

$$E = E_1 e^{j\omega_1 t} + E_2 e^{j\omega_2 t}$$

$$P_{NL} = P_{NL}(\omega_1) e^{j\omega_1 t} + P_{NL}(\omega_2) e^{j\omega_2 t} \\ + P_{NL}(2\omega_1 - \omega_2) e^{j(2\omega_1 - \omega_2)t}$$

$$P_{NL}(\omega_1) = \chi^{(3)} \left\{ |E_1|^2 + 2|E_2|^2 \right\} E_1$$

$$P_{NL}(\omega_2) = \chi^{(3)} \left\{ 2|E_1|^2 + |E_2|^2 \right\} E_2$$

$$\Delta n_j \approx n_2 \left\{ |E_j|^2 + 2|E_{3-j}|^2 \right\}$$

Cross Phase Modulation (XPM)

$$E_j = F_j(x, y) A_j(z) e^{-j\beta_{0j}z}$$

NLS :

$$\frac{\partial A_j}{\partial z} + \underbrace{\beta_{1j} \frac{\partial A_j}{\partial t}}_{\text{G. vel}} + j \frac{\beta_{2j}}{2} \frac{\partial^2 A_j}{\partial t^2} + \frac{\alpha_j}{2} A_j = -j \frac{n_2 \omega_j}{c} \left\{ \underbrace{f_{jj}}_{\substack{\downarrow \\ \text{Overlap integral}}} |A_j|^2 + 2 f_{jk} |A_k|^2 \right\}$$

Overlap integral

Overlap integral

$$f_{jk} = \frac{\iint |F_j(x, y)|^2 |F_k(x, y)|^2 dx dy}{\iint |F_j(x, y)|^2 dx dy \iint |F_k(x, y)|^2 dx dy}$$

for $j = k$, $\frac{1}{A_{\text{eff}}}$

$$\frac{\partial A_1}{\partial z} + \frac{1}{v_{g1}} \frac{\partial A_1}{\partial t} - j \frac{\beta_{21}}{2} \frac{\partial^2 A_1}{\partial t^2} + \frac{\alpha_1}{2} A_1$$
$$= -\gamma_1 \left\{ |A_1|^2 + 2|A_2|^2 \right\} A_1$$

$$\gamma = \frac{n_2 \omega_1}{c A_{\text{eff}}}$$

$$P_{NL} = \epsilon_0 \chi^{(3)} : E_1 E_2 E_3$$

$$\omega_4 = \omega_1 \pm \omega_2 \pm \omega_3$$

$$k_4 = k_1 \pm k_2 \pm k_3$$

$$\omega_1 \quad \omega_2 \quad \omega_3$$

$$\omega_1 + \omega_2 - \omega_3, \quad \omega_1 + \omega_3 - \omega_2, \quad \omega_3 + \omega_2 - \omega_1$$

$$2\omega_1 - \omega_2, \quad 2\omega_1 - \omega_3, \quad 2\omega_2 - \omega_1, \quad 2\omega_2 - \omega_3$$

$$2\omega_3 - \omega_1, \quad 2\omega_3 - \omega_2$$

Four wave mixing
(Inter modulation)



$$\omega_2 = \omega_1 + 2\Delta\omega$$

$$\omega_3 = \omega_1 + \Delta\omega$$

$$\begin{aligned} \omega_4 &= \omega_1 + \omega_2 - \omega_3 = \omega_1 + \omega_1 + 2\Delta\omega \\ &\quad - \omega_1 - \Delta\omega \\ &= \omega_1 + \Delta\omega = \omega_3 \end{aligned}$$

Cross talk between the WDM channels

