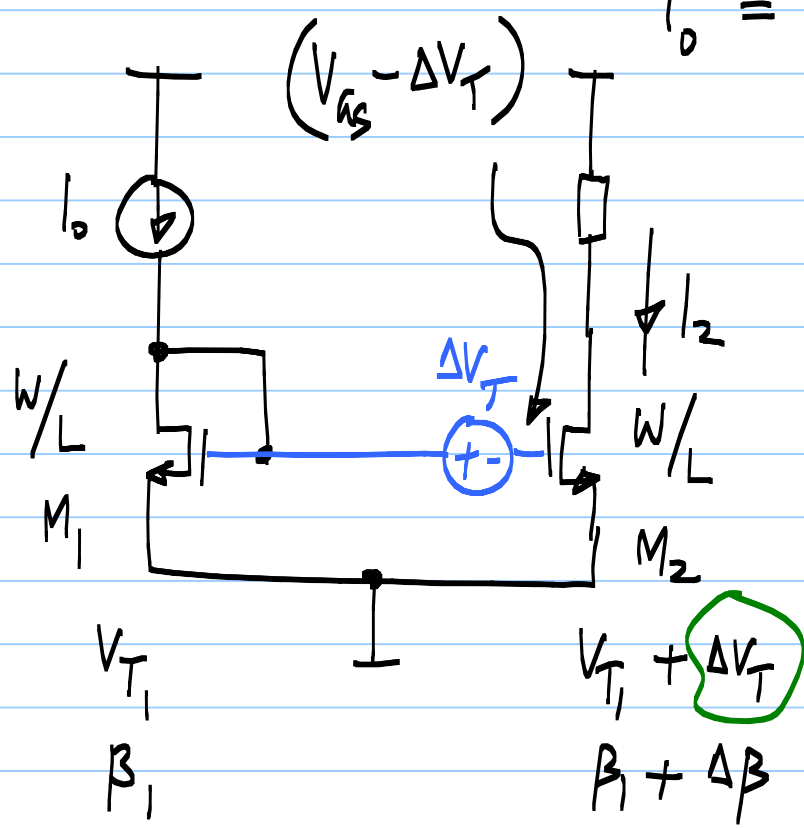
Lecture 23

$$I_2 = \left(\frac{\beta_1 + \Delta\beta}{2}\right)\left(V_{GS} - (V_T + \Delta V_T)\right)^2$$

$$I_0 = \frac{\beta_1}{2}\left(V_{GS} - V_T\right)^2$$



* Expand this expression
* neglect higher order terms : ΔV_T^2, $\Delta V_T \cdot \Delta\beta$

$$\Delta I_D = \frac{\partial I_D}{\partial V_{GS}} \cdot \Delta V_{GS} = g_m \cdot \underset{(-\Delta V_T)}{\Delta V_{GS}}$$

$$I_2 = \left(\frac{\beta_1 + \Delta\beta}{2}\right)\left(V_{GS} - (V_T + \Delta V_T)\right)^2 \longrightarrow \left(\frac{\beta_1 + \Delta\beta}{2}\right) \cdot \left(V_{GS} - V_T\right)^2$$

$$\frac{\beta_1}{2} \cdot \left(V_{GS} - \Delta V_T - V_T\right)^2$$

$$I_0 = \frac{\beta_1}{2}\left(V_{GS} - V_T\right)^2$$

$\Delta\beta$:

$$I_2 = \frac{\beta_1 + \Delta\beta}{\beta_1} \cdot I_0 = \left(1 + \frac{\Delta\beta}{\beta_1}\right) I_0$$

ΔV_T:

$$I_2 = I_0 - g_m \cdot \Delta V_T$$

$\Delta\beta_1, \Delta V_T$:

$$\underline{I_2 = I_0 - g_m \Delta V_T + \frac{\Delta\beta}{\beta_1} \cdot I_0}$$

$$I_2 = I_o - g_m \Delta V_T + \frac{\Delta\beta}{\beta_1} \cdot I_o$$

$$g_m = \frac{2 I_o}{V_{GS} - V_T}$$

$$\frac{\Delta I}{I_o} = \frac{I_2 - I_o}{I_o} = \frac{-g_m \Delta V_T}{I_o} + \frac{\frac{\Delta\beta}{\beta_1} \cdot I_o}{I_o}$$

