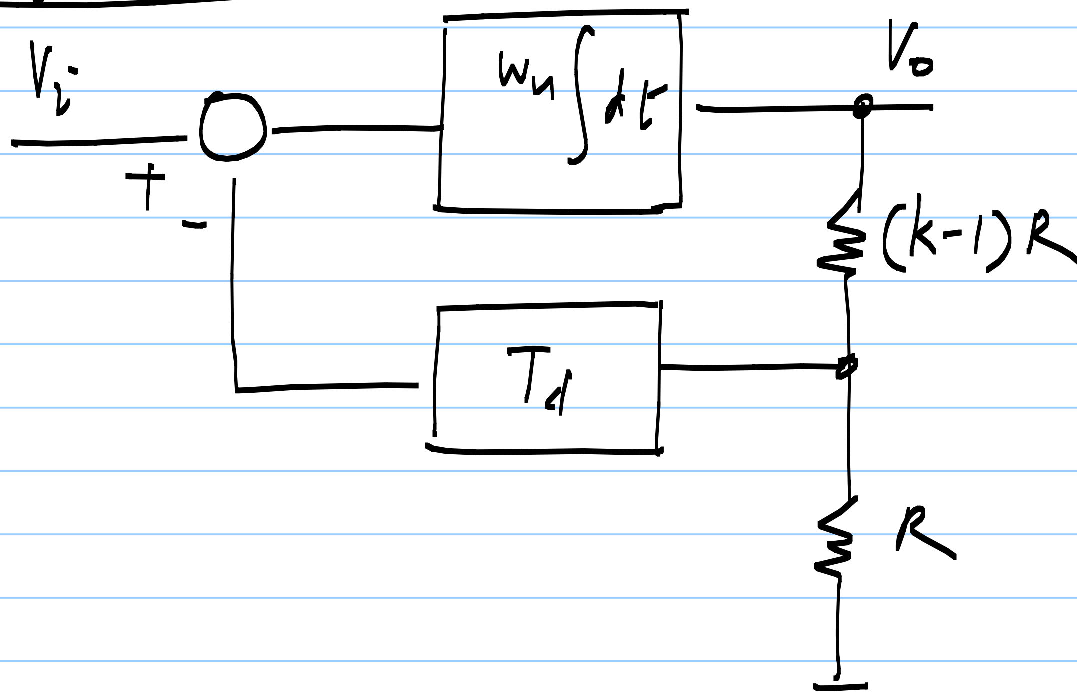
Note Title 1/19/2011

Negative feedback amplifier with delay



$$\frac{dV_o(t)}{dt} = -\frac{\omega_u}{k} V_o(t - T_d)$$

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$$V_o(t) = V_p \exp(\sigma t)$$

