24. 2. 2011

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Design of higher order amplifiers:

2nd order: $L(s) = \dfrac{\omega_u/k}{s} \dfrac{1}{1+s/p_2}$ $\left(\simeq \text{ excess delay of } 1/p_2\right)$

* Critically damped for $p_2 = 4\omega_u/k$ $\left(\text{delay} \simeq \frac{1}{4}\cdot\frac{k}{\omega_u}\right)$
* For $p_2 > 2\cdot \omega_u/k$, overshoot is tolerable

Ideal delay: $L(s) = \dfrac{\omega_u/k}{s}\cdot \exp(-sT_d)$ $\left(= \text{ excess delay of } T_d\right)$

* critically damped for $T_d = \frac{1}{e}\cdot\frac{k}{\omega_u}$
* For $T_d < \frac{1}{2}\cdot\frac{k}{\omega_u}$, overshoot is tolerable

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2^{nd} order system:

$$L(s) = \frac{\omega_{u,loop}}{s} \cdot \frac{1}{1 + s/p_2}$$

$$p_2 = 4\omega_u. \quad (2\omega_u)$$

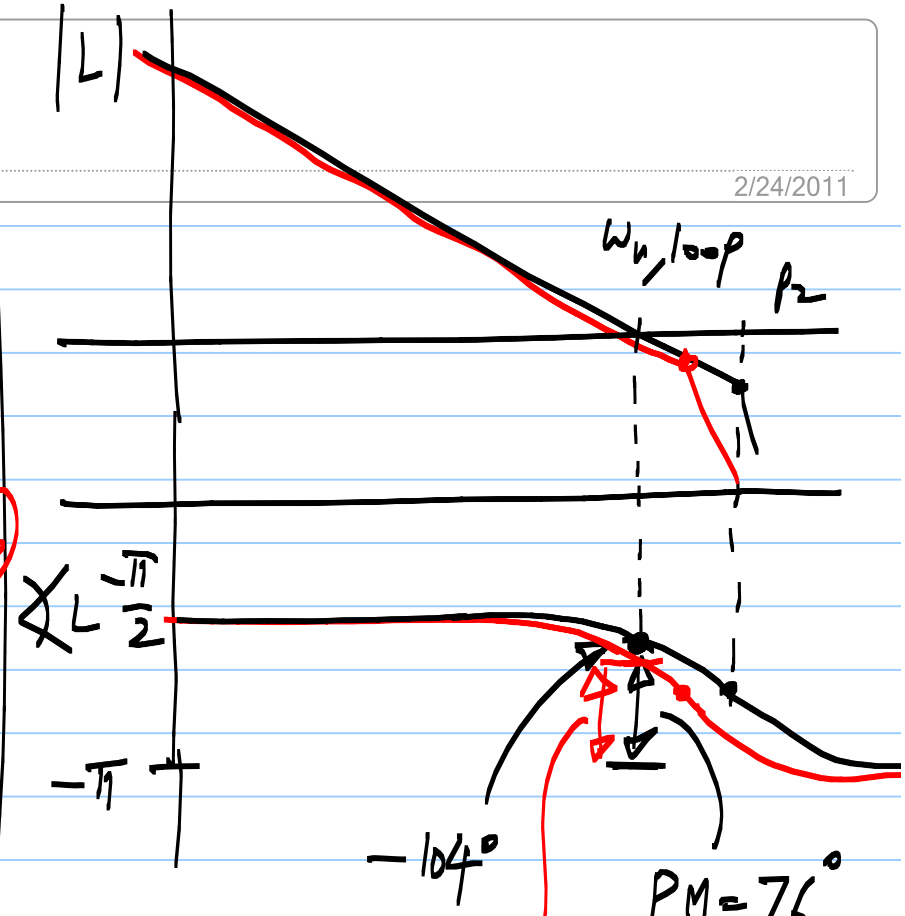
$$|L(j\omega)| = 1, \quad \omega \simeq \omega_{u,loop} \quad (\omega \simeq \omega_{u,loop})$$

$$\angle L(j\omega)\Big|_{\omega = \omega_{u,loop}} = -\frac{\pi}{2} - \tan^{-1}\frac{1}{4} \qquad -\frac{\pi}{2} - \tan^{-1}\frac{1}{2}$$

$$\text{Phase margin } PM = \pi + \angle L(j\omega)$$

$$= \tan^{-1} 4 = 76^\circ$$

$$= \tan^{-1} 2 = 63^\circ$$



$PM = 63^\circ$ for $p_2 = 2 \cdot \omega_{u,loop}$

Ideal delay:

