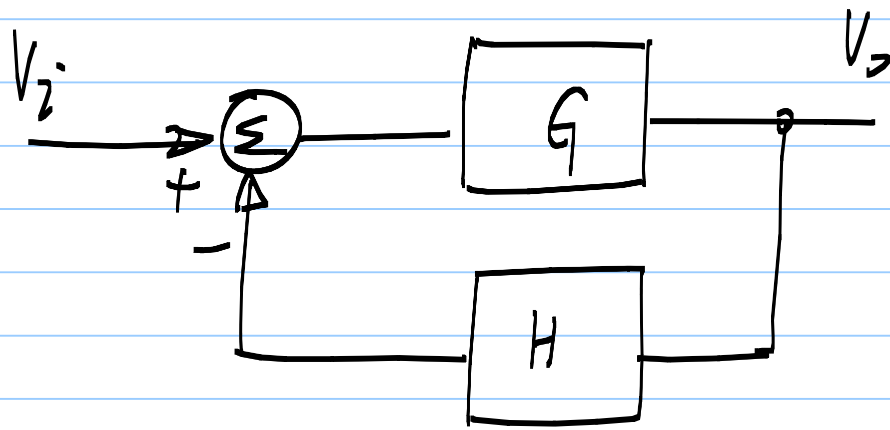
Note Title 2/17/2011

Nyquist stability criterion:



$$\frac{V_o}{V_i} = \frac{G}{1+GH}$$

- $G(s)H(s)$ does not have any poles in the right half plane

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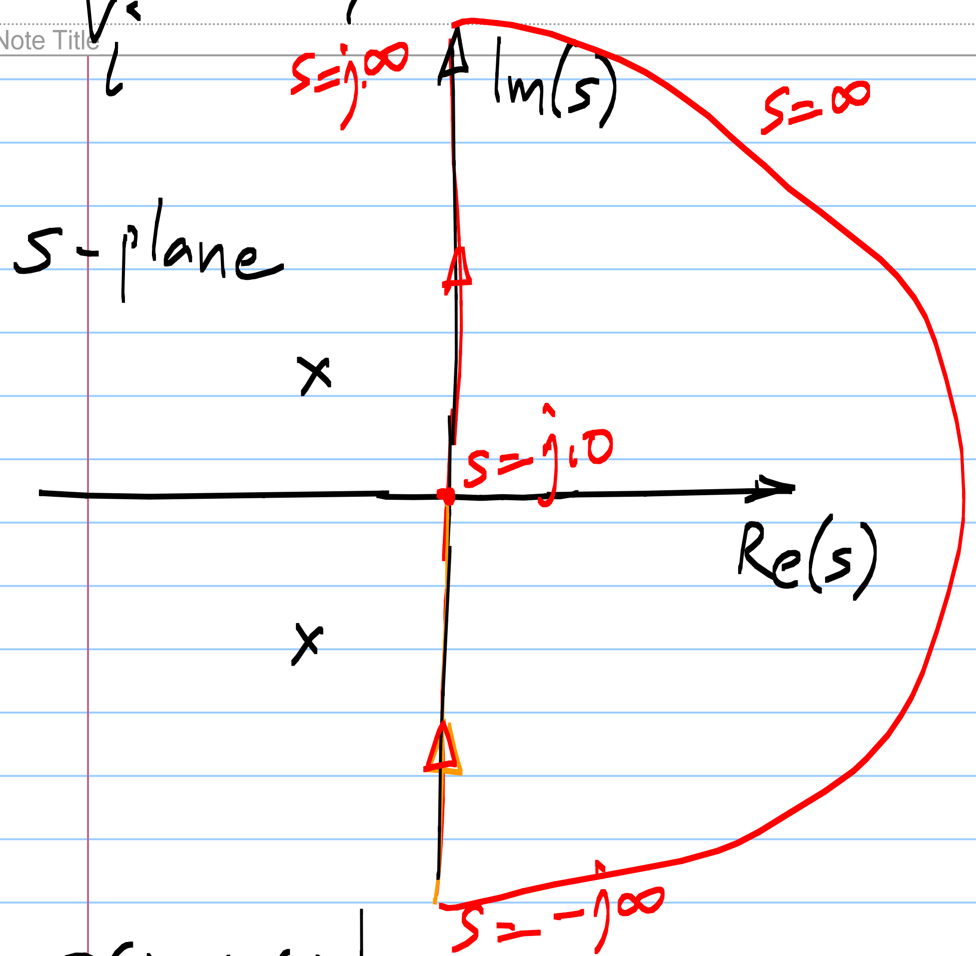
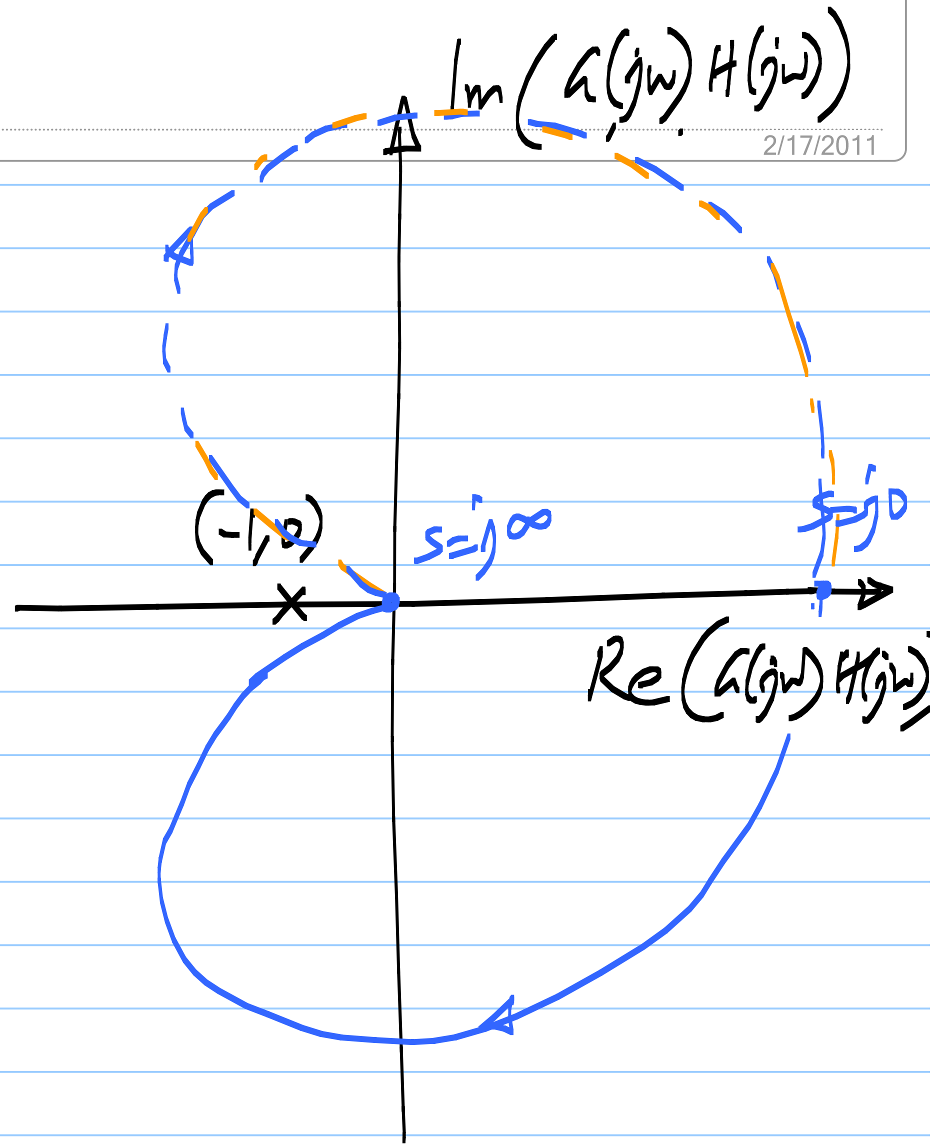
$$\frac{V_o}{V_i} = G / 1 + GH$$