$$\left(\frac{\Delta I}{I_o}\right) = -\frac{g_m}{I_o} \cdot \Delta V_T + \frac{\Delta\beta}{\beta_1}$$

$$= -\frac{2 \cdot \Delta V_T}{V_{GS} - V_T} + \frac{\Delta\beta}{\beta_1}$$

$$\sigma^2\left(\frac{\Delta I}{I_o}\right) = \frac{4\,\sigma_{V_T}^2}{(V_{GS} - V_T)^2} + \sigma_\beta^2$$

Mismatch in a current mirror:

$$\sigma\left\{\left(\frac{\Delta I}{I_0}\right)^2\right\} = \frac{4\sigma_{V_T}^2}{(V_{GS}-V_T)^2} + \sigma_\beta^2$$

$$= \frac{4A_{V_T}^2}{(V_{GS}-V_T)^2 \cdot WL} + \frac{A_\beta^2}{WL}$$

* Mismatch reduces with increasing WL

* Effect of V_T mismatch reduces with increasing $(V_{GS}-V_T)$

For a given current:

$$I_0 = \frac{\beta}{2} \cdot (V_{GS} - V_T)^2$$

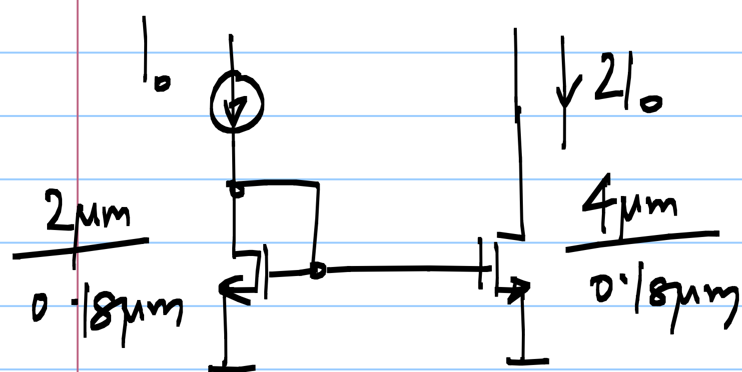
$$(V_{GS} - V_T)^2 = \frac{2 I_0}{\beta} = \frac{2 I_0}{\mu C_{ox} \frac{W}{L}}$$

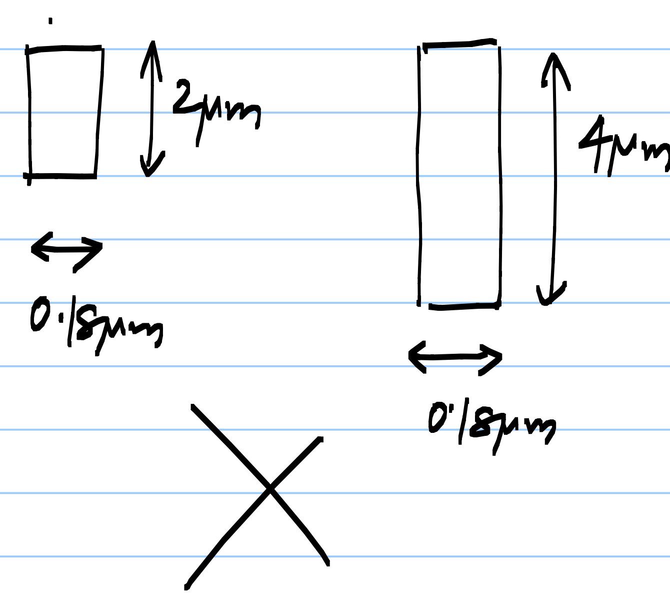
$$\sigma^2\left(\frac{\Delta I}{I_0}\right) = \frac{4 \cdot A_{V_T}^2}{\frac{2 I_0}{\mu C_{ox}} \frac{\cancel{W}}{L} \cdot \cancel{W} L} + \frac{A_\beta^2}{WL}$$