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$$L(s) = \frac{\omega_{u,loop}}{s}\cdot \exp(-sT_d)$$

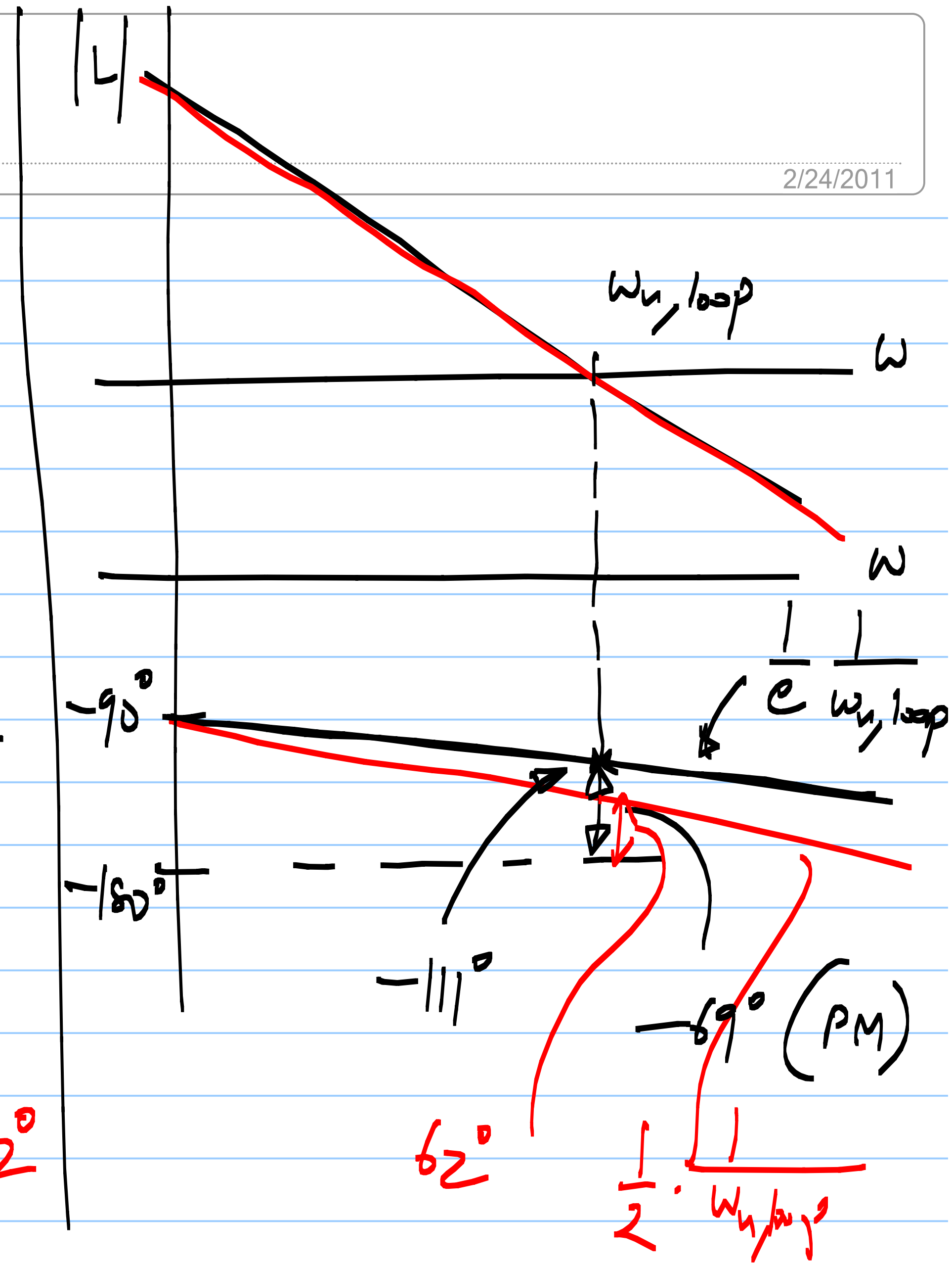
Crit. damping: $T_d = \cancel{\frac{1}{e}}\,\frac{1}{2}\cdot\frac{1}{\omega_{u,loop}}$

$$|L(j\omega)| = 1 \qquad \omega = \omega_{u,loop}$$

$$\angle L(j\omega) = -\frac{\pi}{2} - \cancel{\frac{1}{e}}\,\frac{1}{2} \text{ radians}$$

$$PM = \pi + \angle L(j\omega)\Big|_{\omega=\omega_{u,loop}}$$

$$= \frac{\pi}{2} - \cancel{\frac{1}{e}}\,\frac{1}{2} \rightarrow \cancel{69}^\circ \quad 62^\circ$$