$$G(s)\,H(s)\Big|_{s=j\omega}$$

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Nyquist criterion: For $G(s)H(s)$ which donot have RHP poles, the number of RHP zeros of $1 + G(s)H(s)$ is equal to the number of clockwise encirclements of $G(j\omega)H(j\omega)$ of $(-1, 0)$ [plot of $\text{Im}(G(j\omega)H(j\omega))$ versus $\text{Re}(G(j\omega)H(j\omega))$]

$$\frac{G}{1+GH}$$

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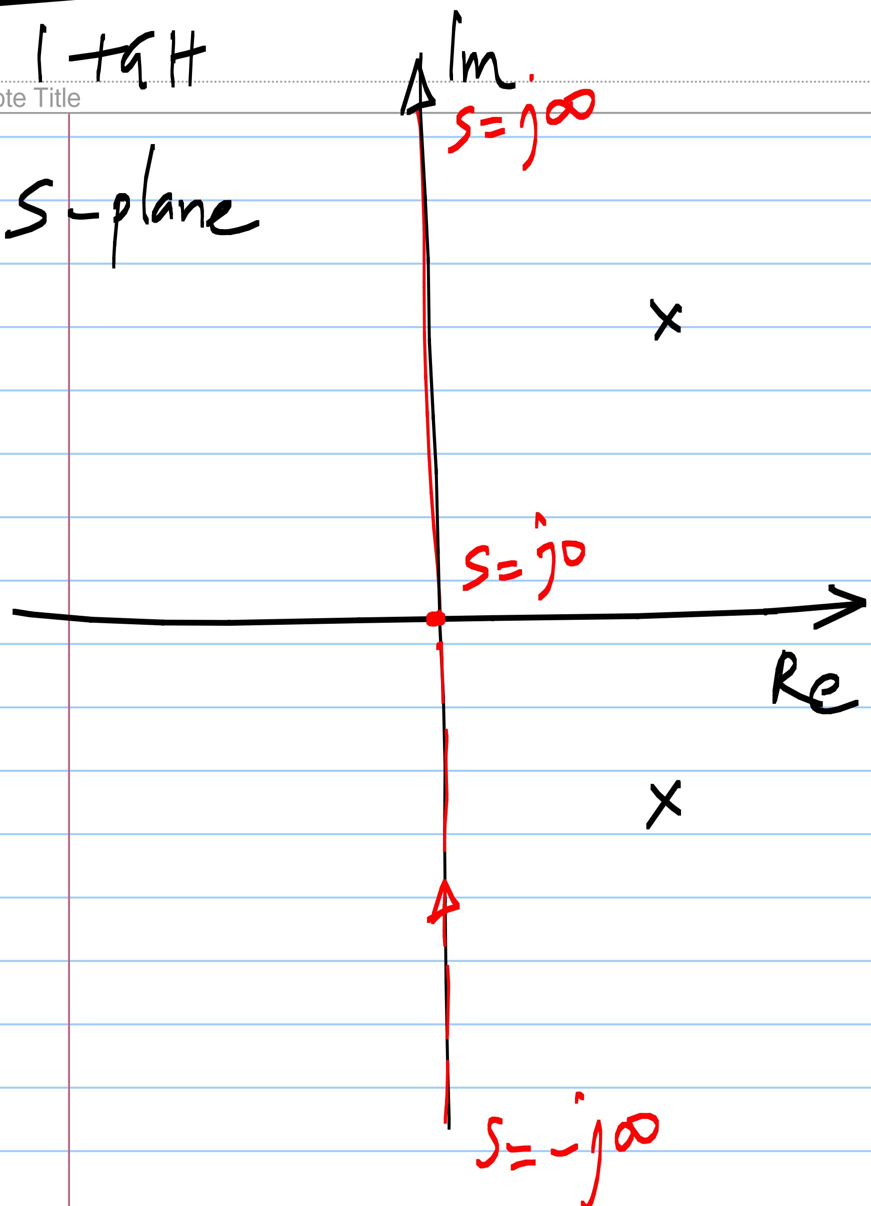
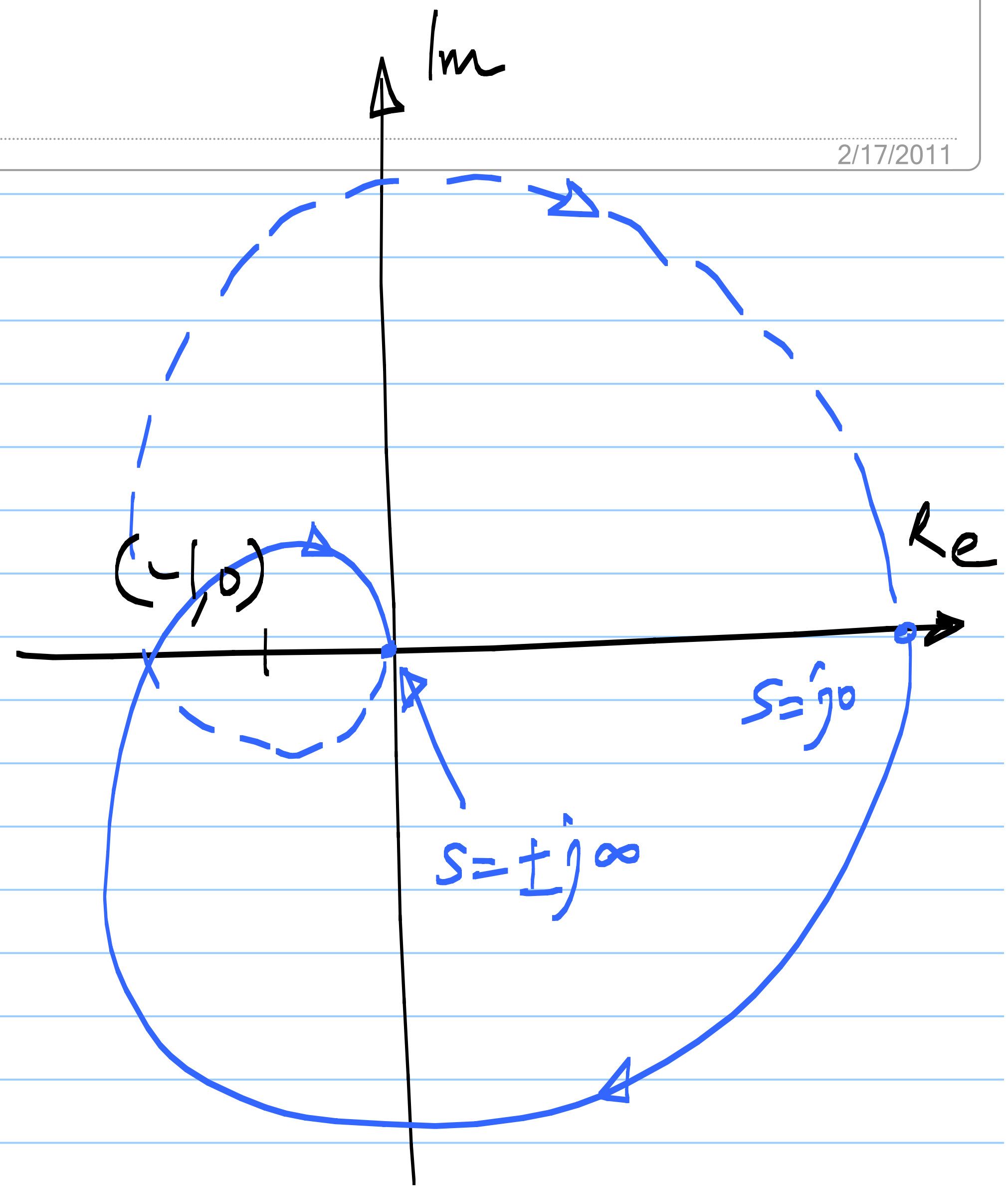
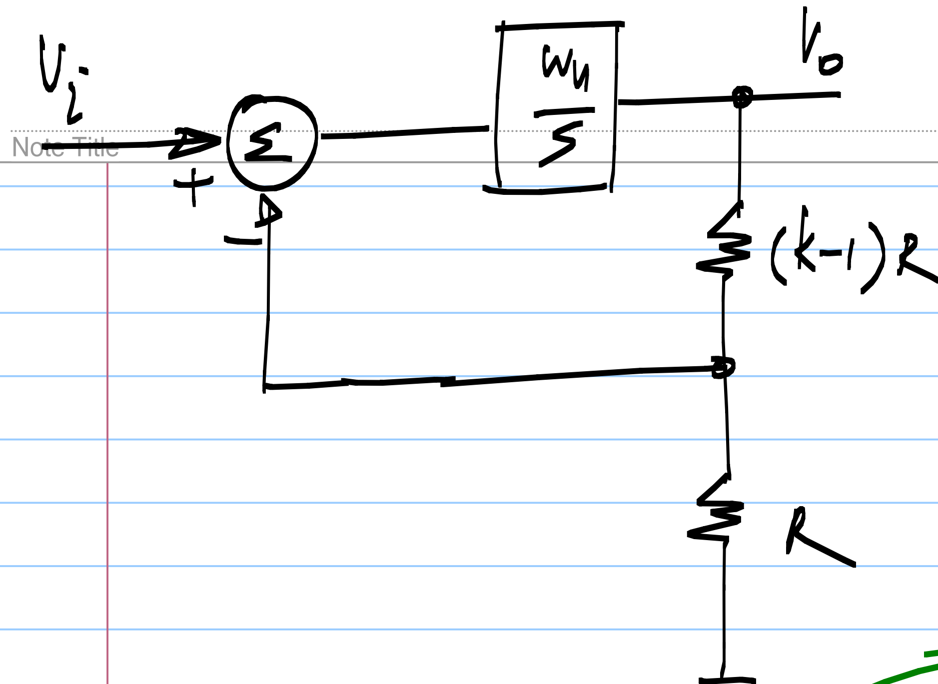
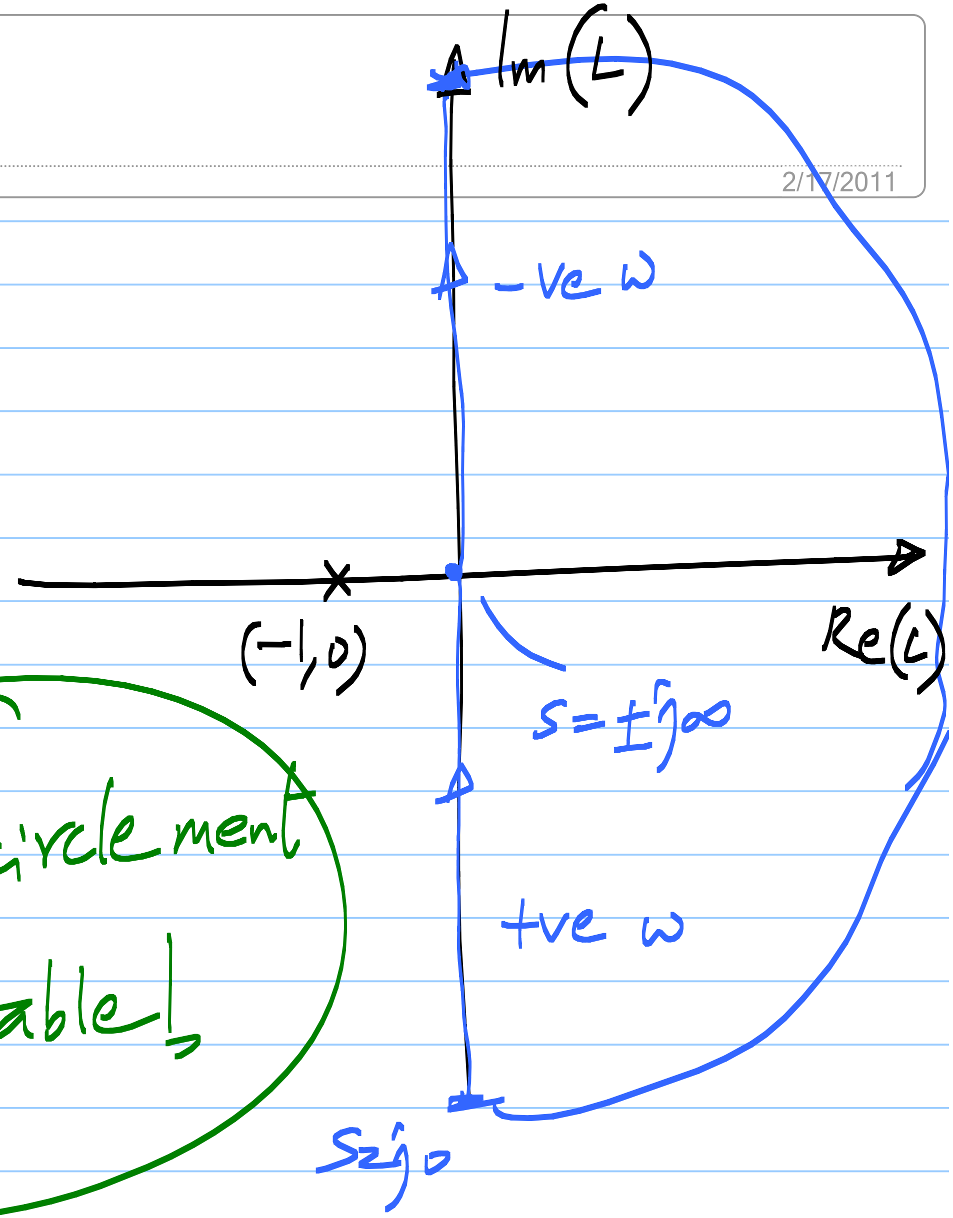
s-plane