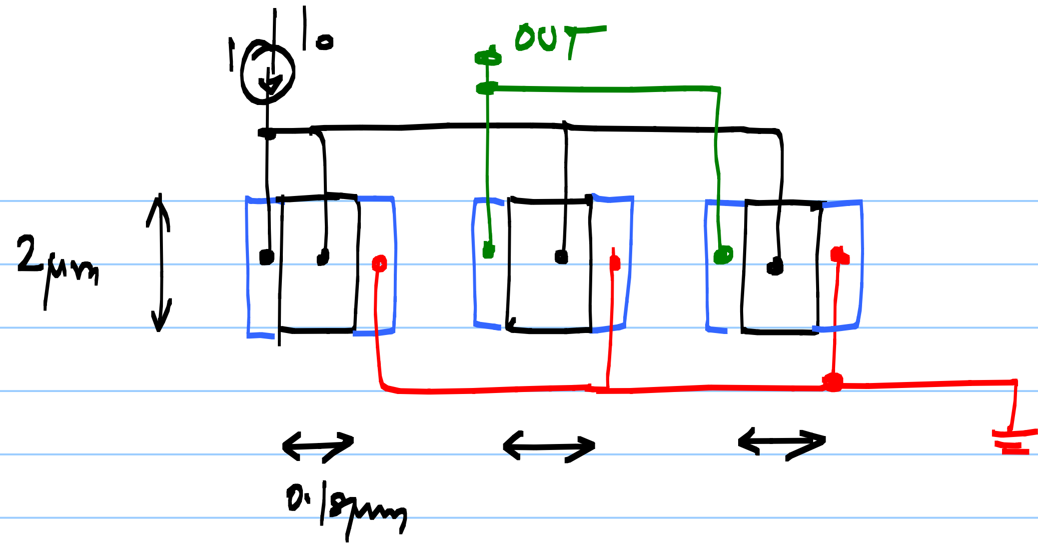
$$= \underbrace{\frac{2 \mu C_{ox}}{I_0} \cdot \frac{A_{V_T}^2}{L^2} + \frac{A_\beta^2}{WL}}$$

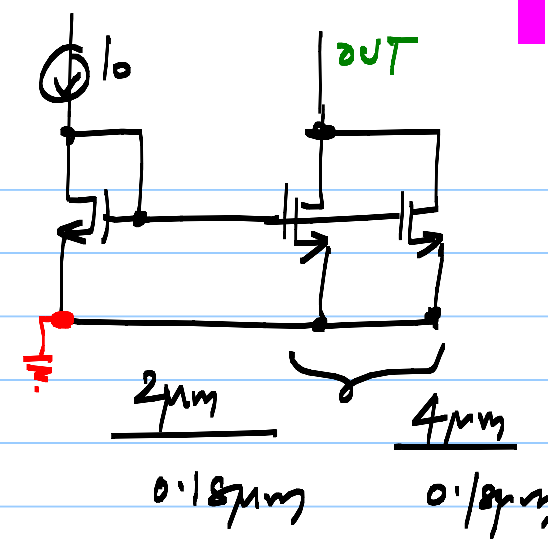
Layout of MOS transistors:

* Two transistors to be matched: Identical L
* For good matching: Multiples of identical units





