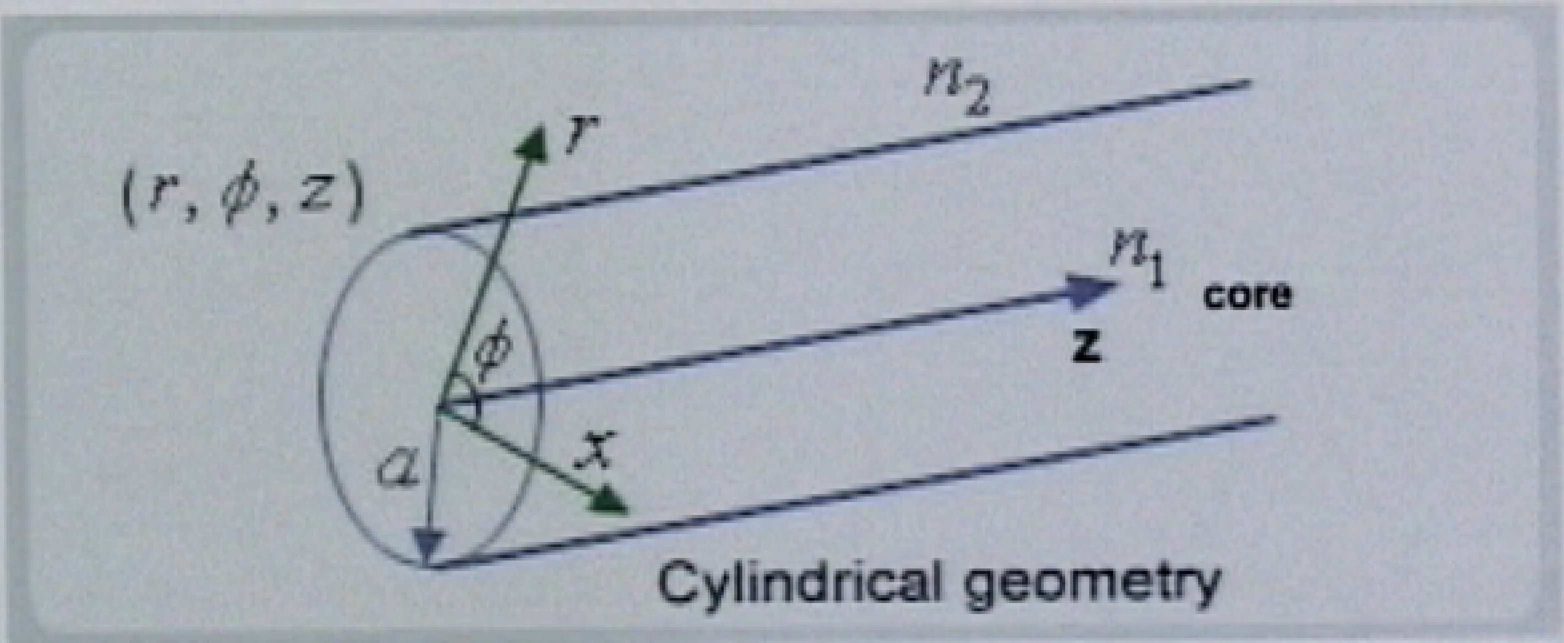
Wave Propagation in Optical Fibre



$$\varepsilon_1 = \varepsilon_0 n_1^{\,2}$$

$$\varepsilon_2 = \varepsilon_0 n_2^{\,2}$$

ε_0 = free space permittivity

$\mu = \mu_0$ = free space permeability

Maxwell's Equation in a source free medium

(a) $\nabla.\bar{D} = 0$

(b) $\nabla.\bar{B} = 0$

(c) $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$

(d) $\nabla \times \bar{H} = -\frac{\partial \bar{D}}{\partial t}$

D = electric displacement vector

B = magnetic flux density

E = electric field

H = magnetic field

$$D = \varepsilon E$$

$$B = \mu H$$

$$\nabla \times \nabla \times \overline{\overline{E}} = -\nabla \times \frac{\partial \overline{B}}{\partial t}$$

$$\nabla \times \nabla \times E = -\frac{\partial}{\partial t} \nabla \times (\mu \overline{H})$$

$$= -\mu \frac{\partial}{\partial t} (\nabla \times \overline{H})$$

$$\nabla \times \nabla \times E = -\mu \frac{\partial}{\partial t} \left(\frac{\partial D}{\partial t} \right)$$

$$= -\mu \varepsilon \frac{\partial}{\partial t} \cdot \frac{\partial}{\partial t} \cdot \overline{E}$$

$$\nabla \times \nabla \times \bar{E} = -\mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\nabla \times \nabla \times \bar{E} = \nabla\left(\nabla \cdot \bar{E}\right) - \nabla^2 \bar{E}$$

$$\nabla\left(\nabla \cdot \bar{E}\right) - \nabla^2 \bar{E} = -\mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\nabla \cdot \bar{E} = 0$$

$$\boxed{\nabla^2 \bar{E} = \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2}}$$