Loop gain

$$G(s)\,H(s) = \frac{\omega_u}{k \cdot s}$$

$\underline{\underline{L(s)}}$

No encirclement

⇒ stable!

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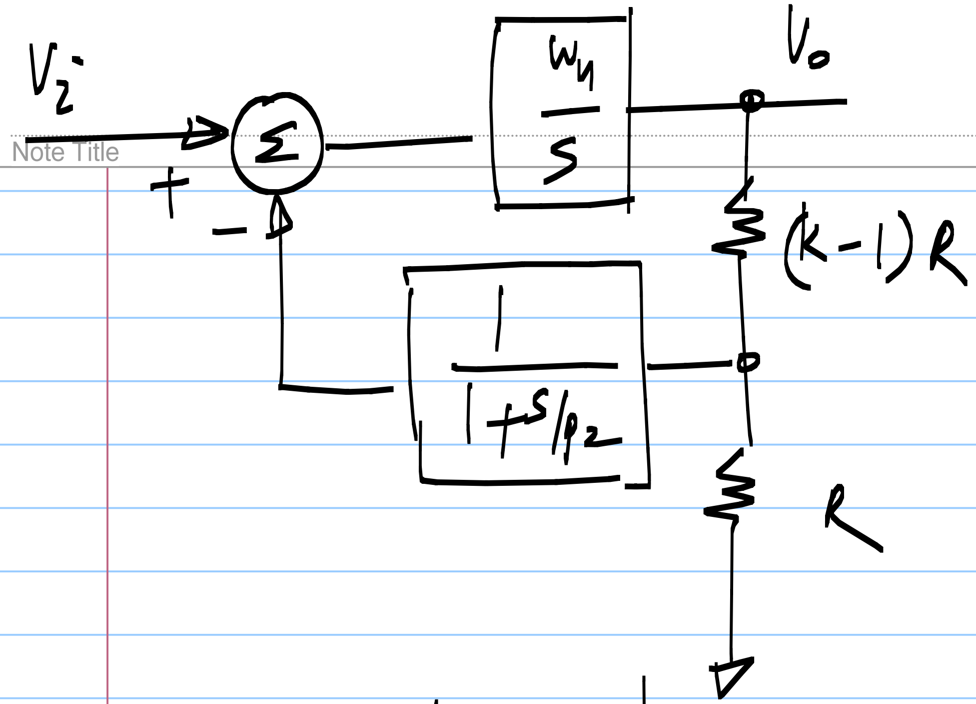
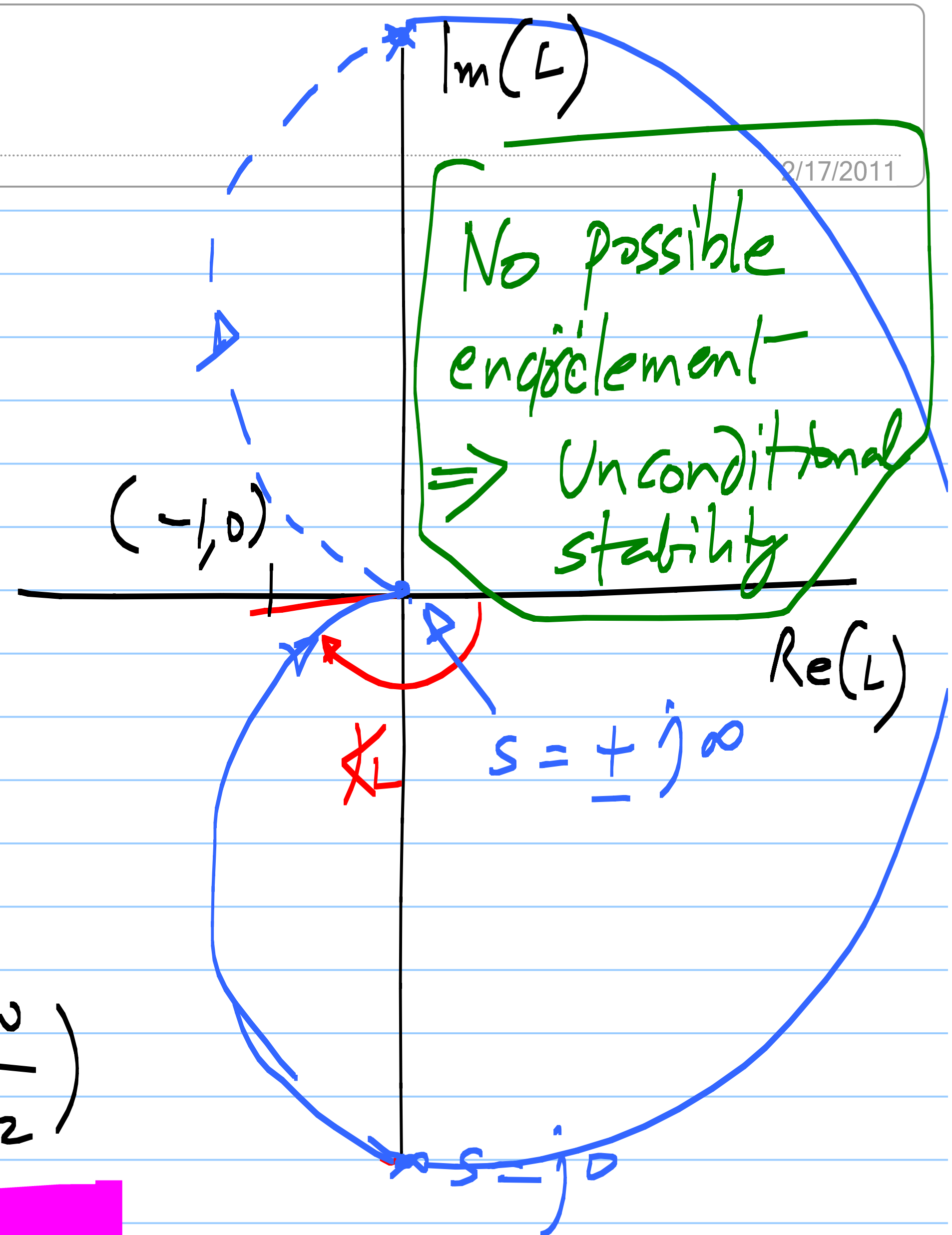
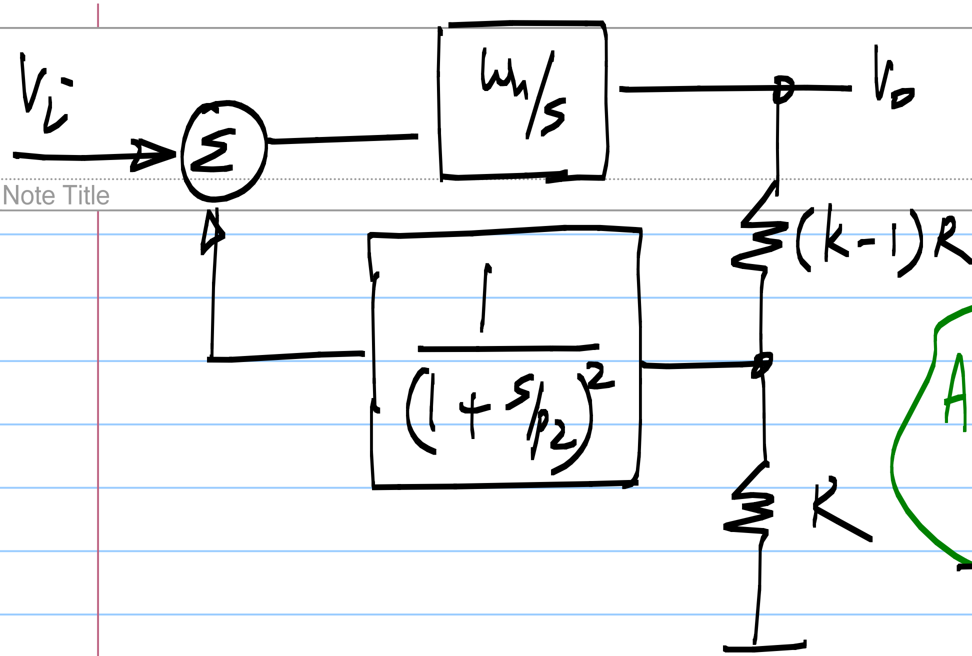