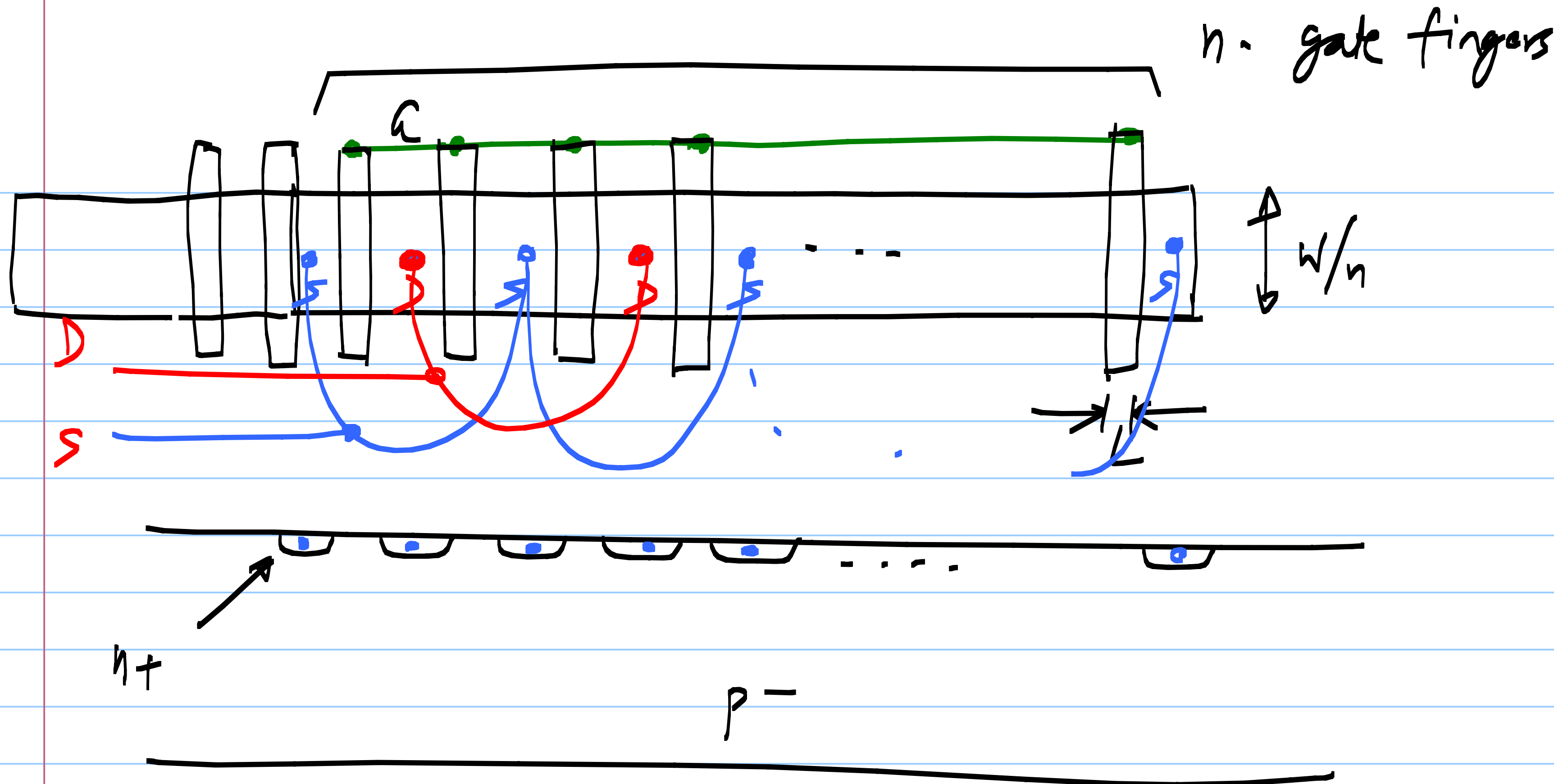


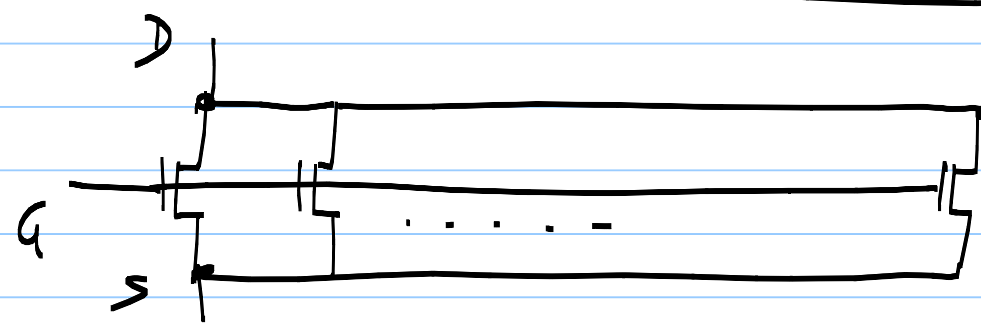


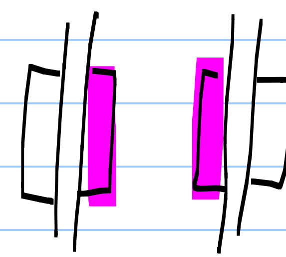
$$\frac{W}{L} = n \cdot \left(\frac{W/n}{L}\right)$$