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Higher order systems:

Order	$L(s)$	Closed loop instability?	Critically damped	Phase margin (crit. damp)
1	$\frac{\omega_{u,loop}}{s}$	NO	—	always 90°
2	$\frac{\omega_{u,loop}}{s} \cdot \frac{1}{1+s/p_2}$	NO	$p_2 = 4\omega_{u,loop}$	76°
3	$\frac{\omega_{u,loop}}{s} \frac{1}{(1+s/p_2)^2}$	$p_2 > \frac{\omega_{u,loop}}{2}$	$\boxed{p_2 = 8.1\,\omega_{u,loop} \text{ for } 76^\circ \text{ PM}}$	
4	$\frac{\omega_{u,loop}}{s} \cdot \frac{1}{(1+s/p_2)^3}$	$p_2 > 1.125\,\omega_{u,loop}$	$\boxed{p_2 = 12.2\,\omega_{u,loop} \text{ for } 76^\circ \text{ PM}}$	
5			$[p_2 = 16.3\,\omega_{u,loop} \text{ for } 76^\circ \text{ PM}]$	

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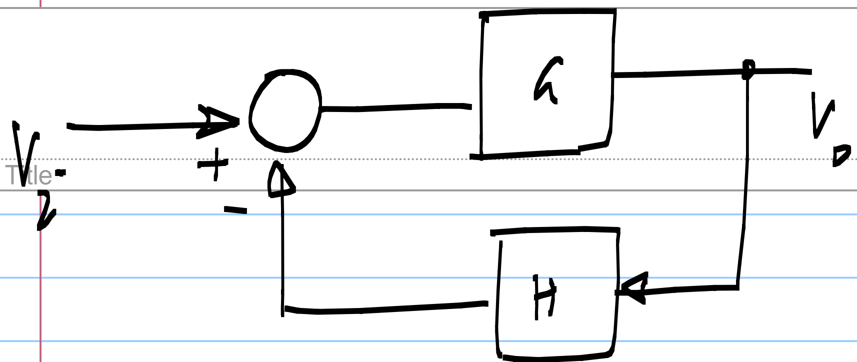


$$\frac{V_o}{V_i} = \frac{G}{1+GH} = \frac{1}{H}\cdot\underline{\underline{\frac{1}{\left(1+\frac{1}{GH}\right)}}}.$$

$$1 + \frac{1}{GH} = 1 + \frac{s}{\omega_{u,loop}}\left(1+\frac{s}{p_2}\right)^2$$

$$= 1 + \frac{s}{\omega_{u,loop}} + 2\cdot\frac{s^2}{\omega_{u,loop}\cdot p_2} + \frac{s^3}{\omega_{u,loop}\cdot p_2^2}\Bigg|_{s=j\omega}$$

$$= \left(1 - \frac{2\omega^2}{\omega_{u,loop}\cdot p_2}\right) + \frac{j\omega}{\omega_{u,loop}} - \frac{j\omega^3}{\omega_{u,loop}\cdot p_2^2}$$

