Negative feedback amplifier with delay

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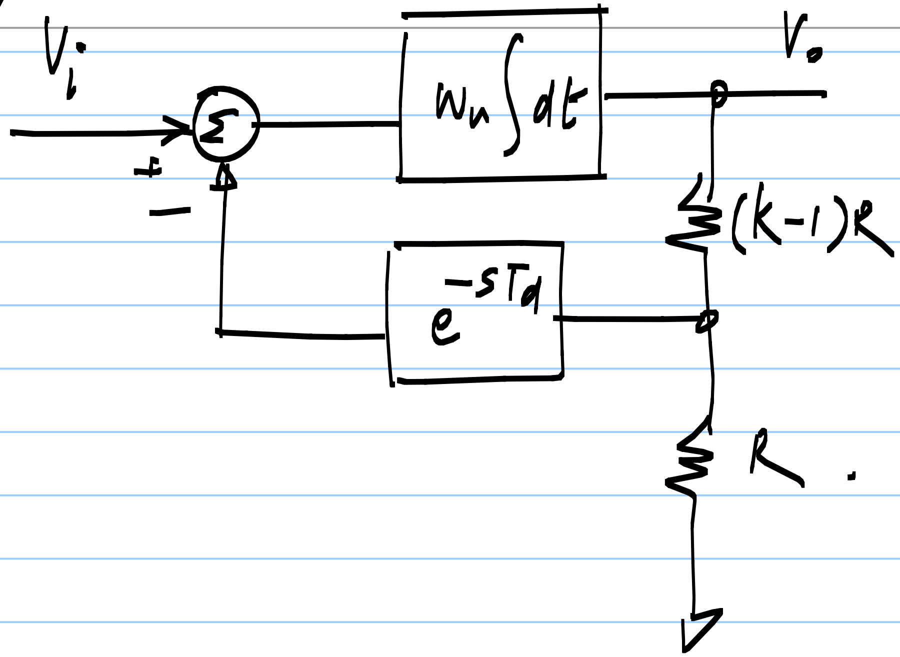

$$\text{Loop gain} = \frac{w_u/k}{s} = \frac{w_{u,loop}}{s}$$

$T_d < \frac{1}{e}\left(\frac{1}{w_{u,loop}}\right)$, no overshoot in the step response (overdamped system)

$T_d = \frac{1}{e}\frac{1}{w_{u,loop}}$, critically damped system

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$\frac{1}{e} \cdot \frac{1}{\omega_{u,loop}} < T_d < \frac{\pi}{2} \cdot \frac{1}{\omega_{u,loop}}$: ringing in the step response; ringing eventually dies out

$\frac{\pi}{2} \cdot \frac{1}{\omega_{u,loop}} < T_d$: Sustained oscillations

For a good (well behaved) amplifier:

can tolerate a little ringing:

$$T_d < \frac{1}{2} \cdot \frac{1}{\omega_{u,loop}}$$

tolerable.

Extra pole in the loop:

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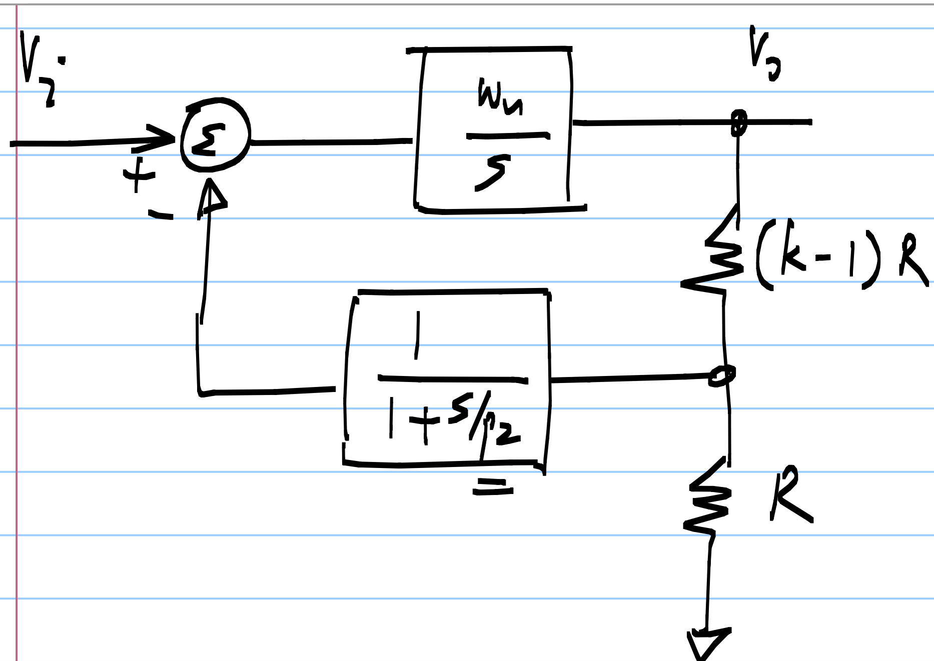