$$\sigma' + \exp(-\sigma' T) = 0$$

* Has two solutions for $T < 1/e$

T	σ_1'	σ_2'
0	-1	$-\infty$
0.1	-1.1	-35.8
⋮		
$1/e$	$-e$	$-e$

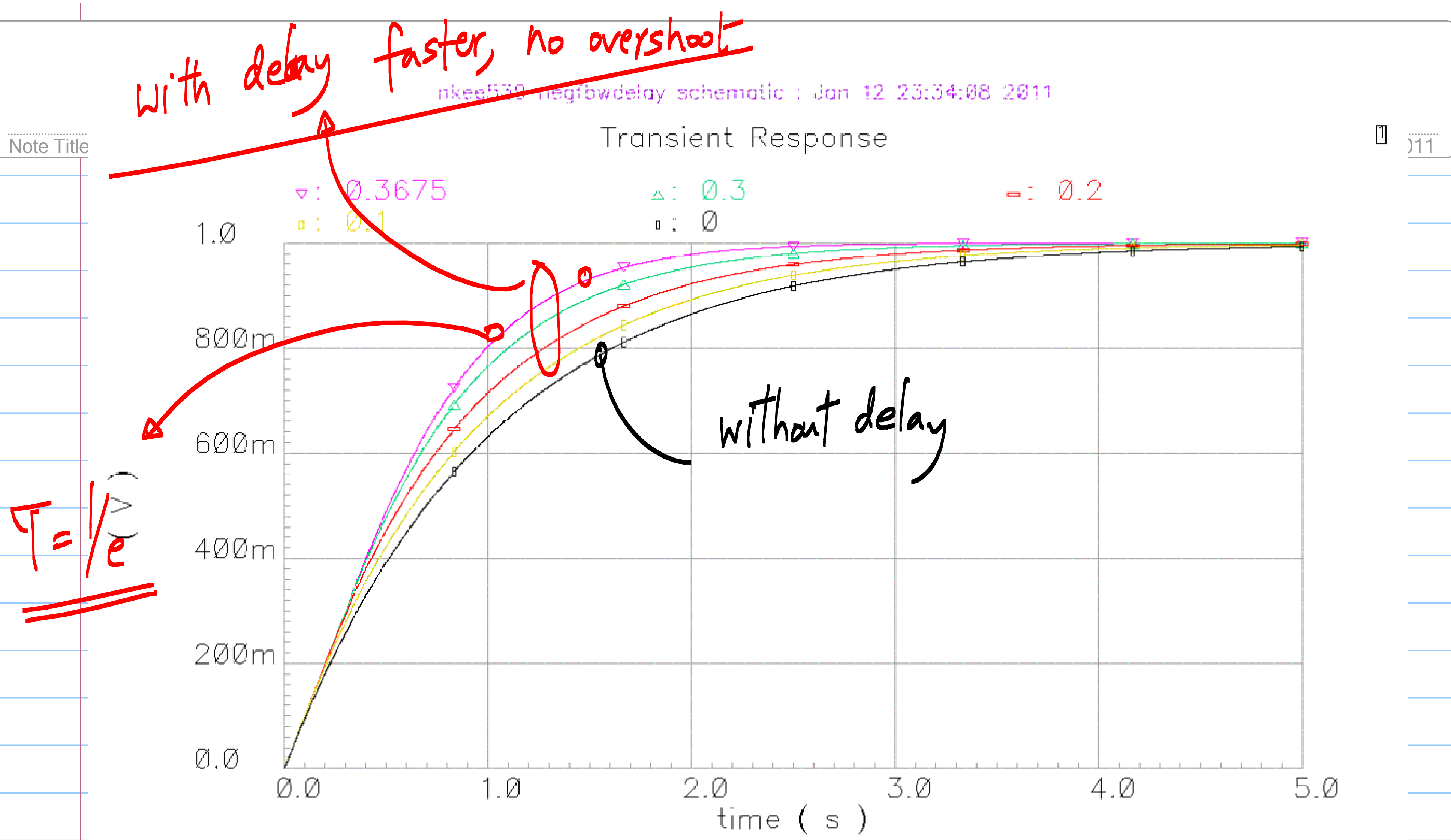
$$\sigma' = \frac{\sigma}{w_u/k}$$

$$T = \frac{T_d}{k/w_u}$$

$$\sigma = e \cdot \frac{w_u}{k}$$

$$2.718\,\frac{w_u}{k}$$

Note Title


with delay faster, no overshoot
nkee539 negfbwdelay schematic : Jan 12 23:34:08 2011
Transient Response
▽: 0.3675
△: 0.3
▭: 0.2
▯: 0.1
▯: 0
1.0
800m
600m
400m
200m
0.0
(V)
0.0
1.0
2.0
3.0
4.0
5.0
time (s)
without delay
T = 1/e

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$T > \frac{1}{e}$

$$\frac{dV_o}{dt} = -\frac{\omega_u}{k} \cdot V_o(t - T_d)$$

$V_o(t) = \exp(\sigma t)$ σ: real

$V_o(t) = \exp((\sigma + j\omega) t)$

$$(\sigma + j\omega) \cdot \exp((\sigma + j\omega) t) = -\frac{\omega_u}{k} \exp((\sigma + j\omega) t) \cdot \exp(-(\sigma + j\omega) \cdot T_d)$$

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$$\sigma + j\omega = -\frac{\omega_u}{k}\cdot \exp\left(-(\sigma+j\omega)\cdot T_d\right)$$

$$= -\frac{\omega_u}{k}\cdot \exp(-\sigma T_d)\cdot \exp(-j\omega T_d)$$

$$= -\frac{\omega_u}{k} \exp(-\sigma T_d)\left[\cos(\omega T_d) - j\sin(\omega T_d)\right]$$

$$\sigma = -\frac{\omega_u}{k} \exp(-\sigma T_d)\cos(\omega T_d)$$

$$\omega = \frac{\omega_u}{k} \exp(-\sigma T_d)\cdot \sin(\omega T_d)$$

Solve for σ, ω

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$$\sigma = -\frac{\omega_n}{k} \exp(-\sigma T_d) \cos(\omega T_d)$$