Nagendra Krishnapura, Dept. of EE
Indian Institute of Technology, Madras

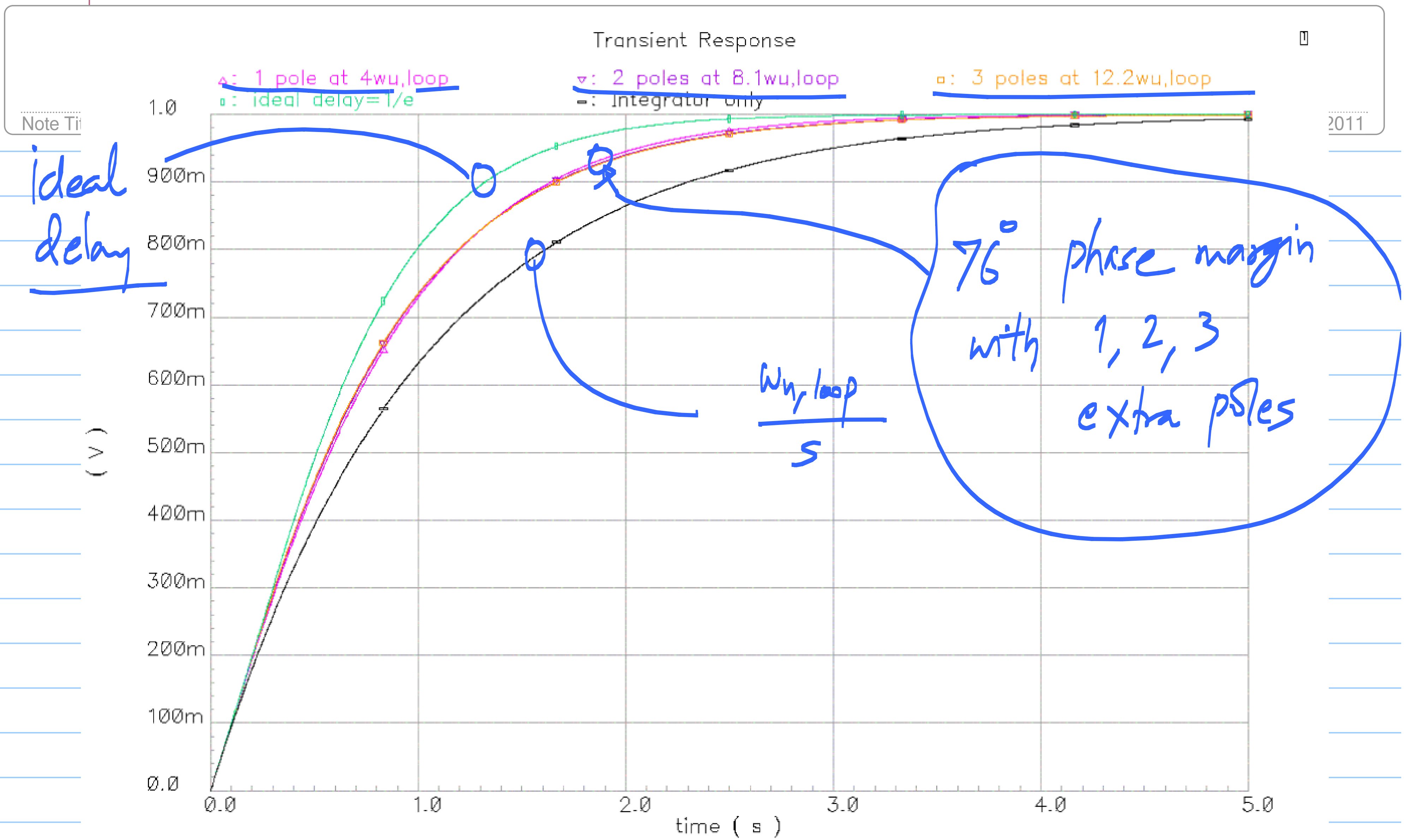
$$= \left(1 - \frac{2\omega^2}{\omega_{u,loop}\cdot p_2}\right) + \underbrace{\frac{j\omega}{\omega_{u,loop}} - \frac{j\omega^3}{\omega_{u,loop}\cdot p_2^2}}_{}$$

$$\underbrace{j\frac{\omega}{\omega_{u,loop}}\left(1 - \frac{\omega^2}{\cdot p_2^2}\right)}_{}$$