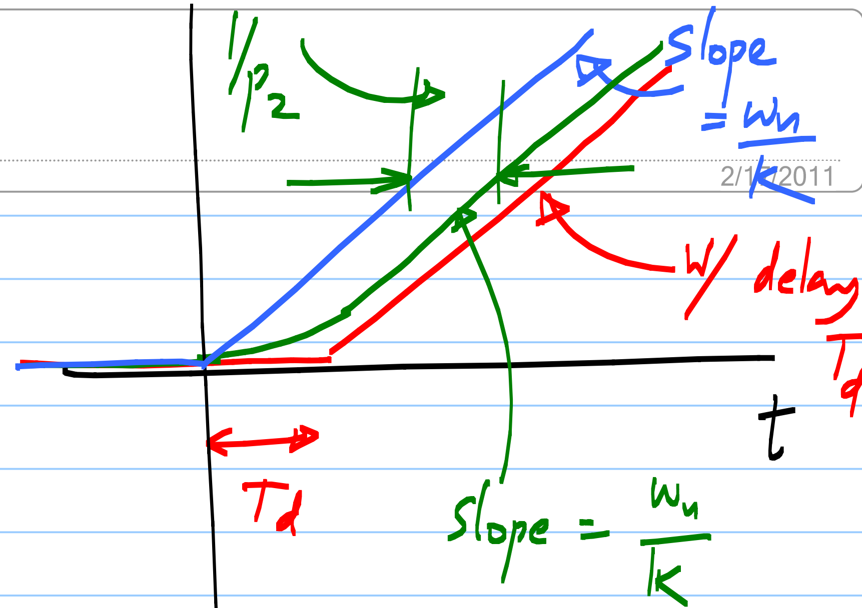

A pole $p_2 \simeq$ a delay of $\frac{1}{p_2}$

$$\xi = \frac{1}{2}\sqrt{\frac{p_2}{\omega_u/K}}$$

$\xi \geq 1$ if $p_2 \geq 4\cdot\left(\frac{\omega_u}{K}\right)$

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* with a single extra pole p_2,

- the system is unconditionally stable.
- the system is critically damped for $p_2 = 4\left(\frac{\omega_n}{K}\right)$

$$= \underline{\underline{4\,\omega_{u,loop}}}$$

- the system is overdamped for $\underline{\underline{p_2 \geq 4 \cdot \omega_{u,loop}}}$

$$\underline{p_2 = 2 \cdot \omega_{u,loop}}$$

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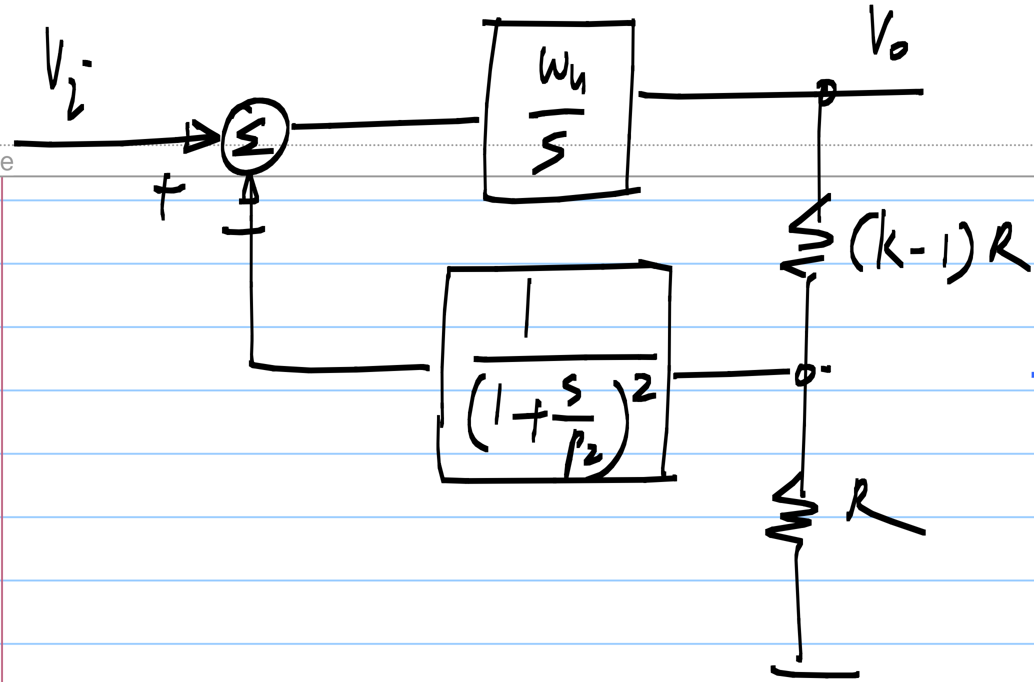