$$\omega = \frac{\omega_n}{k} \exp(-\sigma T_d) \sin(\omega T_d)$$

$$\sigma' = -\exp(-\sigma' T) \cos(\omega' T)$$

$$\omega' = \exp(-\sigma' T) \cdot \sin(\omega' T)$$

$$\sigma' = \frac{\sigma}{\omega_n/k}$$

$$\omega' = \frac{\omega}{\omega_n/k}$$

$$T = \frac{T_d}{k/\omega_n}$$

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$$\sigma' = -\exp(-\sigma' T)\cos(\omega' T) \quad (1)$$

$$\omega' = \exp(-\sigma' T)\cdot\sin(\omega' T) \quad (2)$$

$(1)^2 + (2)^2$:

$$\sigma'^2 + \omega'^2 = \exp(-2\sigma' T) \quad (3)$$

$\frac{(1)}{(2)}$

$$\frac{\sigma'}{\omega'} = -\frac{\cos(\omega' T)}{\sin(\omega' T)} \quad (4)$$

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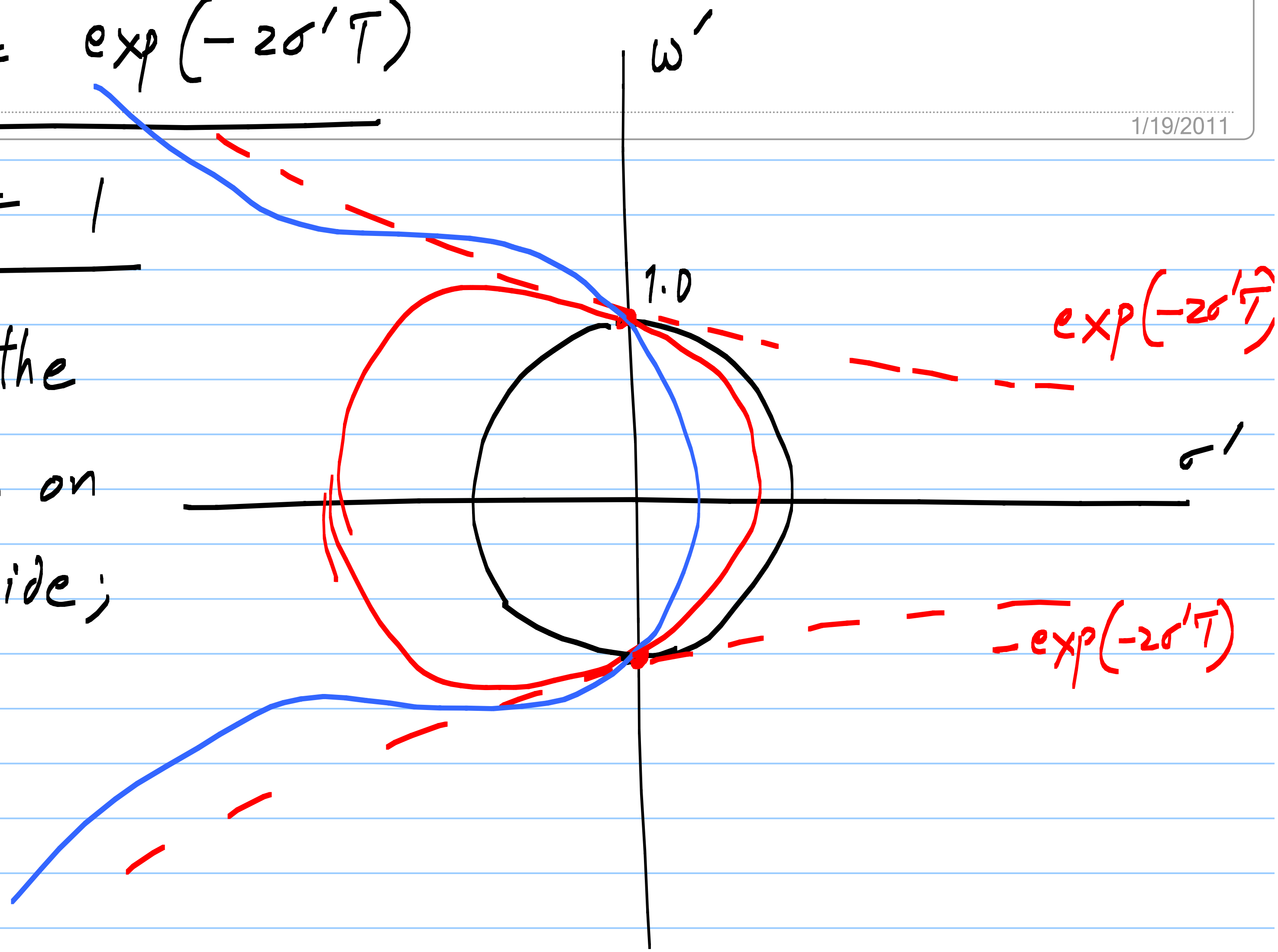
$$\sigma'^2 + \omega'^2 = \exp(-2\sigma' T)$$

$$\sigma'^2 + \omega'^2 = 1$$

For $T > 1/e$, the curve is open on the left side; passes through (0, 1) & (0, -1)