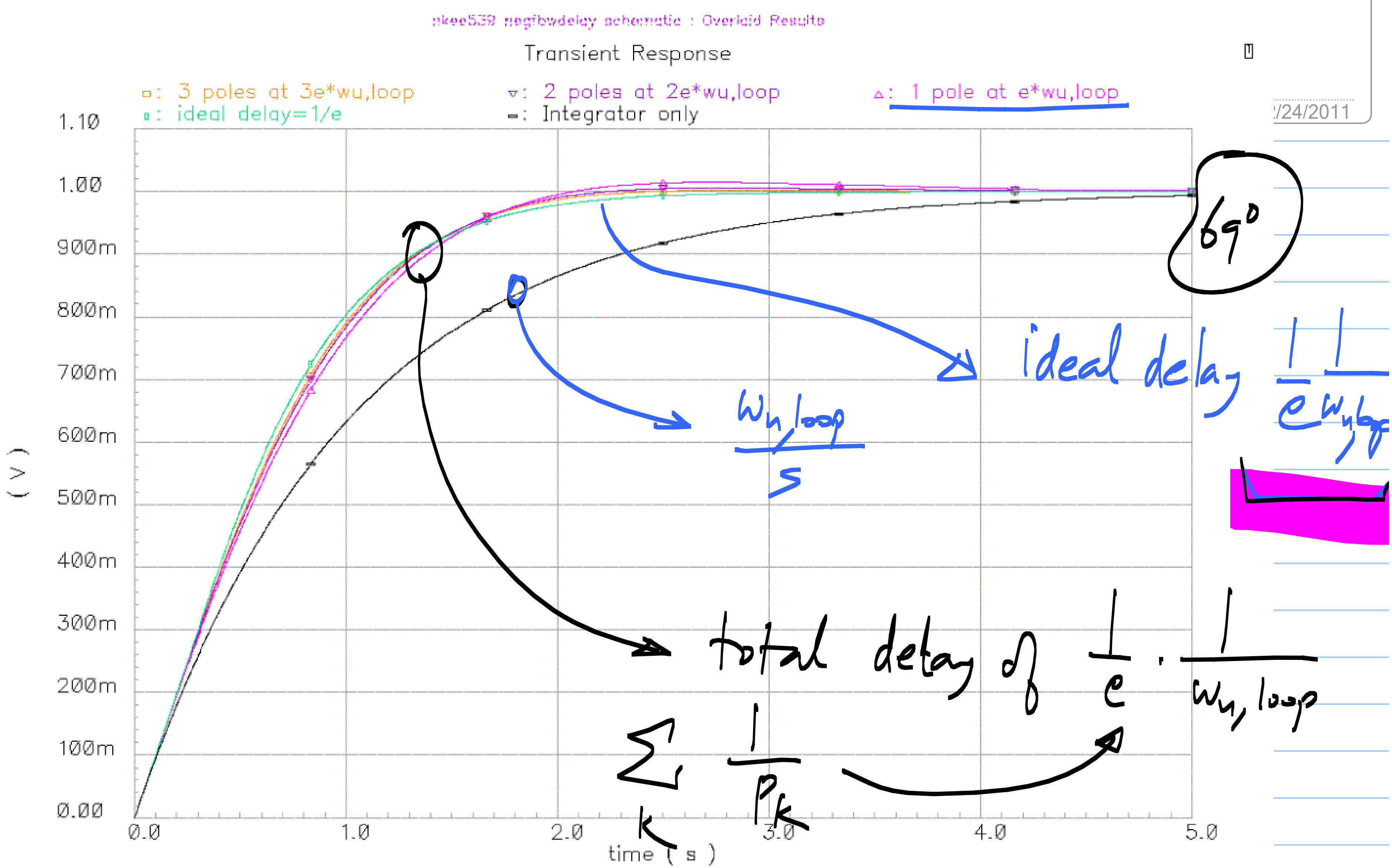
$$1 - \frac{2\omega^2}{\omega_{u,loop}\cdot p_2} = 0 \qquad \omega^2 = \frac{\omega_{u,loop}\cdot p_2}{2}$$

$$1 - \frac{\omega^2}{\cdot p_2^2} = 0 \qquad \omega^2 = \cdot p_2^2$$

$$\underline{p_2 = \frac{\omega_{u,loop}}{2}\ ; \quad \omega = \frac{\omega_{u,loop}}{2}}$$



Transient Response
1 pole at 4wu,loop
ideal delay=1/e
2 poles at 8.1wu,loop
Integrator only
3 poles at 12.2wu,loop
1.0
900m
800m
700m
600m
500m
400m
300m
200m
100m
0.0
(V)
0.0
1.0
2.0
3.0
4.0
5.0
time (s)
ideal delay
$\frac{\omega_{u,loop}}{s}$
76° phase margin with 1, 2, 3 extra poles


Transient Response
3 poles at 3e*wu,loop
2 poles at 2e*wu,loop
1 pole at e*wu,loop
ideal delay=1/e
Integrator only
1.10
1.00
900m
800m
700m
600m
500m
400m
300m
200m
100m
0.00
(V)
0.0
1.0
2.0
3.0
4.0
5.0
time (s)
69°
ideal delay 1/e · 1/w_u,loop
w_u,loop / s
total delay of 1/e · 1/w_u,loop
Σ_k 1/p_k