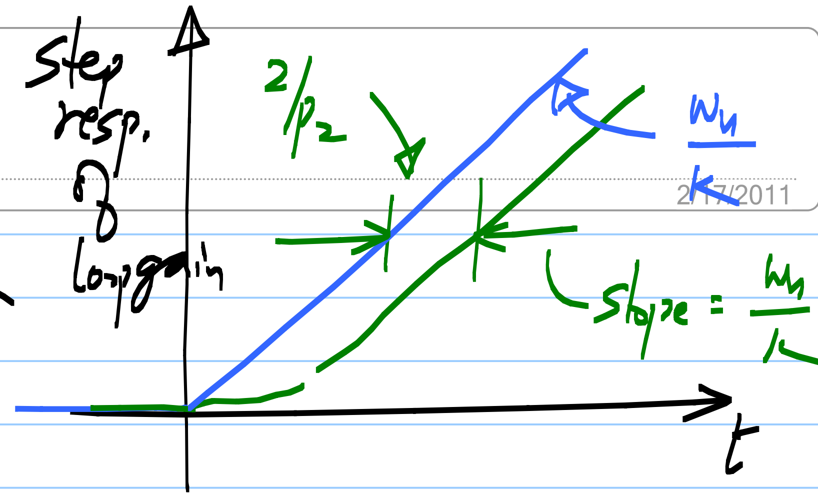

$$\frac{V_o}{V_i} = \frac{\omega_u/s}{1 + \frac{\omega_u}{s}\cdot\frac{1}{k}\cdot\frac{1}{\left(1+\frac{s}{p_2}\right)^2}}$$

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$$\frac{V_o}{V_i} = \frac{k \cdot (1 + s/p_2)^2}{\left[(1 + s/p_2)^2 \cdot \frac{s}{\omega_{u,loop}} + 1\right]}$$

$D(s) = 0$ for $s = j\omega$

Unstable system: Poles are on the $j\omega$ axis OR in the RHP
(of the closed loop system)

$$\frac{V_o}{V_i} = \infty \text{ for some } s = j\omega .$$

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$$\left.\left(1+\frac{s}{p_2}\right)^2\cdot\frac{s}{\omega_{u,loop}}+1\right|_{s=j\omega}=0$$

$$\left.\frac{s^3}{p_2^2\cdot\omega_{u,loop}}+2\cdot\frac{s^2}{p_2\cdot\omega_{u,loop}}+\frac{s}{\omega_{u,loop}}+1\right|_{s=j\omega}=0$$

$$\underline{\underline{-j\frac{\omega^3}{p_2^2\,\omega_{u,loop}}}}\;\underline{\underline{-\,2\cdot\frac{\omega^2}{p_2\cdot\omega_{u,loop}}}}+\underline{\underline{j\frac{\omega}{\omega_{u,loop}}}}+\underline{\underline{1}}=0$$

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$$1 - \frac{2\omega^2}{p_2 \cdot \omega_{u,loop}} = 0 \qquad \underline{p_2 = \left[\omega_{u,loop}/2\right]}$$

$$-\frac{\omega^3}{p_2^2 \cdot \omega_{u,loop}} + \frac{\omega}{\omega_{u,loop}} = 0 \Rightarrow \omega = p_2$$

when there are two identical parasitic poles @ p_2:

when $p_2 = \omega_{u,loop}/2$

$\boxed{\text{If } p_2 < \omega_{u,loop}/2, \text{ the output blows up}}$

$$\frac{V_o}{V_i} = \infty \quad \text{for} \quad \underline{\omega = \left(\frac{\omega_{u,loop}}{2}\right)}$$

$\therefore$ For stability, $p_2 < \dfrac{\omega_{u,loop}}{2}$

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- when $p_2 = \dfrac{\omega_{u,loop}}{2}$, closed loop poles are on the $j\omega$ axis
- when $p_2 > \dfrac{\omega_{u,loop}}{2}$, closed loop poles are in the RHP

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	Loop gain	Unstable
No parasitic poles	$\frac{\omega_u/k}{s} = \frac{\omega_{u,loop}}{s}$	Never
1 parasitic pole @ p_2	$\frac{\omega_{u,loop}}{s} \cdot \frac{1}{\left(1+\frac{s}{p_2}\right)}$	Never (Underdamped if $p_2 < 4\omega_{u,loop}$)
2 " @ p_2	$\frac{\omega_{u,loop}}{s} \frac{1}{\left(1+\frac{s}{p_2}\right)^2}$	$p_2 < 0.5\,\omega_{u,loop}$
3 " @ p_2	$\frac{\omega_{u,loop}}{s} \frac{1}{\left(1+\frac{s}{p_2}\right)^3}$	$p_2 < 1.13\,\omega_{u,loop}$
4 " @ p_2	$\frac{\omega_{u,loop}}{s} \frac{1}{\left(1+s/p_2\right)^3}$	$p_2 < 1.76\,\omega_{u,loop}$

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Closed form expressions for step response are too complicated for higher order systems.

