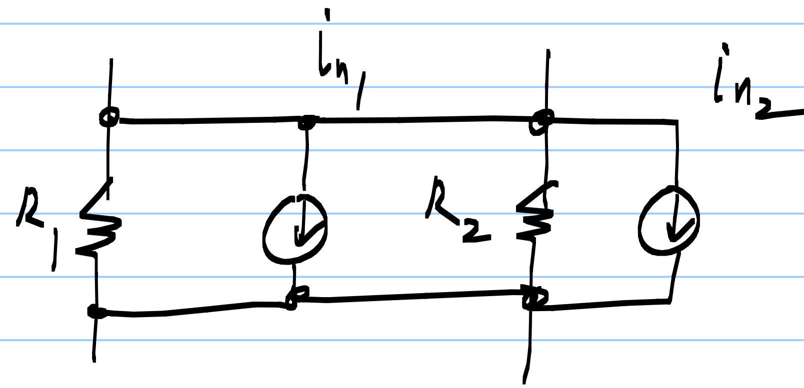
$$S_{V_o} = S_{V_1} \left|H_1(f)\right|^2 + S_{V_2} \left|H_2(f)\right|^2 + S_{I_3} \left|H_3(f)\right|^3 + \ldots$$



$$\frac{4kT}{R_1}$$

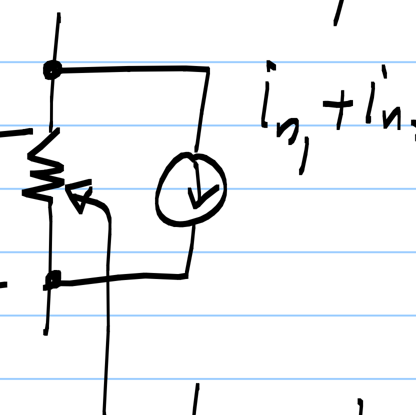


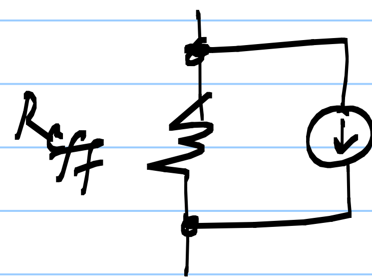
$$\frac{4kT}{R_2}$$



$$= \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$$

$$4kT\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

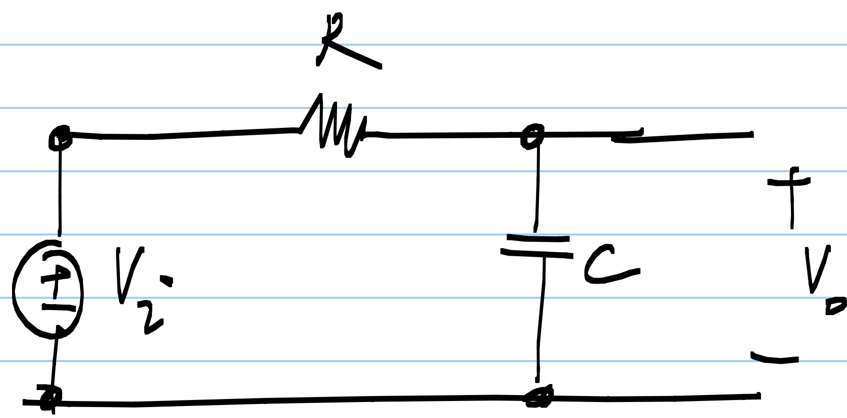




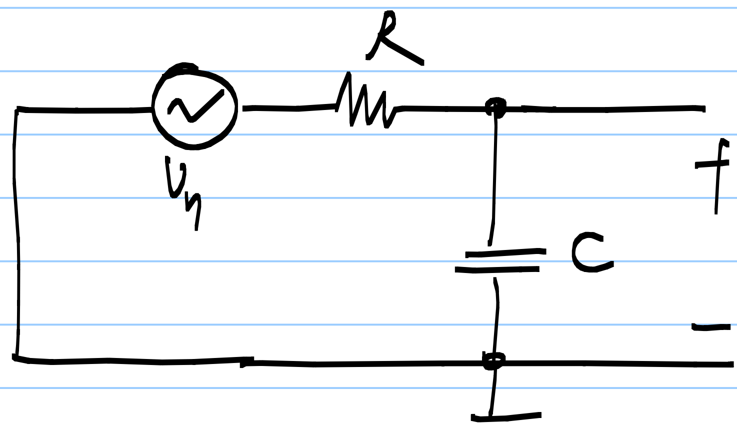
$$\frac{4kT}{R_{eff}}$$

$$\frac{1}{R_{eff}} = \frac{1}{R_1}+\frac{1}{R_2}$$