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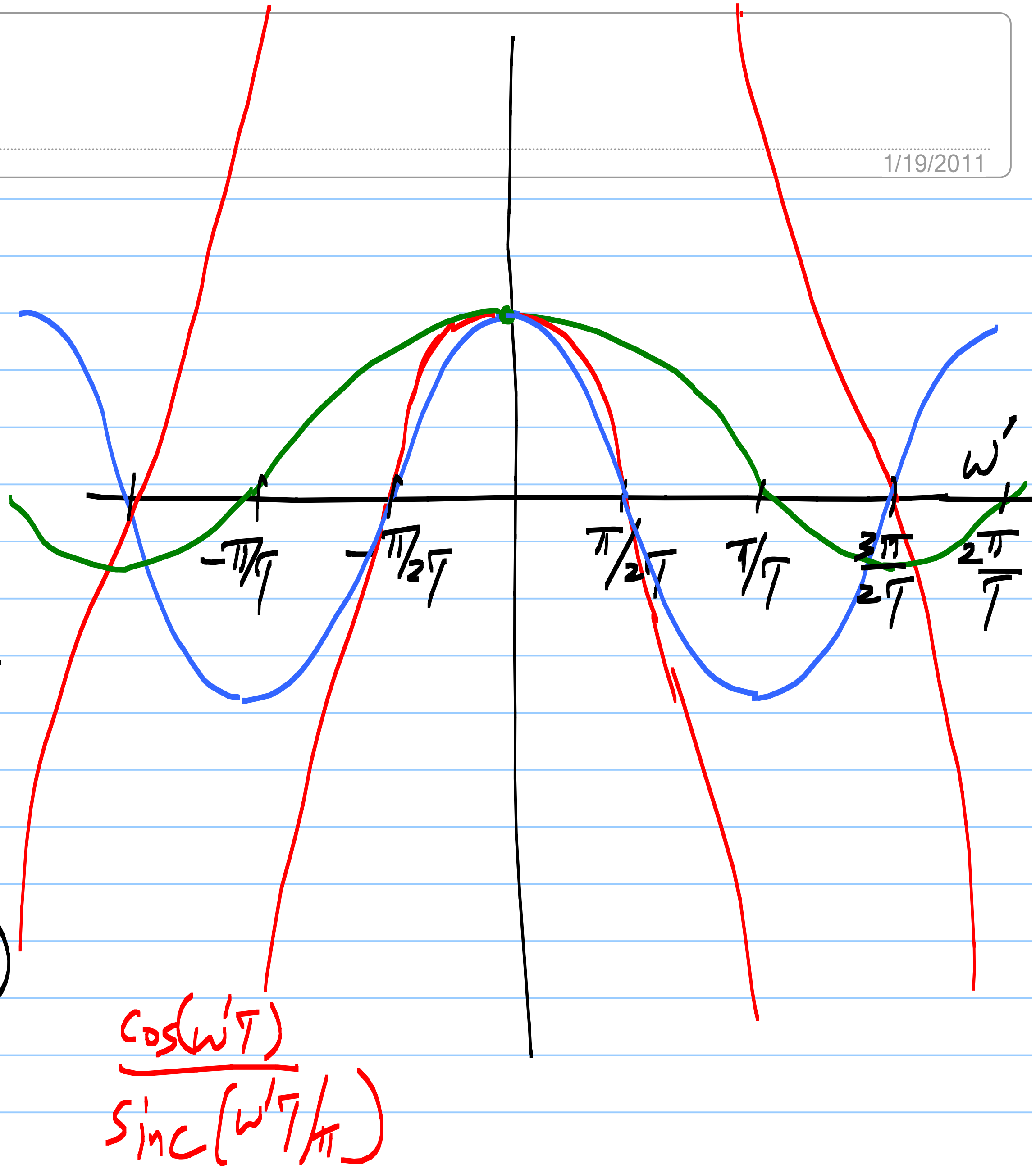
$$\frac{\sigma'}{\omega'} = -\frac{\cos(\omega' T)}{\sin(\omega' T)}$$

$$\sigma' = -\omega' \frac{\cos(\omega' T)}{\sin(\omega' T)}$$

$$= -\frac{\omega' T}{T} \cdot \frac{\cos(\omega' T)}{\sin(\omega' T)}$$

$$= -\frac{1}{T} \frac{\cos(\omega' T)}{\operatorname{sinc}\left(\frac{\omega' T}{\pi}\right)}$$