$\Rightarrow$ We need an alternative

Nyquist criterion for stability of feedback amplifiers;

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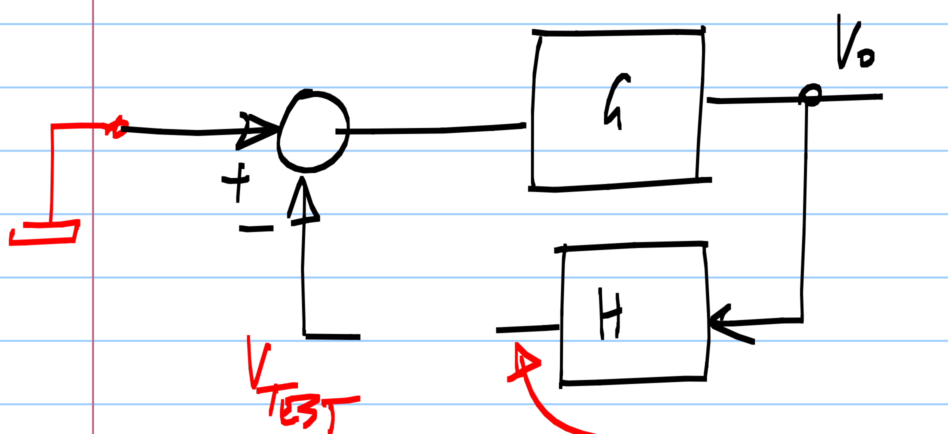

$$G = \omega_u / s$$

$$H = 1/k$$

$-(GH)V_{TEST}$

$$\frac{V_o}{V_i} = \frac{G}{1 + \underline{GH}}$$

Loop gain

If $GH = -1$, $\dfrac{V_o}{V_i} = \infty$

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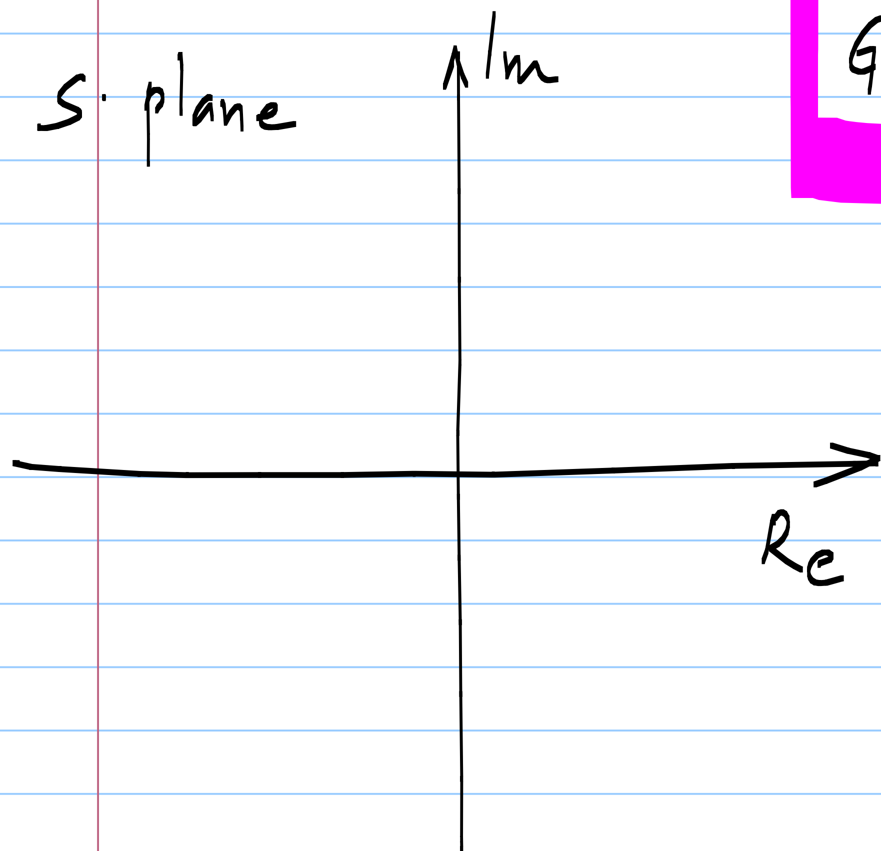
Instability:

$$\frac{V_o}{V_i} = \frac{G}{1 + GH}$$

S-plane



$G(s)H(s)\Big|_{s=j\omega}$