First order RC lowpass filter:



$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1+sCR}$$



$$\frac{V_o(s)}{V_n(s)} = \frac{1}{1+sCR}$$

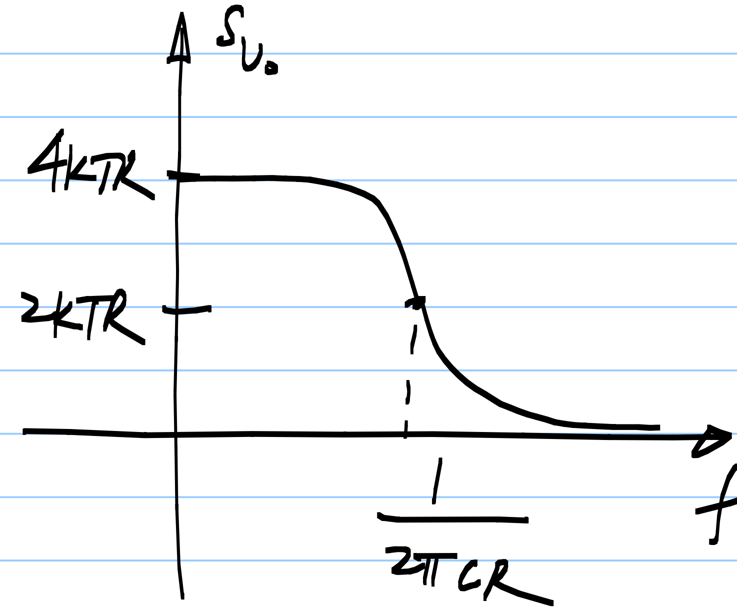
$$\frac{V_o(j2\pi f)}{V_n(j2\pi f)} = \frac{1}{1+j2\pi fCR}$$

$$S_{v_o} = S_{v_n} \cdot |H(f)|^2 \qquad \frac{1}{1 + j2\pi fCR}$$

$$= 4kTR \cdot \frac{1}{1 + 4\pi^2 f^2 C^2 R^2} \qquad f = \frac{1}{2\pi CR}$$

$$\overline{v_n^2} = \int_0^\infty S_{v_o} \cdot df$$

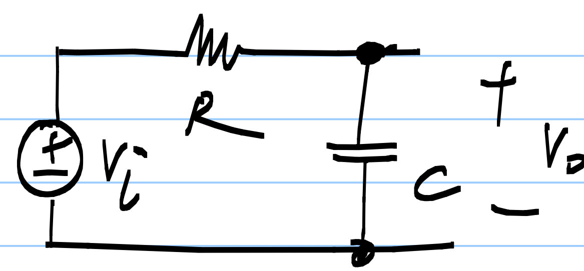
$$= \int_0^\infty \frac{4kTR \quad df}{1 + 4\pi^2 f^2 C^2 R^2}$$