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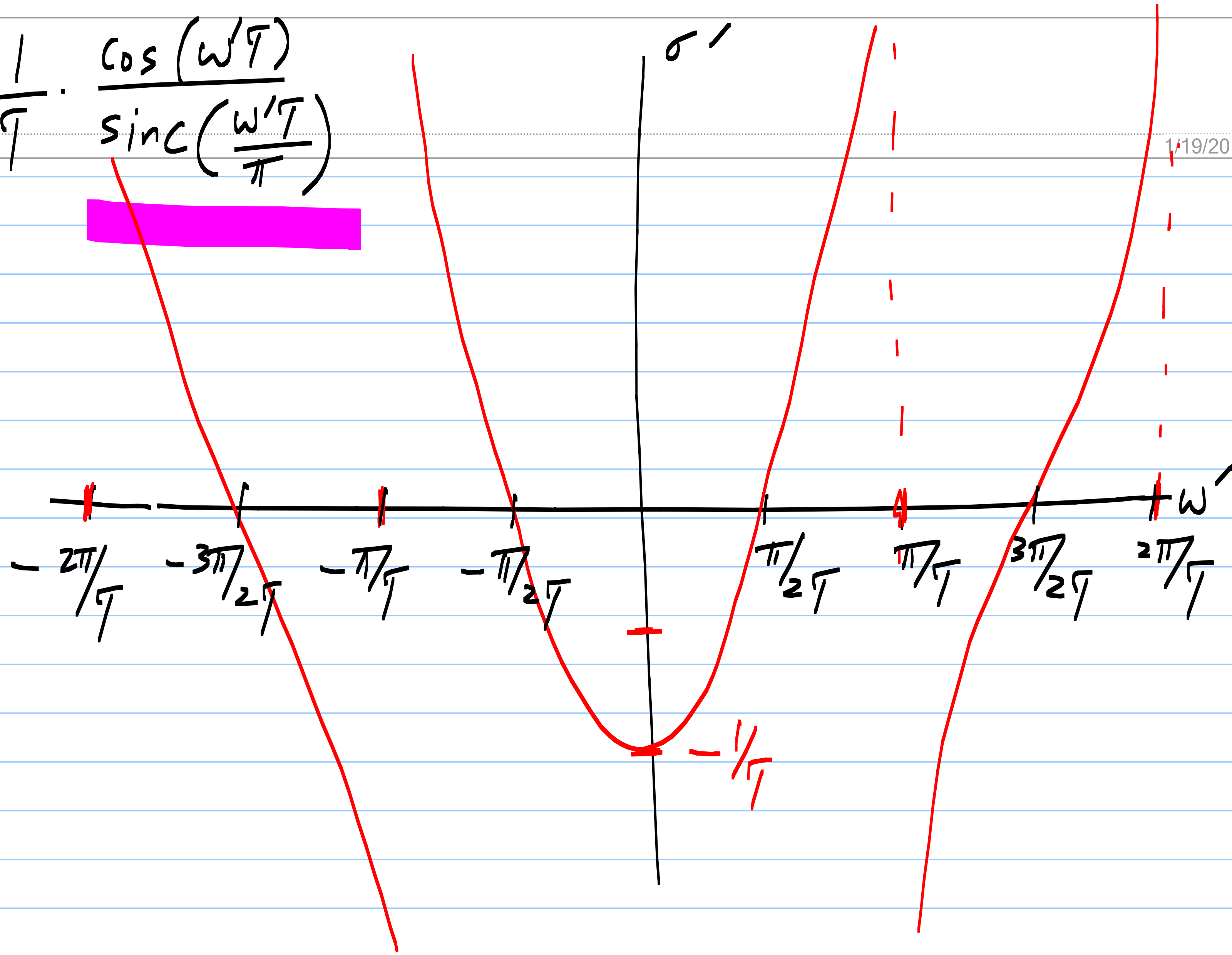
$$\sigma' = -\frac{1}{T} \cdot \frac{\cos(\omega' T)}{\mathrm{sinc}\left(\frac{\omega' T}{\pi}\right)}$$