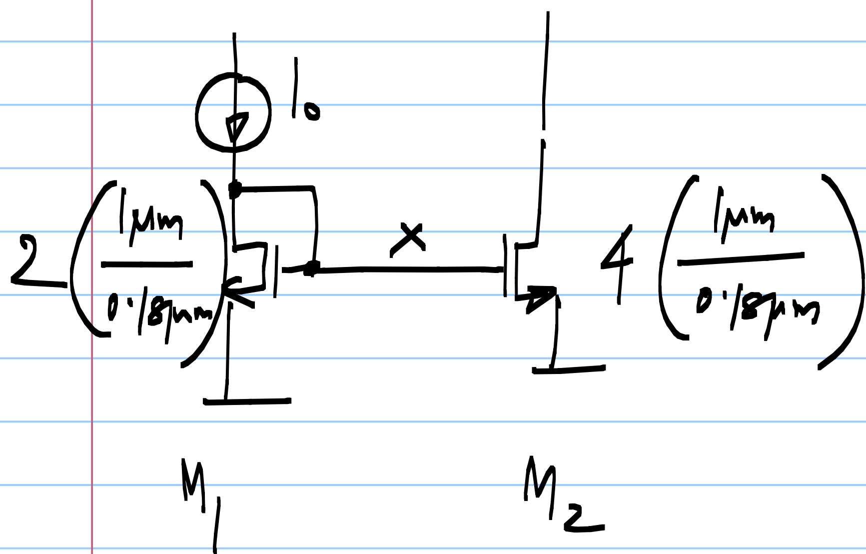
n devices of $\left(\frac{W}{n}\Big/L\right)$ in parallel
(fingers)

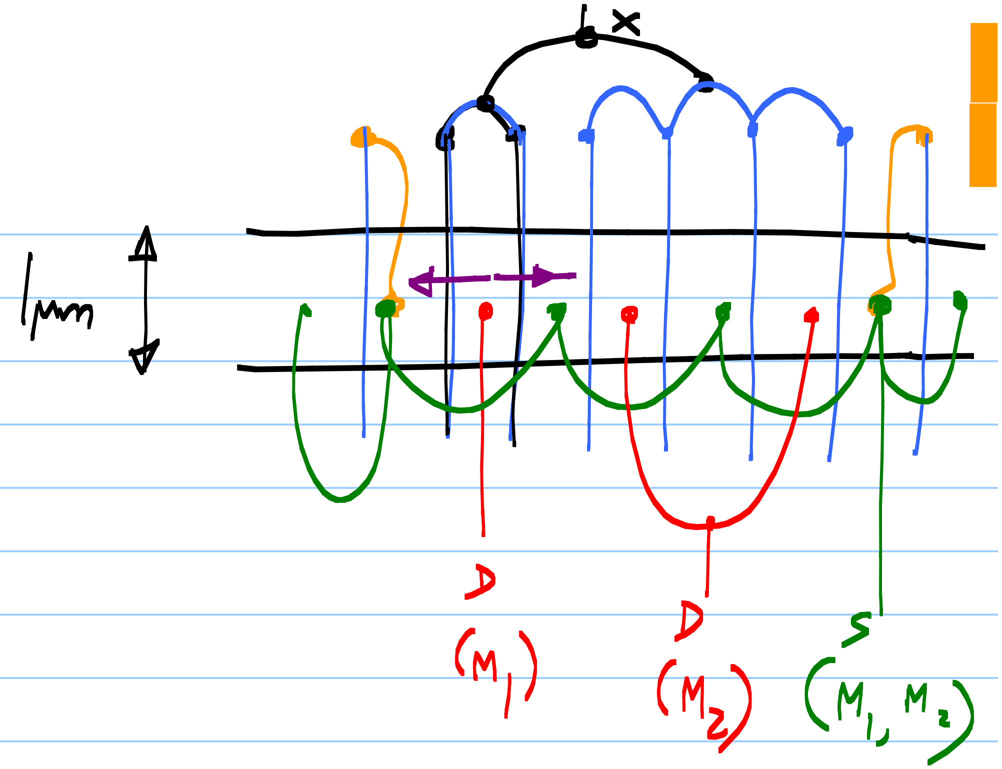
$$\underbrace{\frac{W}{n} \sim 0.5\mu m \text{ to a few } \mu m}$$