Wave Equation

$$\boxed{\nabla^2 \bar{H} = \mu\varepsilon \frac{\partial^2 \bar{H}}{\partial t^2}}$$

Transverse Field components

$$E_r = \frac{-j}{q^2}\left\{\beta\frac{\partial E_z}{\partial r} + \frac{\mu\omega}{r}\frac{\partial H_z}{\partial \phi}\right\}$$

$$E_\phi = \frac{-j}{q^2}\left\{\frac{\beta}{r}\frac{\partial E_z}{\partial \phi} - \mu\omega\frac{\partial H_z}{\partial r}\right\}$$

$$H_r = \frac{-j}{q^2}\left\{\beta\frac{\partial H_z}{\partial r} - \frac{\omega\varepsilon}{r}\frac{\partial E_z}{\partial \phi}\right\}$$

$$H_\phi = \frac{-j}{q^2}\left\{\frac{\beta}{r}\frac{\partial H_z}{\partial \phi} + \omega\varepsilon\frac{\partial E_z}{\partial r}\right\}$$

Independent components

Longitudinal

$$E_z \,, \quad H_z$$

If $E_z = 0$, $H_z \neq 0 \Rightarrow$ TE

If $E_z \neq 0$, $H_z = 0 \Rightarrow$ TM

If $E_z \neq 0$, $H_z \neq 0 \Rightarrow$ Hybrid mode

- Solve wave equation for

E_z and H_z

Define $\psi \equiv E_z$ or H_z

Wave Equation $$\boxed{\nabla^2 \psi = \mu\varepsilon \frac{\partial^2 \psi}{\partial t^2}}$$

Wave Equation in cylindrical coordinates

$$\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\psi}{\partial\phi^2}+\frac{\partial^2\psi}{\partial z^2}=\mu\varepsilon\frac{\partial^2\psi}{\partial t^2}$$

$$\psi - e^{j\omega t}$$

$$\frac{\partial}{\partial t}=j\omega$$

$$\frac{\partial^2}{\partial t^2}=-\omega^2$$

$$\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\psi}{\partial\phi^2}+\frac{\partial^2\psi}{\partial z^2}=-\omega^2\mu\varepsilon\psi$$