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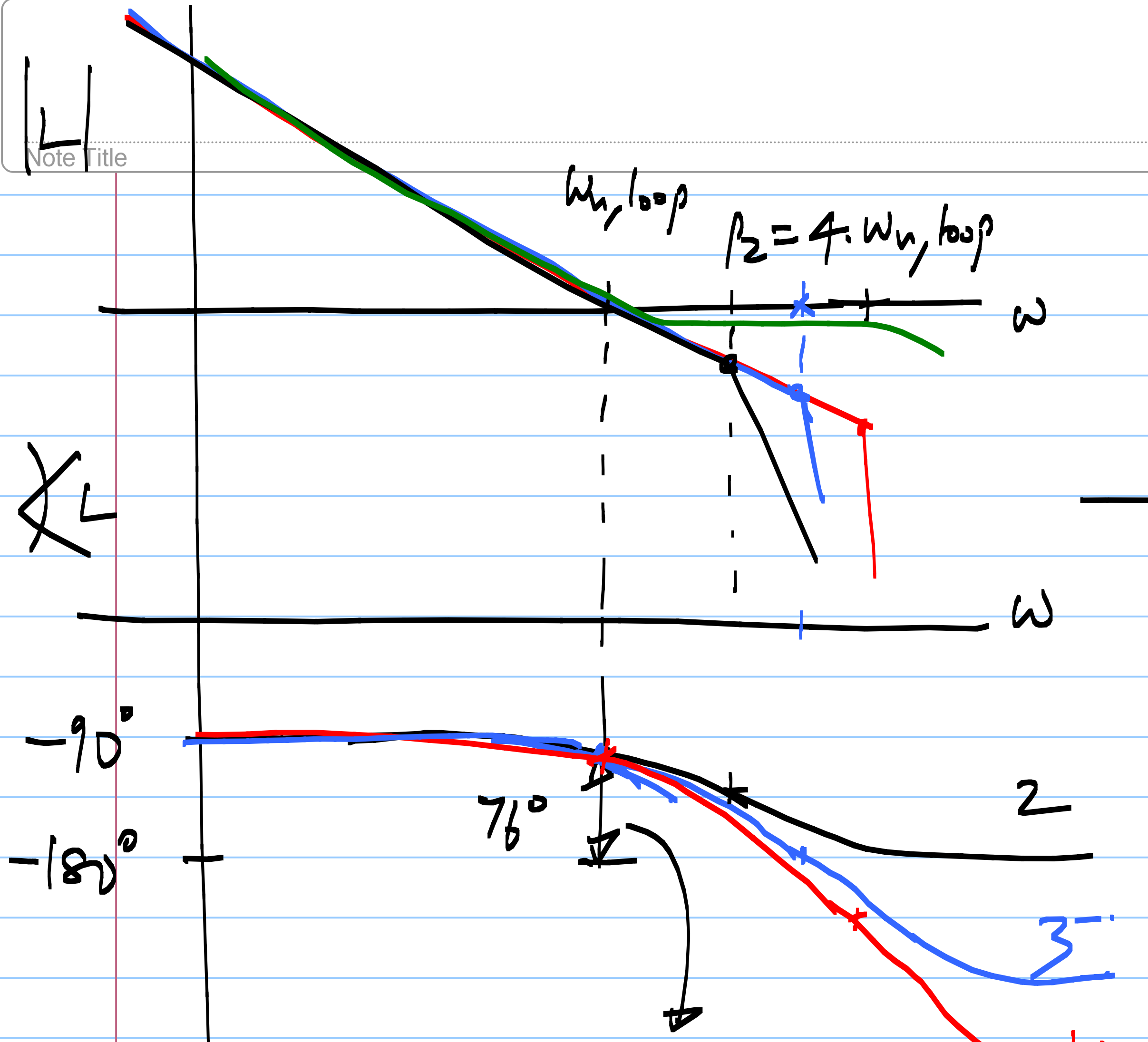