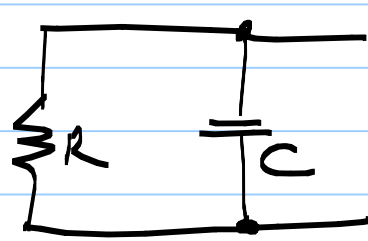


$$\int_0^{\infty} \frac{4kTR \quad df}{1 + 4\pi^2 f^2 C^2 R^2} = 4kTR \cdot \frac{1}{2\pi CR} \cdot \tan^{-1}(2\pi f CR) \Big|_{f=0}^{f=\infty}$$

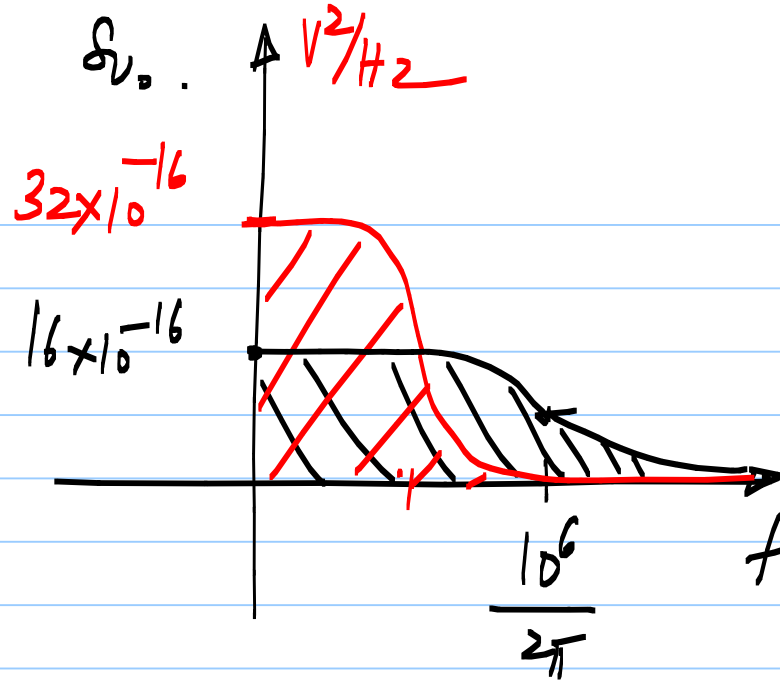
$$= 4kTR \cdot \frac{1}{2\pi CR} \cdot \frac{\pi}{2} = \underline{\underline{\frac{kT}{C}}}$$

$$\boxed{\overline{v_n^2} = \frac{kT}{C}}$$





$$\overline{v_1^2} = \frac{kT}{C}$$



$C = 10\,pF$

$R = 100\,k\Omega$ $\qquad 16\times10^{-16}\ V^2/Hz$

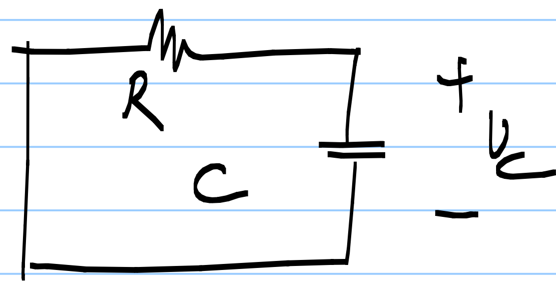
$$\frac{1}{2\pi CR} = \frac{1}{2\pi\cdot10^{-11}\cdot10^{5}} = \frac{10^6}{2\pi}\ Hz$$

$R = 200\,k\Omega$ $\qquad \frac{1}{2\pi CR} = \frac{10^6}{4\pi}\ Hz$

Nagendra Krishnapura, Dept. of EE
Indian Institute of Technology, Madras

Equipartition theorem:

Each degree of freedom: $\frac{kT}{2}$



$$\frac{1}{2} C \overline{v_c^2} = \frac{kT}{2}$$

$$\boxed{\overline{v_c^2} = \frac{kT}{C}}$$

$S_{v_n} = 4kTR$ for all frequencies

$$kT \longleftrightarrow h\nu \Big/ \left[\exp\left(\frac{h\nu}{kT}\right) - 1\right]$$