$$\sigma'^2 + \omega'^2 = \exp(-2\sigma' T)$$

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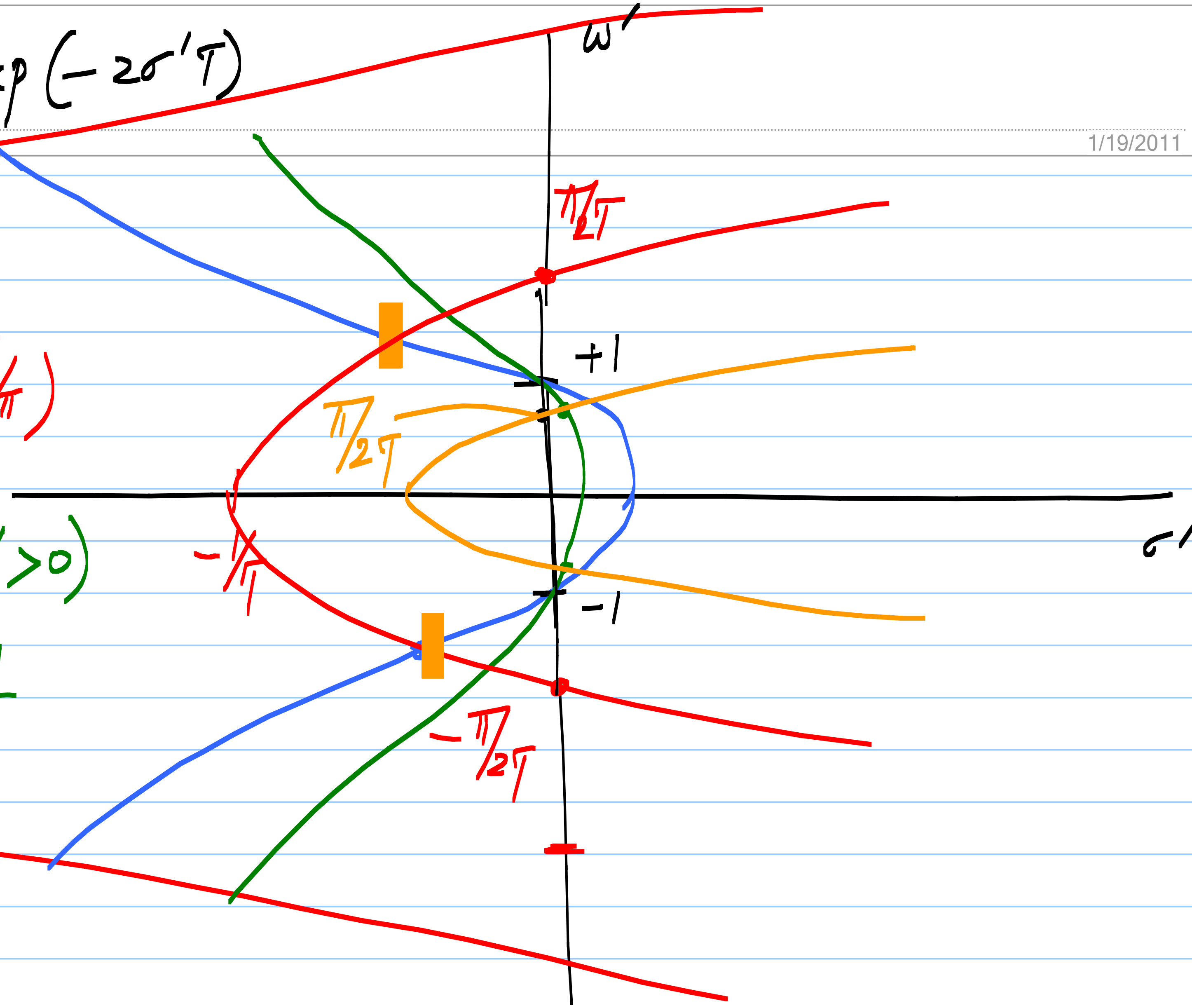
$$\sigma' = -\frac{1}{T} \cdot \frac{\cos \omega' T}{\text{sinc}(\omega' T/\pi)}$$



Instability $(\sigma' > 0)$

for $\pi/2T < 1$

$$T > \pi/2$$

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For $T > 1/e$:

* Infinite number of solutions

$\sigma_1', \sigma_2', \ldots$

$\omega_1', \omega_2', \ldots$

$$\sum_{k=1}^{\infty} A_k \exp((\sigma_k + j\omega_k)t)$$

$$\simeq A_1 \exp((\sigma_1 + j\omega_1)t)$$

σ_1, ω_1: lowest frequency solutions (closest to the origin)

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* $\sigma' > 0$ for $T > \pi/2$

