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Induced Polarization

$$P = \epsilon_0 \left\{ \chi^{(1)} \cdot \bar{E} + \chi^{(2)} : \bar{E}\bar{E} + \chi^{(3)} : \bar{E}\bar{E}\bar{E} + \cdots \right\}$$

- $\chi^{(1)}$ → Dominant term (Dielectric const)
- $\chi^{(2)}$ → For SiO_2 is small
- $\chi^{(3)}$ → Non-linearity

$$\bar{n}(\omega, |E|^2) = \bar{n}(\omega) + n_2 |E|^2$$

n_2 ↑ Non-linearity Coeff (Kerr Non-lin).

$$T = t - \frac{z}{v_g} = t - \beta_1 z \qquad \beta_1 = 1/v_g$$

$$\beta_1 \frac{\partial A}{\partial t} = \beta_1 \frac{\partial A}{\partial T} \cdot \frac{\partial T}{\partial t} = \beta_1 \frac{\partial A}{\partial T}(1 - \beta_1 v_g)$$

$$= 0$$

$$\frac{\partial A}{\partial z} \underbrace{- j \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2}}_{\text{Dispersion}} + \underbrace{\frac{\alpha}{2} A}_{\text{Loss.}} = \underbrace{-j \gamma |A|^2 A}_{\text{Non-linearity}}$$

Non-linear Schrödinger Equation

(NLS)

① $L \ll L_D$, $L \ll L_{NL}$ – Fiber is just a medium to transport light

② $L \gg L_D$, $L \ll L_{NL}$ → Dispersion

Pulse broadening

Group velocity Limited regime.

(GVD)

③ $L \ll L_D$, $L \gg L_{NL}$ → Self Phase Modulation (SPM)

Non-linearity Limited regime

④ $L \gg L_D$, $L \gg L_{NL}$ → 'Soliton'

Gaussian Pulse, st. deviation T_0

$$A(T) = e^{-T^2/2T_0^2}$$

Dispersion Length $L_D \equiv \frac{T_0^2}{|\beta_2|}$

Non-linearity Length $L_{NL} = \frac{1}{\gamma P}$

↑ Pulse power

Physical length L of a fiber with $\alpha = 0$

At 1550 nm, $\beta_2 = -20\ ps^2/km$

$\gamma = 2\ W^{-1}/km$

Data rate ~~&~~ 10 Gbps, $P_0 \sim 3\ dBm$

$= 2\ mW$

$= 2 \times 10^{-3}\ W$

$T_0 = 100\ ps$

$$L_D = \frac{|100\ ps|^2}{20} = 500\ km$$

$$L_{NL} = \frac{1}{\gamma P_0} = \frac{1}{2 \times 2 \times 10^{-3}} \approx 250\ km$$

$L \sim 100\ km$ $L \ll L_D$, $L \ll L_{NL}$

$\Rightarrow$ FT limited spectrum

$40\ Gbps \quad , T_0 = 25\ ps$

$$L_D = \frac{625}{20} \simeq 30\ km$$

$\underline{L >> L_D}$, $L << L_{NL}$

$P_0 \simeq 20\ mW \qquad L_{NL} \approx 25\ km$

Group Velocity Dispersion
(GVD)

Normalized Amplitude $U(z, T)$

$$A(z,T) = \sqrt{P_0}\, e^{-\alpha z/2}\, U(z,T)$$

$$\frac{\partial A}{\partial z} = \sqrt{P_0}\left\{ -\frac{\alpha}{2} e^{-\alpha z/2} U + e^{-\alpha z/2} \frac{\partial U}{\partial z} \right\}$$

$$\frac{\partial U}{\partial z} - j\frac{\beta_2}{2}\frac{\partial^2 U}{\partial T^2} = -j\frac{e^{-\alpha z}}{L_{NL}} |U|^2 U$$

$$L_{NL} \leftarrow \frac{1}{\gamma P_0}$$

$$\frac{\partial U}{\partial z} = j \frac{\beta_2}{2} \frac{\partial^2 U}{\partial T^2}$$

$$\tilde{U}(z, \omega) = \int_{-\infty}^{\infty} U(z, T)\, e^{-j\omega T}\, dT$$

$$U(z, T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{U}(z, \omega)\, e^{j\omega T}\, d\omega$$

$$\frac{\partial \tilde{U}}{\partial z} = j \frac{\beta_2}{2} \left\{ -\omega^2 \tilde{U} \right\}$$

$$\tilde{U}(z, \omega) = \tilde{U}(0, \omega)\, e^{-j \frac{\beta_2}{2} \omega^2 z}$$

