# Induced Polarization

$$P = \epsilon_0 \left\{ \chi^{(1)} \cdot \bar{E} + \chi^{(2)} : \bar{E}\bar{E} + \chi^{(3)} : \bar{E}\bar{E}\bar{E} + \cdots \right\}$$

$\chi^{(1)}$: Dominant term (Dielectric const)

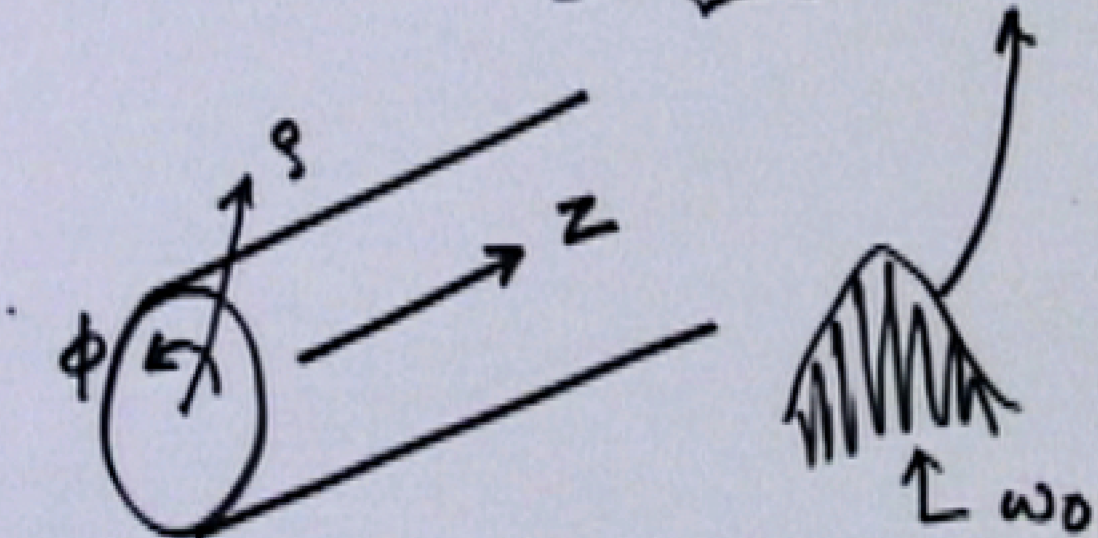
$\chi^{(2)}$: For $SiO_2$ is small

$\chi^{(3)}$: Non-linearity

$$\bar{n}(\omega, |E|^2) = \bar{n}(\omega) + n_2 |E|^2$$

$n_2$: Non-linearity coeff (Kerr Non-lin).

$$\tilde{E}(\bar{r}, \omega-\omega_0) = \underbrace{F(\rho, \phi)}_{} \, \tilde{A}(z, \omega-\omega_0)\, e^{-j\beta_0 z}$$




$$\nabla_\perp^2 \bar{F} + \{\epsilon(\omega) k_0^2 - \tilde{\beta}^2\} F = 0$$

$$-2j\beta_0 \frac{\partial \tilde{A}}{\partial z} + (\tilde{\beta}^2 - \beta_0^2)\tilde{A} = 0$$

$\dfrac{\partial^2 \tilde{A}}{\partial z^2} \leftarrow$ negligible.

$$\tilde{\beta}^2 - \beta_0^2 = (\tilde{\beta} - \beta_0)(\tilde{\beta} + \beta_0)$$

$$\approx 2\beta_0(\tilde{\beta} - \beta_0)$$

$$\frac{\partial \tilde{A}}{\partial z} + j(\tilde{\beta} - \beta_0)\tilde{A} = 0$$

$$\beta(\omega) = \beta_0 + \underbrace{(\omega - \omega_0)\frac{\partial \beta}{\partial \omega}\bigg|_{\omega=\omega_0}}_{\beta_1} + \frac{(\omega-\omega_0)^2}{2}\underbrace{\frac{\partial^2 \beta}{\partial \omega^2}\bigg|_{\omega=\omega_0}}_{\beta_2} + \cdots$$

$$\beta_n \triangleq \frac{\partial^n \beta}{\partial \omega^n}\bigg|_{\omega=\omega_0}$$

$$\cdots$$

$$\epsilon = (n + \Delta n)^2 \simeq n^2 + 2n\Delta n$$

$$\Delta n = n_2 |E|^2 - j\frac{\alpha}{2k_0}$$

$$\tilde{\beta}(\omega) = \beta(\omega) + \Delta\beta(\omega)$$

$$\Delta\beta(\omega) = k_0^2 \frac{n(\omega)}{\beta(\omega)} \frac{\iint \Delta n(\omega) |F|^2 d\Omega}{\iint |F|^2 d\Omega}$$

$$A(z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{A}(z, \omega - \omega_0) e^{j(\omega-\omega_0)t} d\omega$$