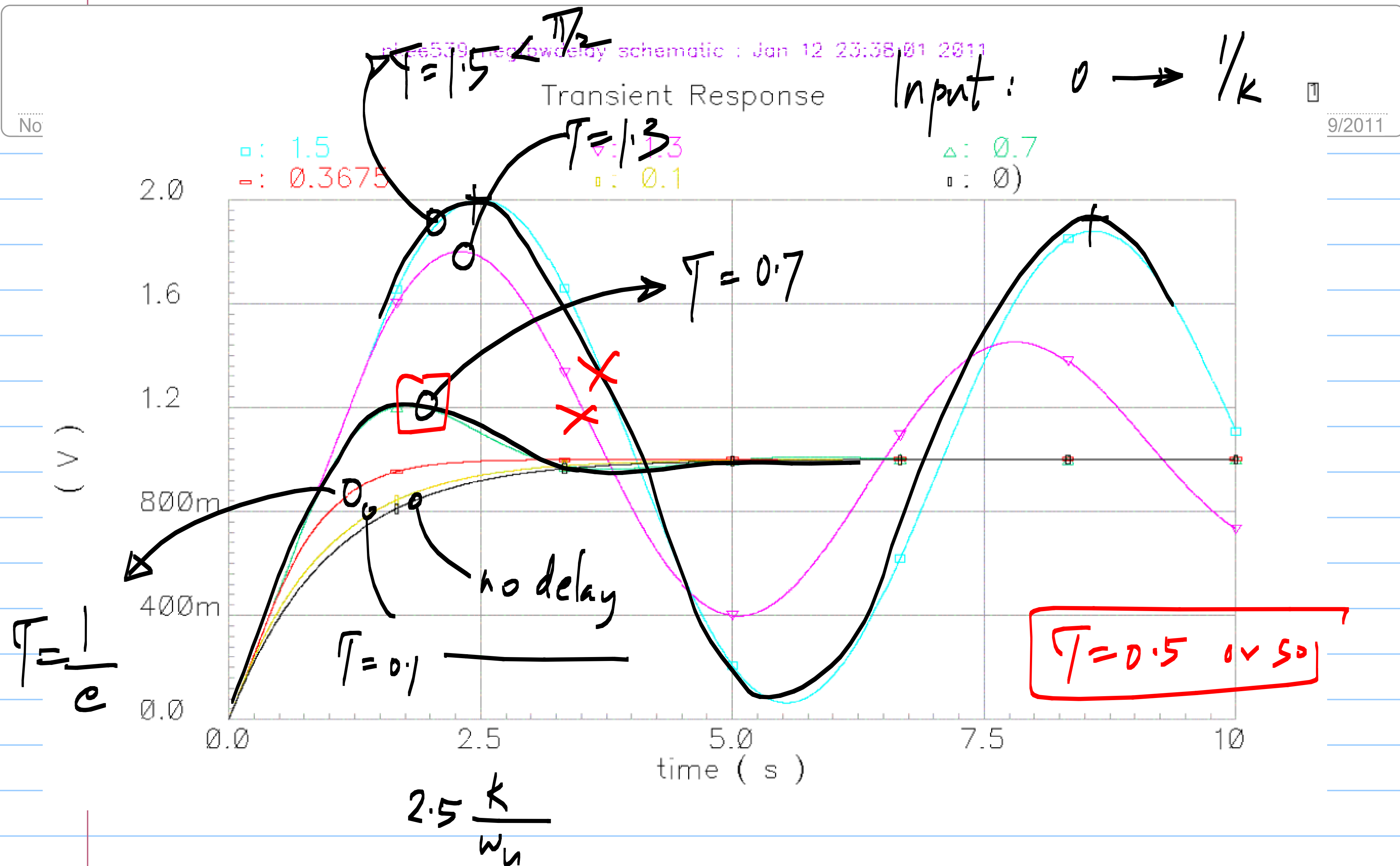
$$T_d > \frac{\pi}{2} \cdot \left(\frac{k}{\omega_n}\right)$$

$$\simeq 1.5 \left(\frac{k}{\omega_n}\right)$$

$$A_1 \exp((\sigma_1 + j\omega_1)t) + A_2 \exp((\sigma_1 - j\omega_1)t)$$

$$\exp(\sigma_1 t) \cdot 2 \cos(\omega_1 t)$$

$$a_1 \exp(\sigma_1 t) \cos(\omega_1 t) + a_2 \exp(\sigma_1 t) \sin(\omega_1 t)$$