Phase $\propto \omega^2$

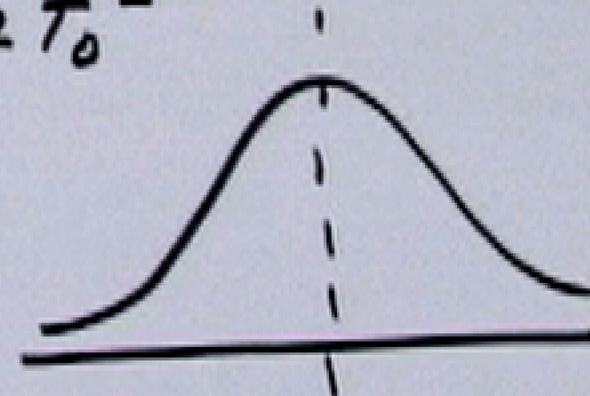
$\propto z$

$$U(z,T) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{U}(z,\omega) \exp\left\{ j\omega T - j\frac{\beta_2}{2}\omega^2 z \right\} d\omega$$

<u>Gaussion pulse</u>

$$U(0,T) = e^{-T^2/2T_0^2}$$

$$\tilde{U}(0,\omega) = \sqrt{2\pi}\, T_0\, e^{-\frac{T_0^2\omega^2}{2}}$$



$T_{FWHM} \approx 1.66\, T_0$

$$\int_{-\infty}^{\infty} \exp\left\{-ax^2 + bx\right\} dx = \sqrt{\frac{\pi}{a}}\, \exp\left\{-\frac{b^2}{4a}\right\}$$

$$u(z,T) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \sqrt{2\pi}\, T_0\, e^{-\frac{\omega_0^2 T_0^2}{2}} \cdot e^{-j\frac{\beta_2 \omega^2 z}{2}}\, e^{j\omega T}\, d\omega$$

$$a = \frac{T_0^2}{2} + j\frac{\beta_2}{2} z$$

$$b = jT$$

$$u(z,T) = \frac{\sqrt{2\pi}\, T_0}{2\pi}\sqrt{\frac{2\pi}{T_0^2 + j\beta_2 z}}\, \exp\left\{\frac{-(jT)^2}{2(T_0^2 + j\beta_2 z)}\right\}$$

$$u(z,T) = e^{-T^2/2T_1^2} \cdot e^{j\phi}$$

$$T_1(z) = T_0\left\{1 + \left(\frac{z}{L_D}\right)^2\right\}^{1/2}, \quad L_D \equiv \frac{T_0^2}{|\beta_2|}$$

at $L = L_D$ $\quad T_1 = \sqrt{2}\, T_0$

$$\phi = \frac{sgn(\beta_2)(z/L_D)}{1+(z/L_D)^2}\frac{T^2}{2T_0^2} - \frac{1}{2}\tan^{-1}\left(\frac{z}{L_D}\right)$$