$$L(s) = \frac{\omega_u/k}{s\left(1+s/p_2\right)}$$

$$|L| = \frac{\omega_u/k}{\omega\sqrt{1+\left(\omega/p_2\right)^2}}$$

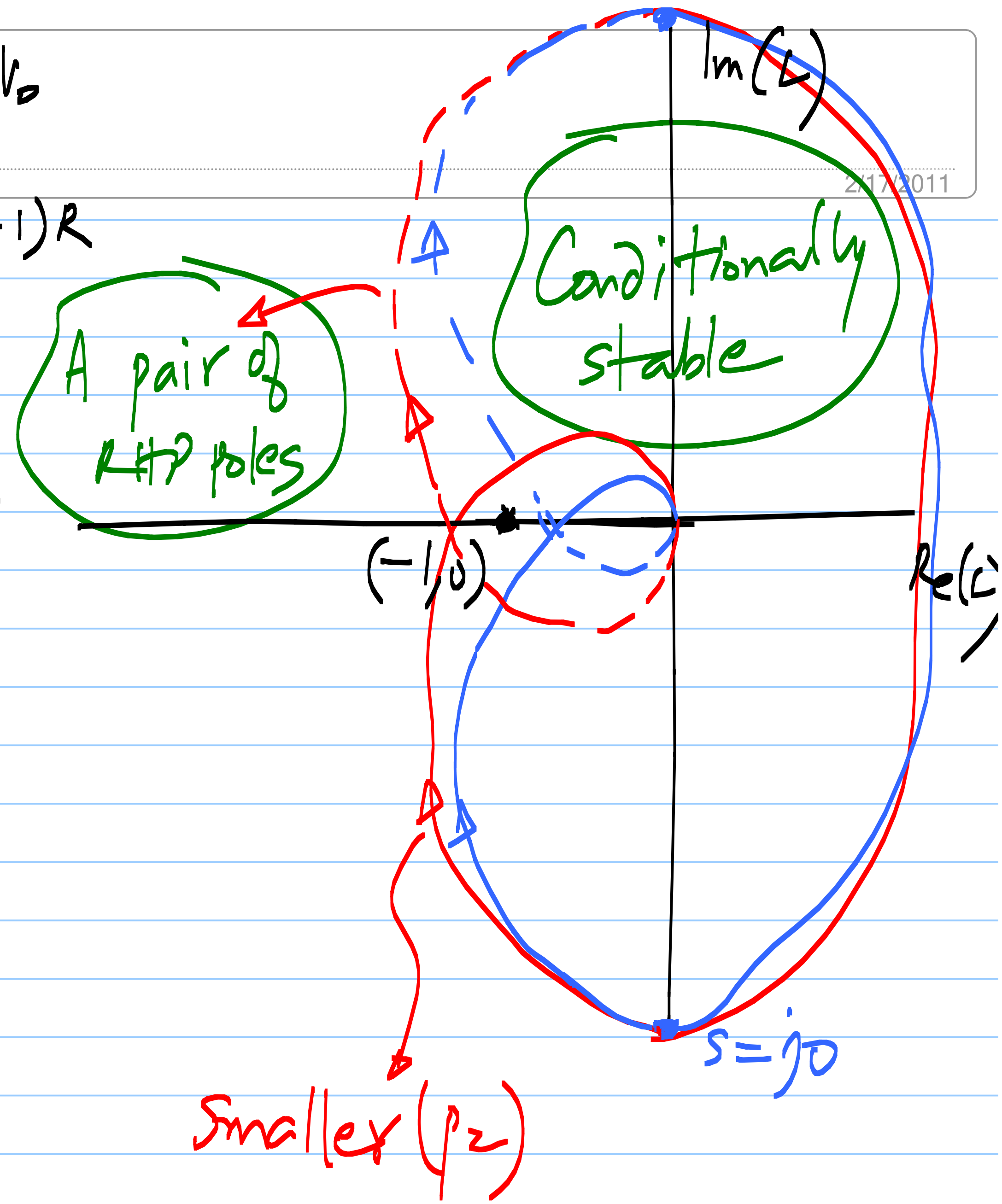
$$\angle L = -\frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{p_2}\right)$$

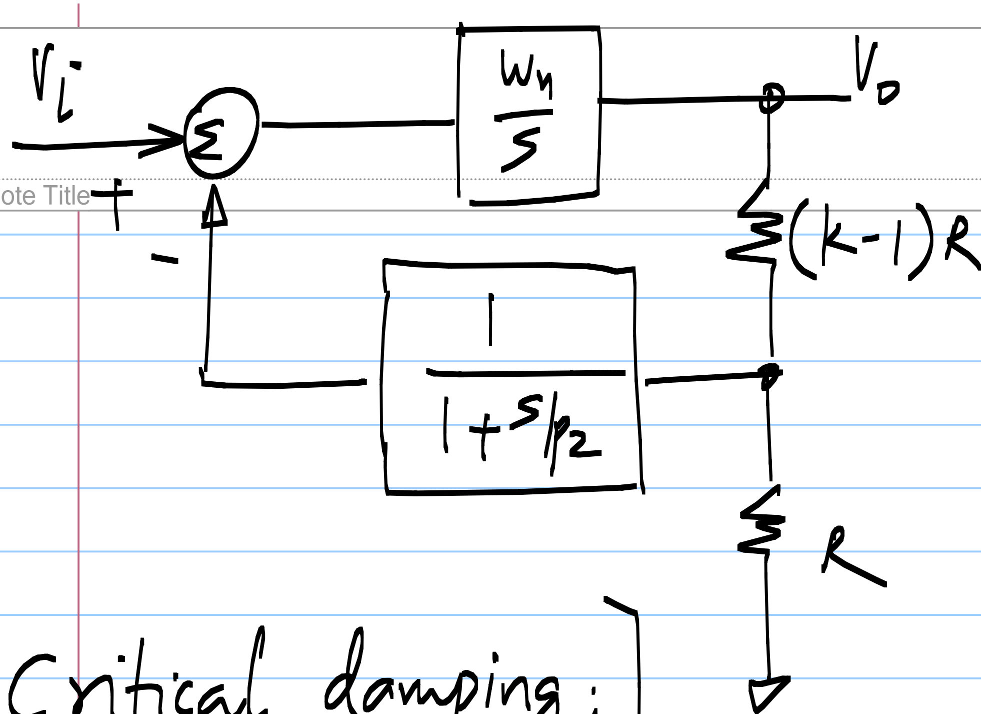




$$L(s) = \frac{\omega_u/k}{s} \cdot \frac{1}{\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)}$$