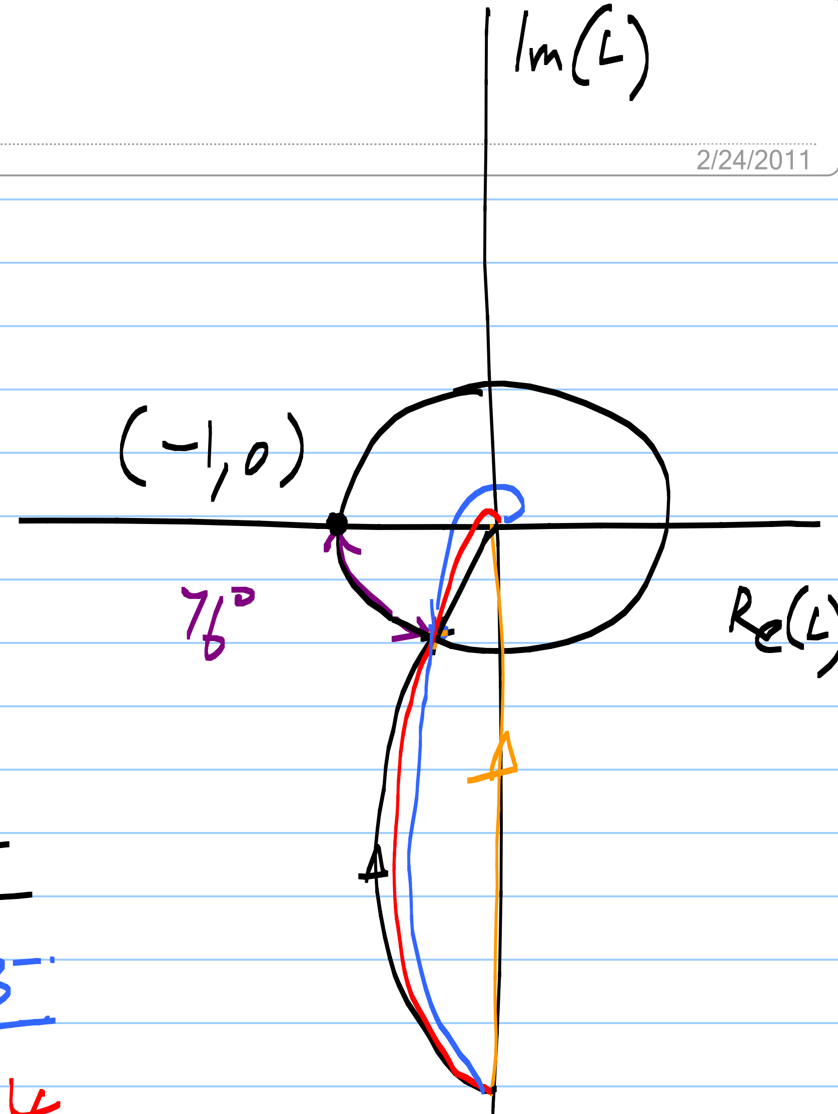

Same phase margin for all

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$$L(s) = \frac{\omega_{u,loop}}{s} \frac{1}{\prod_{k=1}^{N-1}\left(1+\frac{s}{p_k}\right)}$$

$$p_k > \omega_{u,loop}$$

$$|L(j\omega)| = 1, \quad \omega = \omega_{u,loop}$$

$$PM = +\frac{\pi}{2} - \underbrace{\sum_{k=1}^{N-1} \tan^{-1}\frac{\omega_{u,loop}}{p_k}}_{14^\circ} = \underbrace{76^\circ}_{\text{more than } 60^\circ}$$

$$L(s) = \frac{\omega_{u,loop}}{s} \frac{1}{\prod_{k=1}^{N-1}\left(1+\frac{s}{p_k}\right)}$$

$$\text{Excess delay} = \sum_{k=1}^{N-1} \frac{1}{p_k} < \frac{1}{e}\cdot\frac{1}{\omega_{u,loop}}$$

$$\frac{1}{2}\cdot\frac{1}{\omega_{u,loop}}$$

Negative feedback amplifier implementation:

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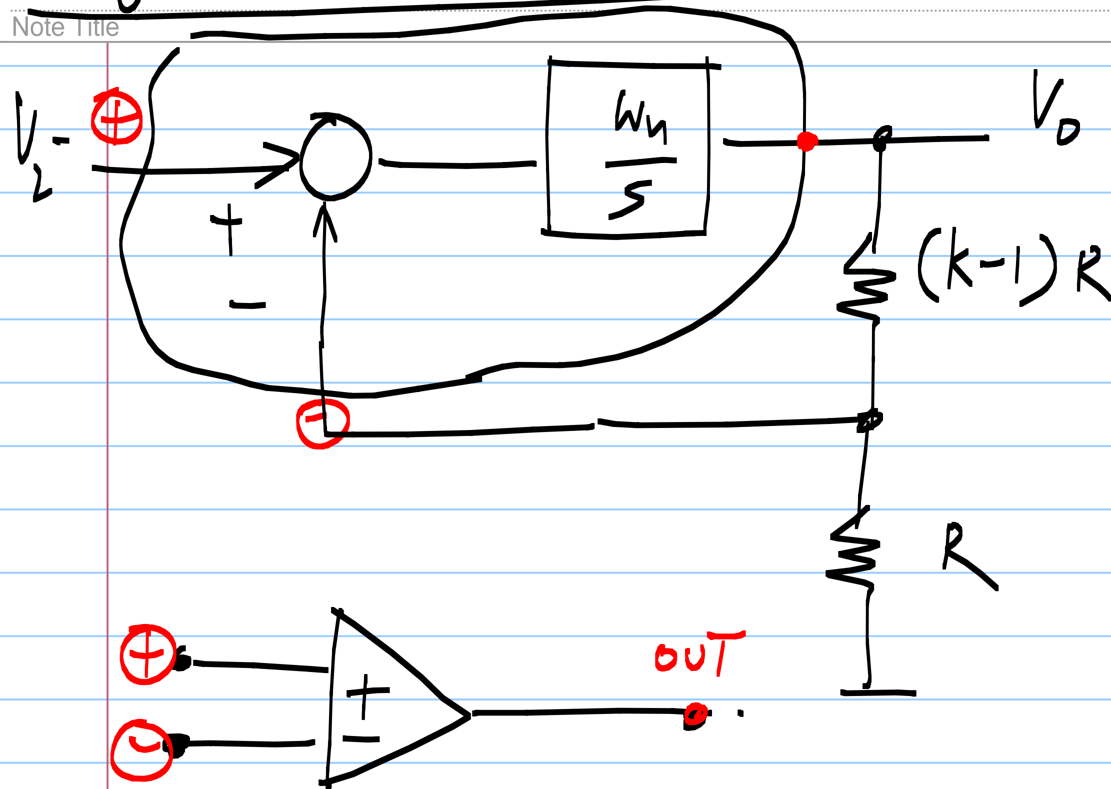
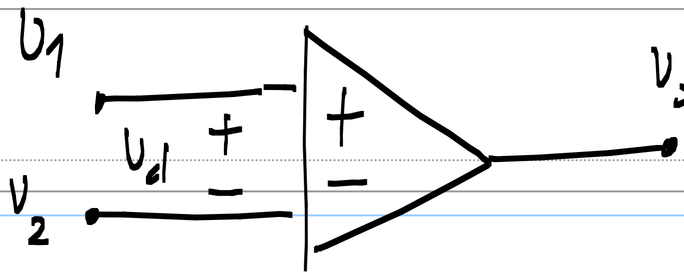


$$\underbrace{R, \quad L, \quad C}_{\text{integration}}$$

$$\boxed{V_c = \frac{1}{C}\int i_c \cdot dt}$$

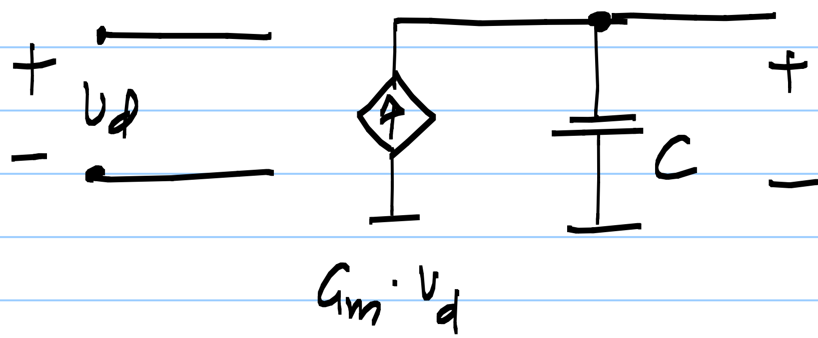
$$i_L = \frac{1}{L}\int V_L \cdot dt$$



$$v_o = \omega_u \int v_d \cdot dt$$

$$v_c = \frac{1}{C} \int i_c \cdot dt$$

$$\boxed{\omega_u = G_m/C}$$



$$\frac{1}{C} \int G_m \cdot v_d \cdot dt$$

$$= \frac{G_m}{C} \int v_d \cdot dt$$