Change in frequency

$$\delta\omega = \frac{d\phi}{dT}$$

$$\delta\omega = \frac{sgn(\beta_2)(z/L_D)}{1+(z/L_D)^2}\cdot\frac{T}{T_0^2}$$

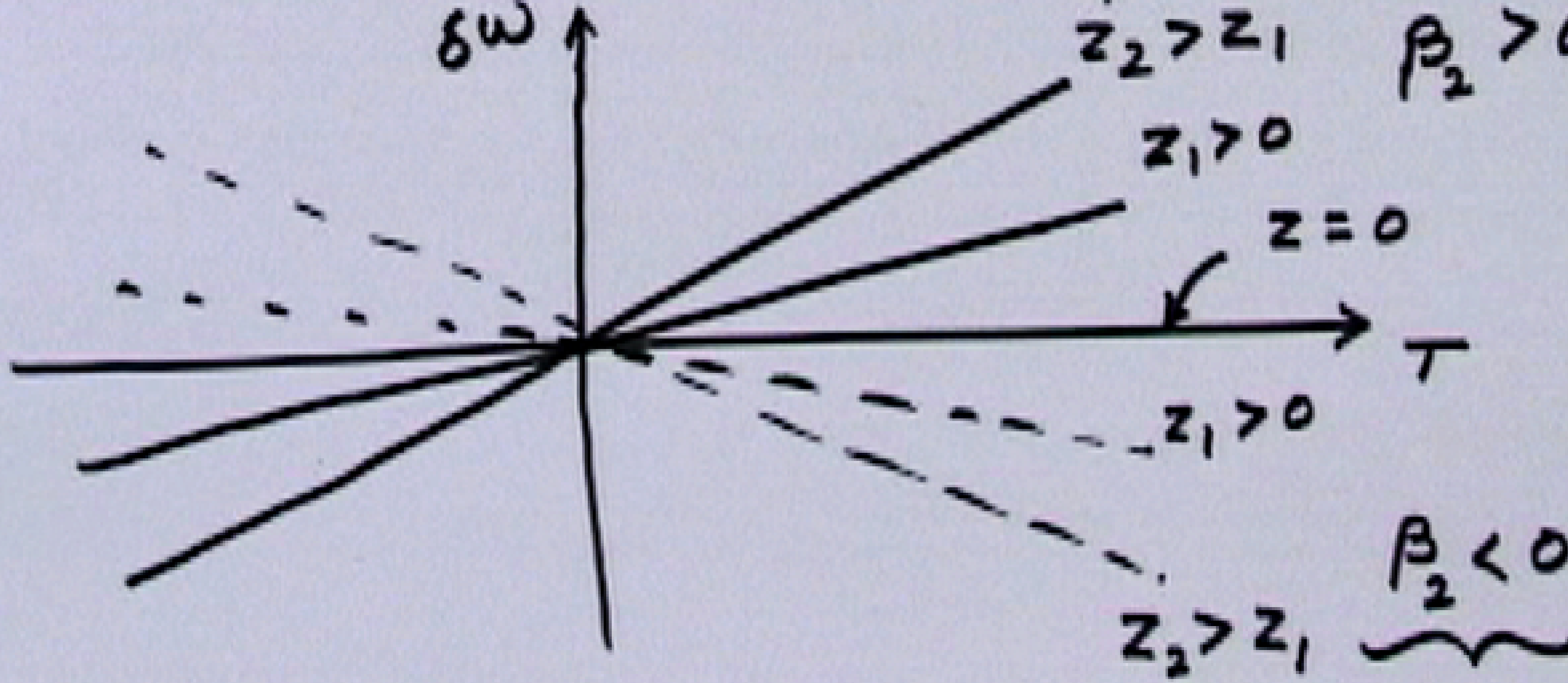
$$\delta\omega \propto T \longrightarrow \text{frequency chirp}$$