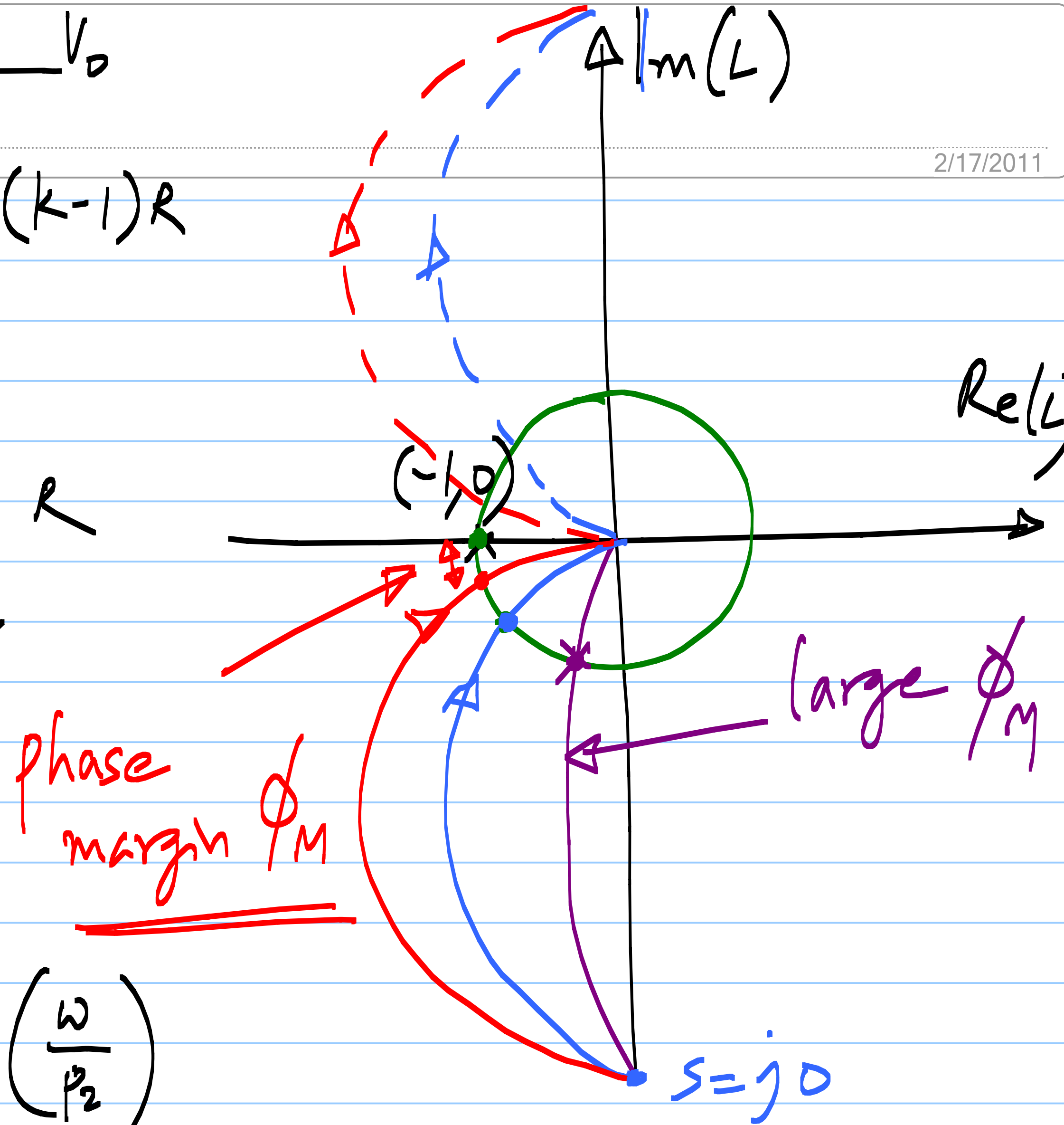
$$\angle L = -\pi/2 - \tan^{-1}\left(\frac{\omega}{p_2}\right) - \tan^{-1}\left(\frac{\omega}{p_3}\right)$$





Critical damping:

$$p_2 = 4\cdot\left(\omega_u/k\right)$$



$$\angle L(j\omega) = -\frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{p_2}\right)$$

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Phase margin ϕ_M = angle between the point of intersection of the Nyquist plot with the unit circle & the negative real axis.

$$|L| = 1 \; ; \qquad \angle L \Big|_{|L|=1} + \pi = \phi_M$$

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2^{nd} order critically damped system.

$$L(s) = \frac{\omega_u/k}{s\left(1 + s/p_2\right)} \qquad p_2 = 4\cdot\left(\frac{\omega_u}{k}\right)$$

$$\left|L\right|_{s=j\frac{\omega_u}{k}} = \left|\frac{\omega_u/k}{j\,\omega_u/k\left(1 + \frac{j\omega_u/k}{4\cdot\omega_u/k}\right)}\right| = \frac{1}{\left|1 + j\frac{1}{4}\right|}$$

$$= \frac{1}{\sqrt{1 + \frac{1}{16}}} \simeq 1$$

$|L| = 1$ when $\boxed{\omega \simeq \frac{\omega_u}{k}}$

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$$\angle L\Big|_{s=j\frac{\omega_u}{k}} = -\frac{\pi}{2} - \tan^{-1}\frac{\omega_u/k}{p_2}$$

$$= -\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$$

$$\pi + \angle L\Big|_{s=j\frac{\omega_u}{k}} = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right) = \tan^{-1}(4)$$

$$= 76^\circ$$