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} - j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A = -j\gamma |A|^2 A$$

$$\gamma = \frac{n_2 \omega_0}{c A_{eff}} \leftarrow \text{Non-linearity parameter}$$

$$A_{eff} = \frac{\left(\iint |F|^2 d\Omega\right)^2}{\iint |F|^4 d\Omega} \quad - \quad 1\text{-}100\ \mu m^2$$

$$\gamma = 1 - 100\ W^{-1}/km.$$

$$T = t - \frac{z}{v_g} = t - \beta_1 z \qquad \beta_1 = 1/v_g$$

$$\beta_1 \frac{\partial A}{\partial t} = \beta_1 \frac{\partial A}{\partial T} \cdot \frac{\partial T}{\partial t} = \beta_1 \frac{\partial A}{\partial T}(1 - \beta_1 v_g)$$

$$= 0$$

$$\frac{\partial A}{\partial z} \underbrace{- j \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2}}_{\text{Dispersion}} + \underbrace{\frac{\alpha}{2} A}_{\text{Loss.}} = \underbrace{-j \gamma |A|^2 A}_{\text{Non-linearity}}$$

Non-linear Schrödinger Equation

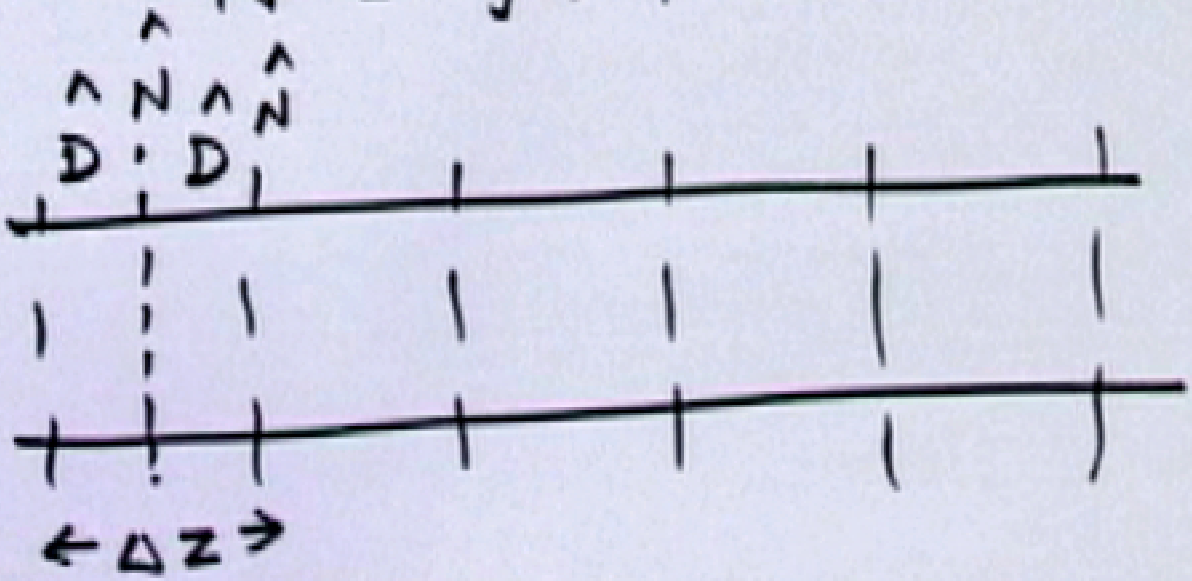
(NLS)

Dispersion operator

$$\hat{D} = j\,\frac{\beta_2}{2}\,\frac{\partial^2}{\partial T^2} - \frac{\alpha}{2} \quad \leftarrow \text{frequency domain}$$

Non-linearity operator

$$\hat{N} = -j\,\gamma\,|A|^2 \quad \leftarrow \text{Time domain}$$



Gaussian Pulse, st. deviation $T_0$

$$A(T) = e^{-T^2/2T_0^2}$$

Dispersion Length $L_D \equiv \dfrac{T_0^2}{|\beta_2|}$

Non-linearity Length $L_{NL} = \dfrac{1}{\gamma P}$

Pulse power

Physical length $L$ of a fiber with $\alpha = 0$

① $L << L_D$, $L << L_{NL}$ – Fiber is just a medium to transport light

② $L >> L_D$, $L << L_{NL}$ → Dispersion

Pulse broadening

Group velocity Limited regime.

(GVD)

③ $L << L_D$, $L >> L_{NL}$ → Self Phase Modulation (SPM)

Non-linearity Limited regime

④ $L >> L_D$, $L >> L_{NL}$ → 'Soliton'