$\beta_2 > 0$ Normal dispersion

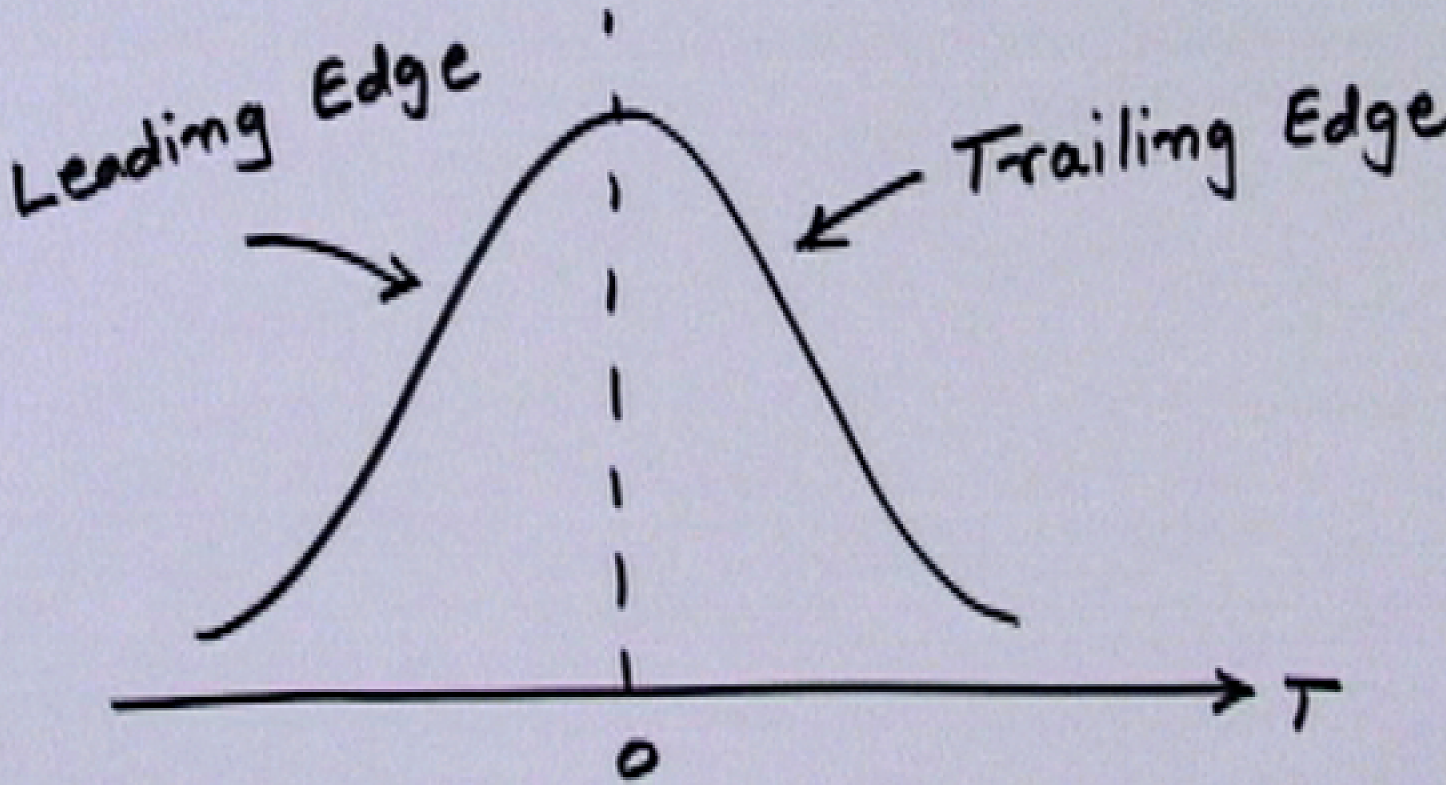
For $\lambda < 1300$ nm

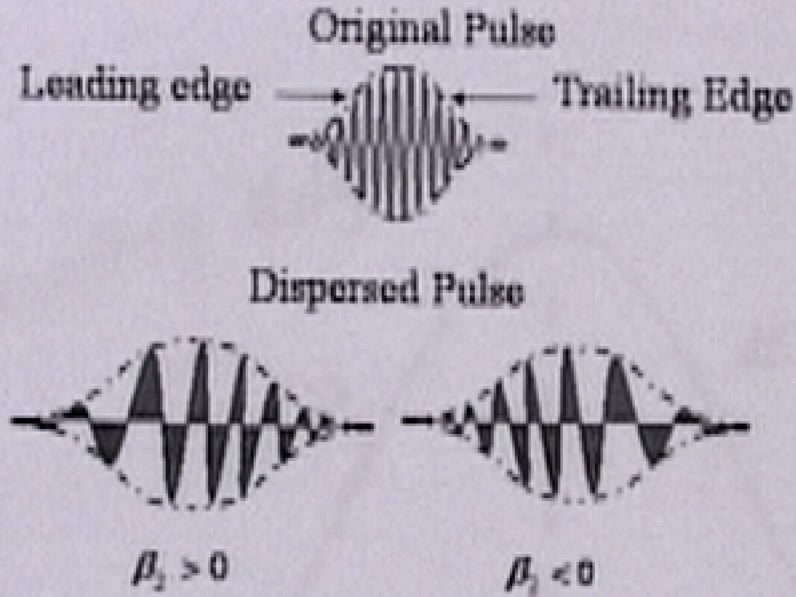
$\beta_2 < 0$ $\lambda > 1300$ nm

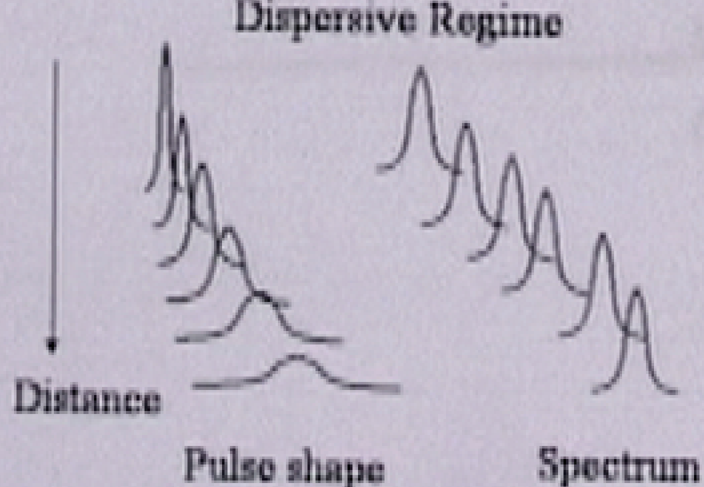
Anomalous dispersion

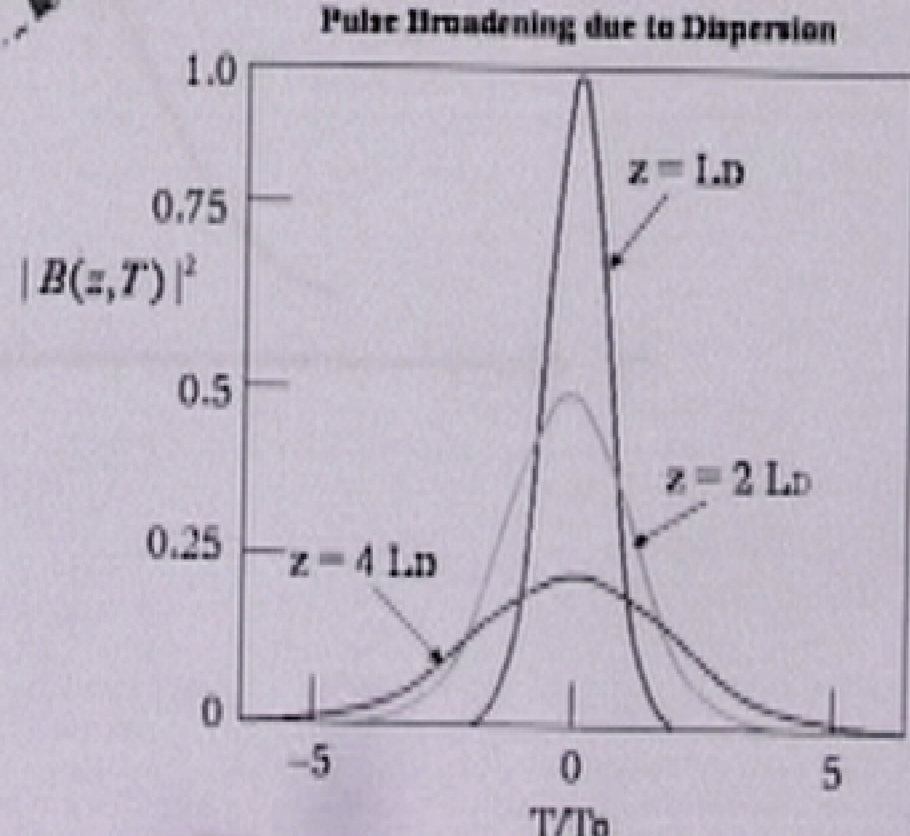


Anomalous dispersion 1550 nm


Leading Edge
Trailing Edge
0
T


Original Pulse
Leading edge
Trailing Edge
Dispersed Pulse
$\beta_2 > 0$
$\beta_2 < 0$


Dispersive Regime
Distance
Pulse shape
Spectrum


Pulse Broadening due to Dispersion
$|B(z,T)|^2$
1.0
0.75
0.5
0.25
0
−5
0
5
T/T0
z = LD
z = 2 LD
z = 4 LD