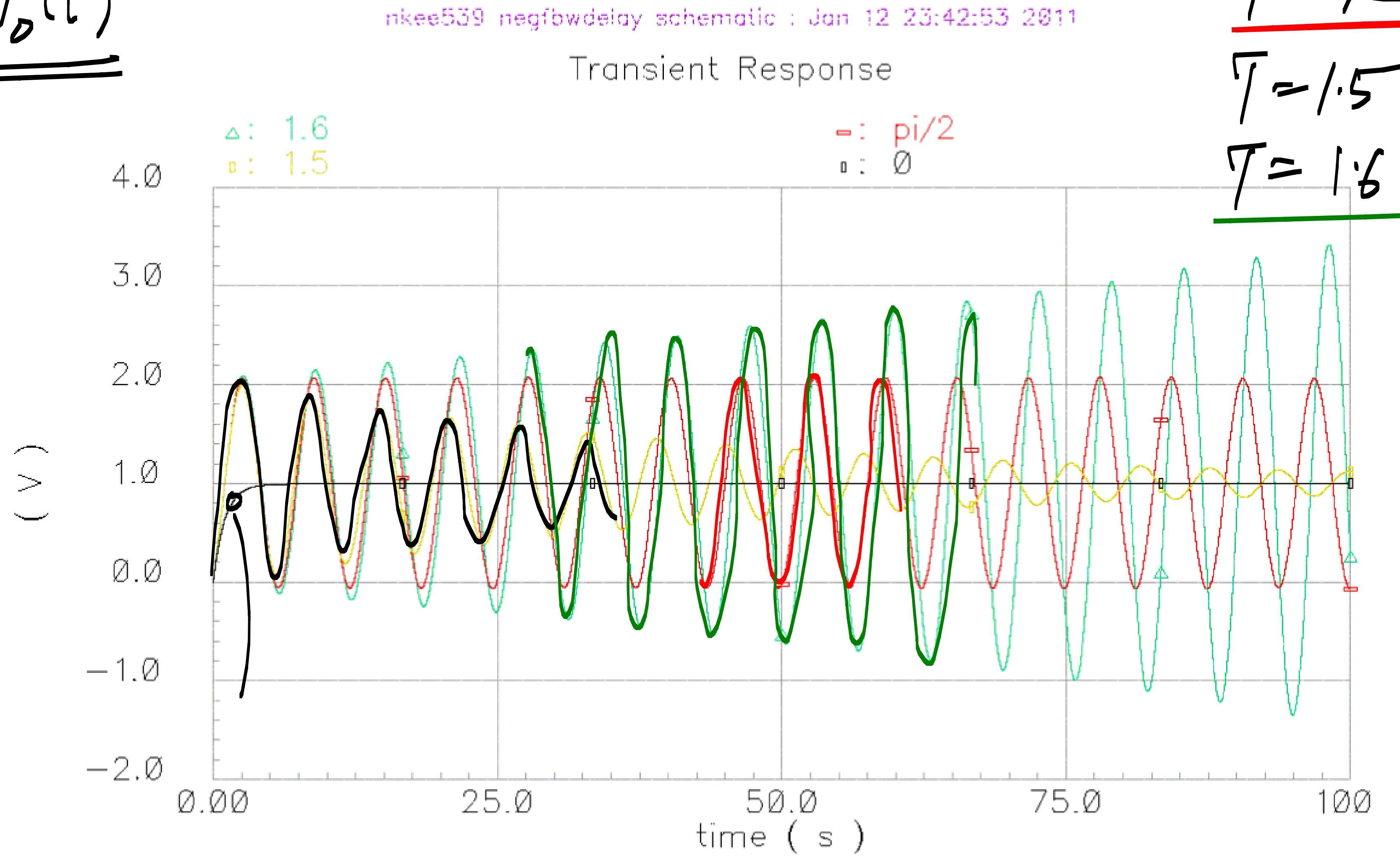

schematic : Jan 12 23:38:01 2011
Transient Response
1.5
0.3675
1.3
0.1
0.7
0)
2.0
1.6
1.2
800m
400m
0.0
(V)
0.0
2.5
5.0
7.5
10
time (s)
T=1.5 < π/2
T=1.3
T = 0.7
Input: 0 → 1/k
no delay
T=0.1
T=1/e
T=0.5 or 50
2.5 k/ωu

$V_o(t)$

$T = \pi/2$

$T = 1.5$

$T = 1.6$



$T = \pi/2$: $\sigma' = 0$, $\omega' = \pm 1$

Negative feedback amplifier with delay

* Small delays speed up the response
* $\tau = 1/e$: $\left(T_d = \frac{1}{2.718}\frac{k}{\omega_u}\right)$, fastest response without overshoot
* $\frac{1}{e} < \tau < \pi/2$: stable response, with ringing
* $\tau > \pi/2$: Unstable
* In practice $\tau < 0.5$