$$\psi = R(r)\Phi(\phi)Z(z)$$

$\phi(\phi) = e^{jv\phi}$ *where* v is integer

$$\Rightarrow \partial/\partial\phi = jv,$$

$$\partial^2/\partial\phi^2 = -v^2$$

$Z(z) = e^{-j\beta z}$ **Traveling wave in +ve z direction**

$$\partial/\partial z = -j\beta,$$

$$\frac{\partial^2}{\partial z^2} = -\beta^2$$

$$\psi = R(r)\Phi(\phi)Z(z)$$

$\phi(\phi) = e^{j\nu\phi}$ *where* ν is integer

$$\Rightarrow \partial/\partial\phi = j\nu,$$

$$\partial^2/\partial\phi^2 = -\nu^2$$

$Z(z) = e^{-j\beta z}$ **Traveling wave in +ve z direction**

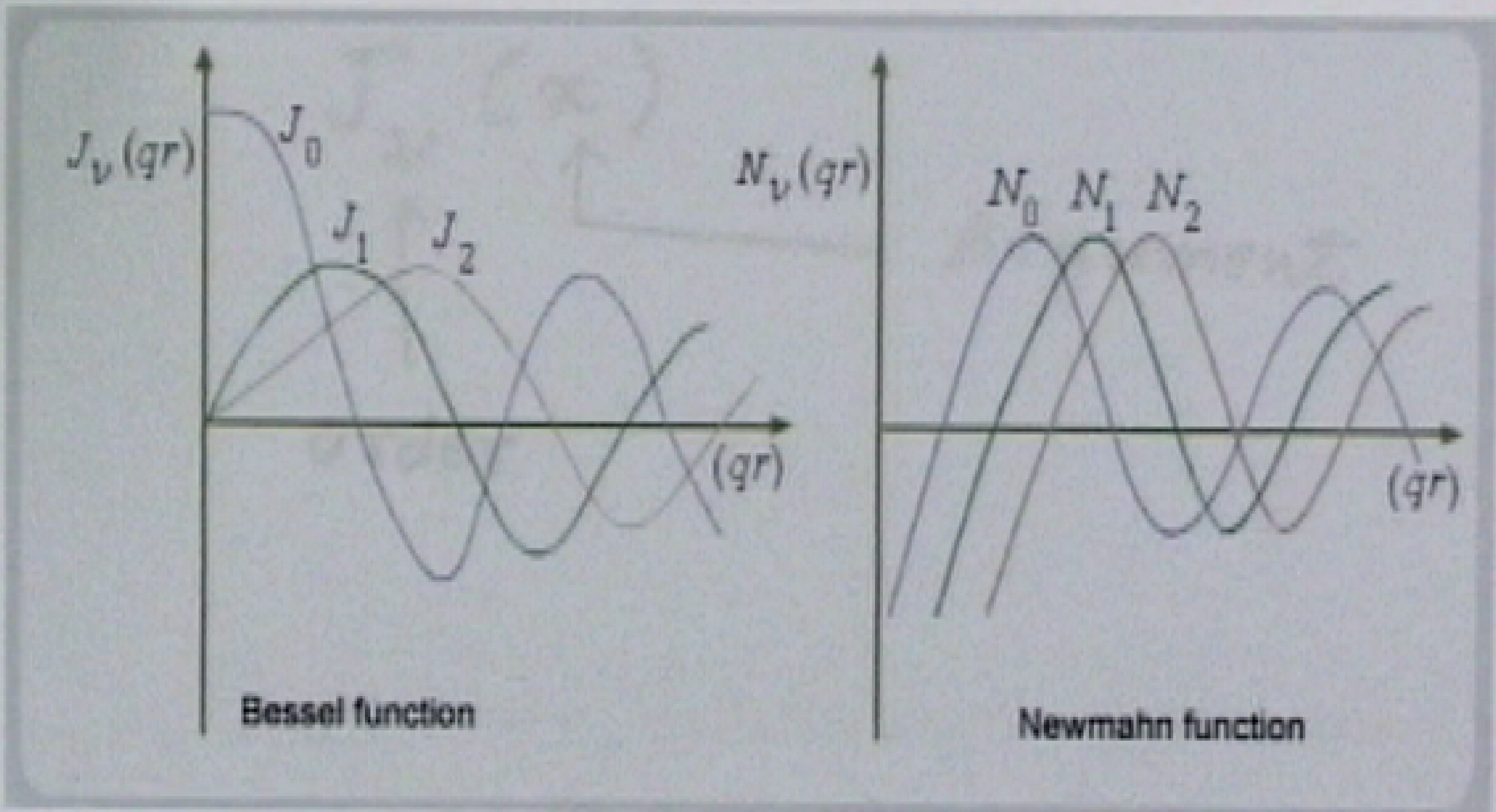
$$\partial/\partial z = -j\beta,$$

$$\frac{\partial^2}{\partial z^2} = -\beta^2$$

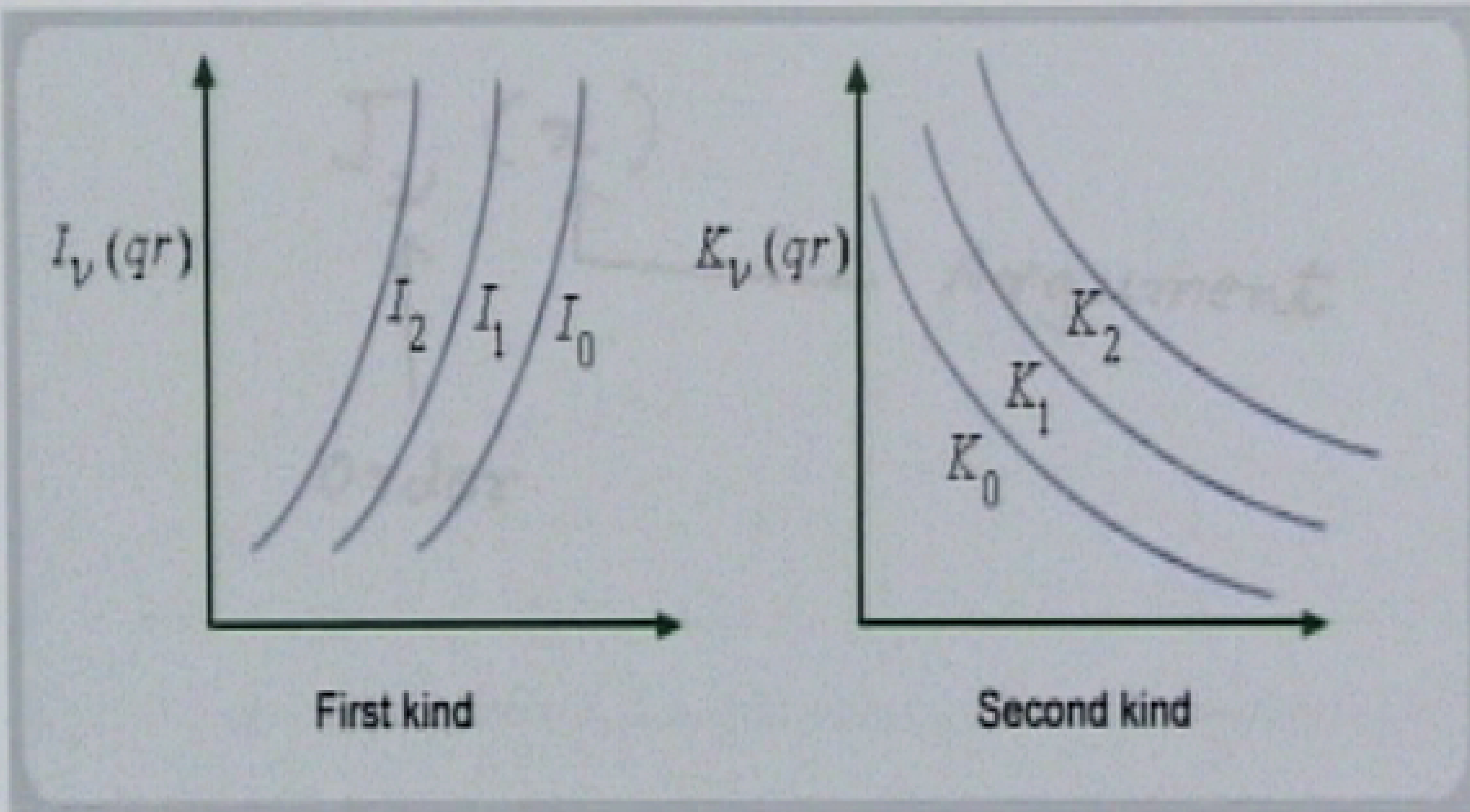
$$\frac{d^2R}{dr^2}+\frac{1}{r}\frac{dR}{dr}+\left\{-\frac{\nu^2}{r^2}-\beta^2+\omega^2\mu\varepsilon\right\}R=0$$

$$\frac{d^2R}{dr^2}+\frac{1}{r}\frac{dR}{dr}+\left(q^2-\frac{\nu^2}{r^2}\right)R=0$$ **Bessel's Equation**