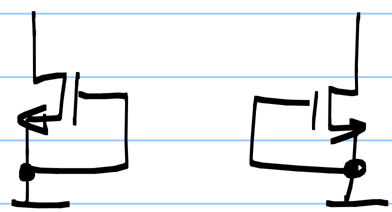

n · gate fingers
G
W/n
D
S
n+
p−

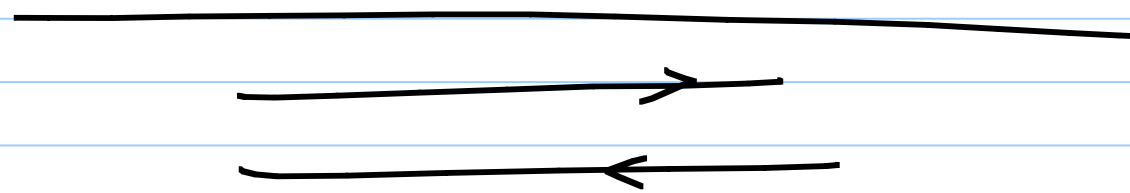

D
G
S





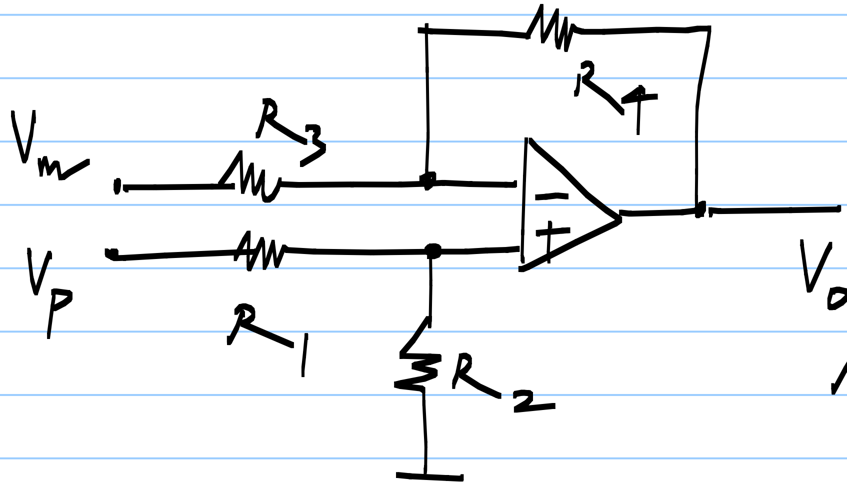






Even number of gate fingers

Differential to single ended converter:



$$V_o = (V_p - V_m)$$

V_o: when the 4 resistors are different from each other.

$R_1 = R + \Delta R_1$

$R_2 = R + \Delta R_2$

⋮

$$V_o = V_p \left(\frac{R_2}{R_1 + R_2}\right)\left(1 + \frac{R_4}{R_3}\right) - V_m \cdot \frac{R_4}{R_3}$$

$$= V_p \cdot \frac{1 + \frac{R_4}{R_3}}{1 + \frac{R_1}{R_2}} - V_m \cdot \frac{R_4}{R_3}$$

$$\frac{R_4}{R_3} = \frac{R + \Delta R_4}{R + \Delta R_3} = 1 + \frac{\Delta R_4 - \Delta R_3}{R}$$

$$\frac{R_1}{R_2} = 1 + \frac{\Delta R_1 - \Delta R_2}{R}$$

$$V_o = V_p \cdot \frac{2\left(1 + \frac{\Delta R_4 - \Delta R_3}{2R}\right)}{2\left(1 + \frac{\Delta R_1 - \Delta R_2}{2R}\right)} - V_m\left(1 + \frac{\Delta R_4 - \Delta R_3}{R}\right)$$

$$= V_p \left\{1 + \frac{\Delta R_4 - \Delta R_3 + \Delta R_1 - \Delta R_2}{2R}\right\} - V_m \left\{1 + \frac{\Delta R_4 - \Delta R_3}{R}\right\}$$