$$\Rightarrow \text{where } q^2=\omega^2\mu\varepsilon-\beta^2$$



$J_\nu(qr)$
J_0
J_1
J_2
(qr)
Bessel function
$N_\nu(qr)$
N_0
N_1
N_2
(qr)
Newmahn function



Modified Bessel Fn.

$$J_\nu(x)$$

ν ← Order

x ← Argument

Inside Core $(r < a)$

Electric field:

$$E_{z1} = AJ_{\nu}(ur)e^{j\nu\phi - j\beta z + j\omega t}$$

Magnetic field:

$$H_{z1} = BJ_{\nu}(ur)e^{j\nu\phi - j\beta z + j\omega t}$$

$$u = \sqrt{\beta_1^2 - \beta^2}$$

$$\beta_1^2 = \omega^2 \mu\varepsilon_1 = \omega^2 \mu\varepsilon_0 n_1^2 = \beta_0^2\, n_1^2$$

In Cladding $(r > a)$

Electric field:

$$E_{z2} = CK_{\nu}(wr)e^{j\nu\phi - j\beta z + j\omega t}$$

Magnetic field:

$$H_{z2} = DK_{\nu}(wr)e^{j\nu\phi - j\beta z + j\omega t}$$

$$w = \sqrt{\beta^2 - \beta_2^2}$$

$$\beta_2^2 = \omega^2 \mu \varepsilon_2 = \omega^2 \mu \varepsilon_0 n_2^2 = \beta_0^2\, n_2^2$$

$$\beta_0 n_2 = \beta_2 < \beta < \beta_1 = \beta_0 n_1$$